Viewpoints using ranking based argumentation semantics

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Abstract.

We introduce a new modular framework that generates viewpoints based on ranking argumentation semantics by considering a selection function, a ranking on arguments and a lifting function as its input parameters. We study the different combinations the instantiation and introduce a set of postulates investigated for the frameworks different classes of output.

1. Introduction

Ranking based semantics are being studied by a large amount of researchers [15, 2, 6, 4, 1, 8, 14, 16]. New semantics are being introduced, as well as the principles they should satisfy. One of the main reasons of their popularity is that they offer a finer evaluation than extension-based semantics [12, 11, 5, 10].

There is a difference in the output format between these two approaches: when using a ranking based semantics, the output is a ranking on the arguments; in the case of extension based semantics, the output is a set of extensions. While the ranking and the scores (which are present in many ranking based semantics) allow to better assess the acceptability degree of each individual argument, the question *"what are the points of view of the argumentation framework?"* stays unanswered when using a ranking based semantics. The main research goal of this paper is to (ask and) answer that question.
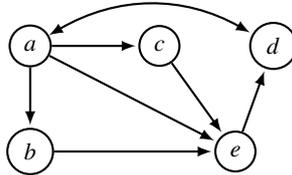

Figure 1. Is one highly ranked argument a stronger point of view? Are several well-ranked arguments another?

Consider the argumentation framework (AF) from Figure 1. Let us use h-categorizer ranking semantics [7]. We obtain $a \succ b, c \succ d \succ e$. What are the possible points of view?
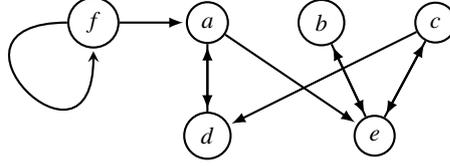
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Figure 2. Is a non admissible set consisting of highly ranked arguments a point of view?

Clearly $S_1 = \{a\}$ is admissible and strong, but should we also accept $S_2 = \{b,c,d\}$? Since S_2 contains three arguments, is it (even) better than S_1? It seems that both S_1 and S_2 are acceptable points of view, but which one is stronger: S_1 having only one highly ranked argument or S_2 having three arguments of medium strength? Consider now the AF from Figure 2. Let us use h-categorizer ranking semantics. The three strongest arguments are a, b and c with scores approximately 0.76, 0.71 and 0.71, respectively. Arguments d and f are much weaker with scores approximately 0.31 and 0.40, respectively. If one looks for conflict-free sets containing highly ranked arguments, two potential candidates are $S_1 = \{a,b,c\}$ and $S_2 = \{d,e\}$. Should one accept S_1, which contains the three strongest arguments but is not admissible (it is attacked by a self-attacking argument)? The two previous examples show that the question *"how to generate points of view when arguments are evaluated by a ranking based semantics?"* is not easy. In particular, the strongest arguments with respect to the ranking based semantics do not always form an admissible (or not even a conflict-free) set.

We do not claim that there is a unique answer to the aforementioned question. Instead, *we propose to define a general and modular framework able to suit different needs.* Our framework is based on three layers.

1. First, a *selection function* selects a set of subsets of arguments. This can be any function; some popular choices would probably be all maximal conflict-free sets or all admissible sets.
2. Second, a *ranking based semantics* is used to calculate the ranking on the set of arguments. In the previous example we mentioned h-categorizer, but any function returning an order on the set of arguments can be used.
3. Third, we need a *lifting operator*, i.e. a function that compares the set of arguments returned by the selection function, based on the individual scores of arguments. A simple criterion would be to compare the strongest arguments of each set. A generalisation of this criterion is so-called *leximax*, which in case that the best arguments are equally strong proceeds to compare the second best argument of each set, and so on. If a ranking based semantics returns numerical scores, one could also compare the sums of scores of all arguments.

Out of all sets returned by the selection function, only the best ones (w.r.t. the ranking function and the lifting operator) are kept and they represent the output of our framework.

Let us give a quick preview of the kind of results our framework can return. In this example we assume the reader is familiar with many argumentation notions including the definitions of several ranking semantics. If this is not the case please refer to Section 2 for the full definitions. In the example from Figure 1 the scores of arguments with respect to h-categorizer ranking semantics are approximately 0.67 for a, 0.60 for b and c, 0.50

for d and 0.35 for e. Let us select all maximal conflict-free sets; we obtain $\mathtt{S}_1 = \{a\}$, $\mathtt{S}_2 = \{b,c,d\}$ and $\mathtt{S}_3 = \{e\}$. Now, if we use leximax or max as lifting, we will obtain $\mathtt{S}_1$ as the output of our framework. If we use the sum of arguments' scores as the lifting function, we obtain $\mathtt{S}_2$. So accordingly to the user's choice of the lifting function, one of the two results is obtained.

Consider now the example from Figure 2. If admissibility is important for the user, they will select only (a subset of) admissible sets, thus $\{a,b,c\}$ will not be a part of the output. On the contrary, if we select all the maximal conflict-free sets, we obtain $\mathtt{S}_1 = \{a,b,c\}$ and $\mathtt{S}_2 = \{d,e\}$. For all reasonable lifting operators $\mathtt{S}_1$ is preferred to $\mathtt{S}_2$, thus the output of the framework is $\mathtt{S}_1$ in this case.

Please note that there is no universally correct answer at the question of "What is the best viewpoint?". Such question heavily relies on its context: what is the application at hand, how are the argumentation graphs obtained, how are the attacks obtained, who is the user and what is their profile etc. Such dilemma does not mean that we cannot advance the state of the art. Indeed, we can propose a modular framework that is generic enough to be able to accommodate various application scenarios. In this case, one important property of the framework lies in its versatility and its capacity to yield different results according to various instantiations.

After defining this general framework in a formal way, we propose and study some of its instantiations. We show that some of the instantiations are comparable in terms of set inclusion and that the vast majority of them return distinct results. We evaluate our framework via a set of postulates and prove their satisfaction for certain instantiations. Furthermore, we also exhibit its non trivial behaviour when instantiated with logical AFs.

The importance of that part of the work is to show that the three layers work independently and that the reasoning machinery does provide, accordingly, different outputs (more details are provided in Section 3). Since not all applications have the same needs, such versatility ensures the significance of our result. Please note that we are not interested here in how the various possible outputs of the framework are intuitive with respect to particular application. Such research question, albeit important to be addressed in the context of the practical applicability of our work is out of th escope of this paper. We consider that the work of studying the various outputs of the framework is, in itself, a self contained piece of research.

The paper is organised as follows: in Section 2, we introduce our new framework aimed at calculating the possible points of views ("extensions") when using ranking based semantics. In Section 3, we show the usages of this framework and highlight the general inclusion, equality and difference results with respect to several possible inputs. In Section 4, we propose several new postulates for the ranking-based selection framework and prove the satisfaction of these postulates for some instantiations of the framework. Finally we show how the use of our framework in a logically instantiated setting yields intuitive results.

2. Ranking-based selection $\mathbb{RSF}$

An AF is a pair $\mathbb{AS}=(\mathscr{A},\mathscr{C})$ where the nodes in $\mathscr{A}$ represent the arguments and the binary relation $\mathscr{C}$ represents the attacks (conflicts) between arguments. The set of all possible AFs is denoted by $\mathscr{K}_{\mathbb{AS}}$. The proposed ranking-based selection framework $\mathbb{RSF}$, given

an AF and three input parameters, returns a set of arguments with particular properties. It consists of three steps:

1. First, a *selection* function $\mathtt{S} : \mathscr{K}_{\mathbb{AS}} \to 2^{2^{\mathscr{A}}}$ selects a set of subsets of $\mathscr{A}$.
2. Second, a ranking $\mathtt{R} : \mathscr{K}_{\mathbb{AS}} \to \mathscr{A} \times \mathscr{A}$ computes a total, reflexive and transitive binary relation on $\mathscr{A}$.
3. Third, we lift the ranking of $\mathtt{R}$ using a lifting function $\mathtt{L}$ on the subsets of $\mathscr{A}$ returned by the selection function and consider its top element(s) (i.e. the "best" sets of arguments according to the total order returned by $\mathtt{L}$).

Definition 1. A *ranking-based selection framework* $\mathbb{RSF}$ is a tuple $(\mathtt{S},\mathtt{R},\mathtt{L})$ where $\mathtt{S}$ is a selection function, $\mathtt{R}$ is a ranking on arguments and $\mathtt{L}$ is a lifting function. The top result of a $\mathbb{RSF} = (\mathtt{S},\mathtt{R},\mathtt{L})$ on an AF $\mathbb{AS}$ is $\mathscr{O}_{(\mathtt{S},\mathtt{R})}^{(\mathtt{L}^{\succeq},\mathbb{AS})} = \{E \in \mathtt{S}(\mathbb{AS}) \mid \text{for all } E' \in \mathtt{S}(\mathbb{AS}), (E,E') \in \mathtt{L}^{\succeq}(\mathbb{AS})\}$, where $\succeq$ is the order induced by $\mathtt{R}$. If the graph is obvious, we denote the result by $\mathscr{O}_{(\mathtt{S},\mathtt{R})}^{\mathtt{L}^{\succeq}}$.

2.1. The $\mathbb{RSF}$ selection

Given an AF, a selection function returns a set of sets of arguments. Before presenting the several selection functions considered in the paper we need some additional notions.

Given a subset of nodes X of $\mathscr{A}$, we denote by X^+, the set of arguments attacked by X, i.e. $X^+ = \{a \in \mathscr{A} \mid \text{there is } x \in X \text{ such that } (x,a) \in \mathscr{C}\}$. Likewise, the set of nodes that attack at least one node of X is denoted by $X^- = \{a \in \mathscr{A} \mid \text{there exists } x \in X \text{ such that } (a,x) \in \mathscr{C}\}$. An argument $a \in \mathscr{A}$ is defended by a set $X \subseteq \mathscr{A}$ in $\mathbb{AS}$ if for each $b \in \mathscr{A}$ with $(b,a) \in \mathscr{C}$, there exists $x \in X$ s.t. $(x,b) \in \mathscr{C}$.

The selection functions investigated by this paper are:

- *conflict-free* sets of $\mathbb{AS}$: $cf(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid \text{for every } x_1, x_2 \in X, (x_1,x_2) \notin \mathscr{C}\}$.
- *maximal for set inclusion conflict-free* sets of $\mathbb{AS}$: $cf_{max}(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in cf(\mathbb{AS}) \text{ and there is no } X' \text{ s.t. } X \subset X' \text{ and } X' \in cf(\mathbb{AS})\}$.
- *maximal for set cardinality conflict-free* sets of $\mathbb{AS}$: $cf_{max}^{card}(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in cf(\mathbb{AS}) \text{ and there is no } X' \text{ s.t. } |X| < |X'| \text{ and } X' \in cf(\mathbb{AS})\}$.
- *admissible extensions* of $\mathbb{AS}$: $ad(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in cf(\mathbb{AS}) \text{ and for each } a \in X, a \text{ is defended by } X\}$.
- *preferred extensions* of $\mathbb{AS}$: $pr(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in ad(\mathbb{AS}) \text{ with no } X' \text{ s.t. } X \subset X' \text{ and } X' \in ad(\mathbb{AS})\}$.
- *stable extensions* of $\mathbb{AS}$: $st(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in cf(\mathbb{AS}), \text{ for every } a \in \mathscr{A}\backslash X, \exists c \in X \text{ s.t. } (c,a) \in \mathscr{C}\}$.
- *complete extensions* of $\mathbb{AS}$: $co(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in ad(\mathbb{AS}), \text{ for each } a \in \mathscr{A} \text{ defended by } X, a \in X\}$.
- *grounded extension* of $\mathbb{AS}$: $gr(\mathbb{AS}) = \{X \subseteq \mathscr{A} \mid X \in co(\mathbb{AS}) \text{ and for each } X' \in co(\mathbb{AS}), X' \not\subset X\}$.

Example 1. Let $\mathbb{AS} = (\mathscr{A}, \mathscr{C})$ with $\mathscr{A} = \{a,b,c,d,e\}$ and $\mathscr{C} = \{(a,e),(b,a),(b,c),(c,e), (d,a),(e,d)\}$. We have $\{a,c\}^+ = \{e\}$, $\{a,c\}^- = \{d,b\}$ and e is defended by $\{d,b\}$. The set of conflict-free sets is $cf(\mathbb{AS}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,c\}, \{d\}, \{b,d\}, \{c,d\}, \{e\}, \{b,e\}\}$. Amongst those sets, we have $cf_{max}(\mathbb{AS}) = cf_{max}^{card}(\mathbb{AS}) = \{\{a,c\}, \{b,d\}, \{c,d\}, \{b,e\}\}$. However, the only admissible set amongst these maximal sets is $\{b,e\}$. Furthermore, we also have $pr(\mathbb{AS}) = st(\mathbb{AS}) = co(\mathbb{AS}) = gr(\mathbb{AS}) = \{\{b,e\}\}$.

2.2. The $\mathbb{RSF}$ ranking

A ranking R is a total, reflexive and transitive binary relation $R \subseteq \mathscr{A} \times \mathscr{A}$. We chose to use two well-known ranking semantics to illustrate our framework: burden-based semantics [1] and h-categorizer semantics [7] reminded below. Please refer to the original papers for the full definitions, here we only give a quick overview.

The score of each argument at a given step with respect to burden-based semantics is calculated as follows. Let $\mathbb{AS} = (\mathscr{A}, \mathscr{C})$ be an AF, $a \in \mathscr{A}$ and $i \in \{1, 2, \ldots, n\}$, then:

$$Bur_i(a) = \begin{cases} 1 & \text{if } i = 1 \\ 1 + \sum_{(b,a) \in \mathscr{C}} \frac{1}{Bur_{i-1}(b)} & \text{otherwise} \end{cases}$$

Please note that an equality-ensuring threshold exists for the burden-based semantics [4]. This ensures an exact computation, despite the fact that the number of steps is infinite. The ranking obtained with this semantics on an AF $\mathbb{AS}$ is computed using the lexicographical order and will be denoted by $R_{BBS}(\mathbb{AS})$.

The h-categorizer semantics is computed as follows. Each argument a is attached a score $Cat(a) > 0$ such that:

$$Cat(a) = \begin{cases} 1 & \text{if } \{a\}^- = \emptyset \\ \frac{1}{1 + \sum_{(b,a) \in \mathscr{C}} Cat(b)} & \text{otherwise} \end{cases}$$

It has been proved that h-categorizer is well-defined, i.e. that for every AF, the score $Cat(.)$ of each argument is unique. The ranking obtained with this semantics on an AF $\mathbb{AS}$ will be denoted $R_{CAT}(\mathbb{AS})$.

Example 2. [Ex. 1 cont.] The ranking obtained using the burden-based semantics is $R_{BBS}(\mathbb{AS}) = b \succ d \succ c \succ e \succ a$ whereas the ranking obtained throughout the h-categorizer semantics is $R_{CAT}(\mathbb{AS}) = b \succ d \succ e \succ c \succ a$.

2.3. The $\mathbb{RSF}$ lifting

A *lifting function* L (also referred to as *lifting operator*, or *lifting*) compares sets of arguments given their individual order and returns a total order on the sets.

Let us first introduce the sort function that will be used in order to define the $L_{leximax}^{\succeq}$ notion below. Given a set of elements $X = \{x_1, x_2, \ldots, x_n\}$ and a total, reflexive and transitive binary relation $\succeq$ on X, $\text{sort}(X, \succeq)$ returns a sorted vector $(x_1, x_2, \ldots, x_n)$ such that for every x_i, x_j, we have that $x_i \succeq x_j$ iff $i \leq j$. The element at position i in the vector $\text{sort}(X, \succeq)$ is denoted by $\text{sort}_i(X, \succeq)$. Note that the returned vector is not necessarily unique due to the fact that some elements might be equivalent, i.e. $x_i \sim x_j$.

In this paper we consider several possible instantiations of the lifting operator L:

- The $L_{max}^{\succeq}$ lifting operator compares the subsets with respect to their maximal elements.
- The $L_{leximax}^{\succeq}$ lifting operator compares the elements after sorting them in decreasing order.

- The $\mathrm{L}_{PST}^{\succeq}$ lifting operator compares the subsets on whether or not they are preferred sub-theories [9].

We recall the reader that a preferred sub-theory (PST) of a stratification $T = (T_1, \ldots, T_n)$ with respect to an order $\succeq$ (i.e. a sorted sequence where $T_i \in X/ \sim$ where $\sim$ is the equivalence relation induced by $\succeq$) is a set $S = S_1 \cup S_2 \cup \cdots \cup S_n$ such that for every $k \in \{1, \ldots, n\}$, $S_1 \cup \cdots \cup S_k$ is a maximal (with respect to $\subseteq$) conflict-free subset of $T_1 \cup \cdots \cup T_k$.

Let $E, E' \in \mathrm{S}(\mathbb{AS})$, $\mathrm{sort}(E, \succeq) = (x_1, x_2, \ldots, x_n)$ and $\mathrm{sort}(E', \succeq) = (x_1', x_2', \ldots, x_m')$. We say that:

- $(E, E') \in \mathrm{L}_{\overline{max}}^{\succeq}(\mathbb{AS})$ iff $max(E) \succeq max(E')$, where $max(X) = \mathrm{sort}_1(X, \succeq)$.
- $(E, E') \in \mathrm{L}_{leximax}^{\succeq}(\mathbb{AS})$ iff one of the following holds:
  * $m = n$ and for every $i \in \{1, \ldots, n\}, x_i \sim x_i'$
  * there exists $i \in \{1, \ldots, min(m,n)\}$ s.t. $x_i \succ x_i'$ and for every $j \in \{1, \ldots, i - 1\}, x_j \sim x_j'$
  * $n > m$ and for every $i \in \{1, \ldots, m\}, x_i \sim x_i'$

- $(E, E') \in \mathrm{L}_{PST}^{\succeq}(\mathbb{AS})$ iff E is a preferred sub-theory of the stratification induced by $\succeq$.

Example 3. [Ex 2 cont.] Let $\succeq$ be the order induced by the burden-based semantics. Then: $b \succ d \succ c \succ e \succ a$. We have $cf_{max}^{card}(\mathbb{AS}) = \{\{a,c\}, \{b,d\}, \{c,d\}, \{b,e\}\}$. We have for every $E' \in cf_{max}^{card}(\mathbb{AS})$, $(\{b,d\}, E') \in \mathrm{L}_{max}^{\succeq}(\mathbb{AS})$ and $(\{b,e\}, E') \in \mathrm{L}_{max}^{\succeq}(\mathbb{AS})$. However, if we use $\mathrm{L}_{leximax}^{\succeq}$ and $\mathrm{L}_{PST}^{\succeq}$, only $\{b,d\}$ is maximal, namely for each $E' \in cf_{max}^{card}(\mathbb{AS})$, $(\{b,d\}, E') \in \mathrm{L}_L^{\succ}(\mathbb{AS})$, $L \in \{leximax, PST\}$.

The *ranking-based selection framework* $\mathbb{RSF}$ is general enough to consider various instantiation of the tuple $(\mathrm{S}, \mathrm{R}, \mathrm{L})$. For example, $\mathscr{O}_{(ad, \mathrm{R}_{CAT})}^{(\mathrm{L}_{\overline{max}}^{\succeq}, \mathbb{AS})}$ is the set of admissible subsets of $\mathbb{AS}$ whose strongest with respect to *Cat* argument is stronger or equal than the strongest with respect to *Cat* argument of every other admissible set of arguments.

Example 4 (Ex. 3 cont.). We have $\mathscr{O}_{(cf_{max}^{card}, \mathrm{R}_{BBS})}^{(\mathrm{L}_{\overline{max}}^{\succeq}, \mathbb{AS})} = \{\{b,d\}, \{b,e\}\}$.

In the next section we investigate how the different combinations of instantiations of the tuple elements $(\mathrm{S}, \mathrm{R}, \mathrm{L})$ relate to each other.

3. $\mathscr{O}_{(\mathrm{S}, \mathrm{R})}^{(\mathrm{L}^{\succeq}, \mathbb{AS})}$ instantiation landscape

This section compares the outputs of an $\mathbb{RSF}$ obtained when varying the selection function and the lifting operator. We show that the output is different when using different parameters, since using a particular lifting function does not make two given semantics coincide. Hence, $\mathbb{RSF}$ allows for a large amount of combinations leading to different results. We investigate how all combinations of the $(\mathrm{S}, \mathrm{R}, \mathrm{L})$ instantiations relate to each other and focus on both (1) inclusion and equality results and (2) difference results.

Inclusion and equality results wrt $\mathscr{O}_{(\mathrm{S}, \mathrm{R})}^{(\mathrm{L}^{\succeq}, \mathbb{AS})}$**.** Obviously, for any given $\mathbb{AS}$, we always have $\mathscr{O}_{(\mathrm{S}, \mathrm{R})}^{\mathrm{L}, \mathbb{AS}} \subseteq \mathrm{S}(\mathbb{AS})$. This is because the top result is always a subset of the set

returned by the selection function S. The first result of this section shows that the output corresponding to the leximax lifting refines the output corresponding to the max lifting.

Proposition 1. Let $\mathbb{AS}$ be an AF, R a ranking semantics and S a selection function. Then $\mathscr{O}^{\mathrm{L}_{leximax}}_{(\mathrm{S,R})} \subseteq \mathscr{O}^{\mathrm{L}_{max}}_{(\mathrm{S,R})}$.

The next result concerns preferred sub-theories. Every preferred sub-theory is a maximal for set inclusion conflict-free subset of $\mathscr{A}$. Thus, comparing all conflict-free subsets or only maximal conflict-free subsets of $\mathscr{A}$ by using L_{PST} will yield the same result. Formally:

Proposition 2. Let $\mathbb{AS}$ be an AF and R a ranking semantics. Then $\mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf_{max},\mathrm{R})} = \mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf,\mathrm{R})}$ and $\mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf_{max},\mathrm{R})} = \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$. For $\mathrm{L} \in \{\mathrm{L}^{\succeq}_{leximax}, \mathrm{L}^{\succeq}_{PST}\}$ then $\mathscr{O}^{\mathrm{L}}_{(cf^{card}_{max},\mathrm{R})} \not\subseteq \mathscr{O}^{\mathrm{L}}_{(cf_{max},\mathrm{R})}$.

Proof. We split the proof in 3 parts:

- *Sketch.* We know that $\mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf_{max},\mathrm{R})} \supseteq \mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf,\mathrm{R})}$. We show $\mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf_{max},\mathrm{R})} \subseteq \mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf,\mathrm{R})}$ by contradiction. Suppose the existence of a maximal conflict free set E that is in $\mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf_{max},\mathrm{R})}$ but not in $\mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf,\mathrm{R})}$. Thus, there is a conflict free set E' such that $(E',E) \in \mathrm{L}^{\succeq}_{leximax}$ and $(E,E') \notin \mathrm{L}^{\succeq}_{leximax}$. Thus, there exists a maximal conflict free set E'' such that $E' \subseteq E''$ and $(E,E'') \notin \mathrm{L}^{\succeq}_{leximax}$. Therefore, $E \notin \mathscr{O}^{\mathrm{L}^{\succeq}_{leximax}}_{(cf_{max},\mathrm{R})}$.

- *Sketch.* We know that $\mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf_{max},\mathrm{R})} \supseteq \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$. We show $\mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf_{max},\mathrm{R})} \subseteq \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$ by contradiction by supposing that $\mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf_{max},\mathrm{R})} \supset \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$. Let $E \in \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf_{max},\mathrm{R})}$ such that $E \notin \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$. There are two cases: either E is a PST of the stratification T induced by $\succeq$ or there is no PST of T. In the two cases, we have $E \in \mathscr{O}^{\mathrm{L}^{\succeq}_{PST}}_{(cf,\mathrm{R})}$.

- The counter-example can be constructed using the argumentation graph of Example 7. Indeed, if the ranking is changed to $a \succ b,c,d$ we have that the output with respect to cf^{card}_{max} is $\{\{c,d\}\}$ whereas the one for cf_{max} is $\{\{a\}\}$.

$\square$

Non-equality results wrt $\mathscr{O}^{(\mathrm{L}^{\succeq},\mathbb{AS})}_{(\mathrm{S,R})}$. Tables 2 and 3 show the results for different instantiations (for brevity reasons a list of output abbreviations is given in Table 1). The elements in the first column and in the first row of those tables represent the result of the corresponding instantiation of the $\mathbb{RSF}$. The entry "=" in row C and column X indicates that C is equal to X in all cases. The entry "$\neq_n$" in row C and column X indicates that C can be different than X and that the counter-example is provided in Example n. For example $\neq_5$ means that the counter example can be found in Example 5.

Example 5. Let us consider the AF $\mathbb{AS} = (\mathscr{A},\mathscr{C})$ with $\mathscr{A} = \{a,b,c,d,e,f\}$ and $\mathscr{C} = \{(d,a),(a,d),(a,f),(a,b),(b,c),(c,e),(e,b)\}$. Note that the ranking obtained with R_{BBS} and R_{CAT} is the same $c \succ a,d,f \succ e \succ b$. Thus:

$\mathscr{O}_1$	$\mathscr{O}^{(\mathrm{L}_{leximax}^{\succeq})}_{(cf,\mathrm{R})}$
$\mathscr{O}_2$	$\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf,\mathrm{R})}$
$\mathscr{O}_3$	$\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(cf_{max},\mathrm{R})}$
$\mathscr{O}_4$	$\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf_{max},\mathrm{R})}$
$\mathscr{O}_5$	$\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(cf_{max}^{card},\mathrm{R})}$
$\mathscr{O}_6$	$\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf_{max}^{card},\mathrm{R})}$

$\mathscr{O}_7$	$\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(pr,\mathrm{R})}$
$\mathscr{O}_8$	$\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(pr,\mathrm{R})}$
$\mathscr{O}_9$	$\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(st,\mathrm{R})}$
$\mathscr{O}_{10}$	$\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(st,\mathrm{R})}$
$\mathscr{O}_{11}$	$\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(gr,\mathrm{R})}$

Table 1. Nomenclature

	$\mathscr{O}_2$	$\mathscr{O}_3$	$\mathscr{O}_4$	$\mathscr{O}_5$	$\mathscr{O}_6$
$\mathscr{O}_1$	$\neq_5$	$=$	$\neq_5$	$\neq_7$	$\neq_7$
$\mathscr{O}_2$		$\neq_5$	$=$	$\neq_5$	$\neq_5$
$\mathscr{O}_3$			$\neq_5$	$\neq_7$	$\neq_7$
$\mathscr{O}_4$				$\neq_5$	$\neq_5$
$\mathscr{O}_5$					$\neq_8$
$\mathscr{O}_6$					

Table 2. (Non-)equality results in the general case, part 1

	$\mathscr{O}_7$	$\mathscr{O}_8$	$\mathscr{O}_9$	$\mathscr{O}_{10}$	$\mathscr{O}_{11}$
$\mathscr{O}_1$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_2$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_3$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_4$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_6$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$	$\neq_5$
$\mathscr{O}_7$		$\neq_6$	$\neq_6$	$\neq_6$	$\neq_5$
$\mathscr{O}_8$			$\neq_6$	$\neq_6$	$\neq_5$
$\mathscr{O}_9$				$\neq_8$	$\neq_5$
$\mathscr{O}_{10}$					$\neq_5$

Table 3. (Non-)equality results in the general case, part 2

- $\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(cf_{max},\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(cf,\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(cf_{max}^{card},\mathrm{R}_{BBS})} =$
  $\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf_{max}^{card},\mathrm{R}_{BBS})} = \{\{c,d,f\}\}$

- $\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf,\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(cf_{max},\mathrm{R}_{BBS})} = \{\{a,c\},\{c,d,f\}\}$

- $\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(pr,\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(pr,\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(st,\mathrm{R}_{BBS})} = \mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(st,\mathrm{R}_{BBS})}$
  $= \{\{a,c\}\}$

Example 6. [Ex 5 cont.] If the ranking R is changed to $d,f,a \succ c,b,e$ for some reason (experts evaluation, use of a new ranking, etc.), we have:

- $\mathscr{O}^{\mathrm{L}_{leximax}^{\succeq}}_{(pr,\mathrm{R})} = \{\{d,f\}\}$

- $\mathscr{O}^{\mathrm{L}_{PST}^{\succeq}}_{(pr,\mathrm{R})} = \{\{a,c\},\{d,f\}\}$

- $\mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(st,\mathsf{R})} = \{\{a,c\}\}$
- $\mathscr{O}^{\mathsf{L}_{PST}^{\succeq}}_{(st,\mathsf{R})} = \{\{a,c\}\}$

Example 7. Let us consider the AF $\mathbb{AS} = (\mathscr{A},\mathscr{C})$ with $\mathscr{A} = \{a,b,c,d\}$ and $\mathscr{C} = \{(b,a),(a,b),\ (b,c),\ (c,b),\ (b,d),\ (a,c),\ (a,d)\}$. Note that the ranking obtained with R_{BBS} or R_{CAT} is $a \succ b,c,d$. Thus, we have that:

- $\mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(cf,\mathsf{R}_{BBS})} = \mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(cf_{max},\mathsf{R}_{BBS})} = \{\{a\}\}$
- $\mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(cf_{max}^{card},\mathsf{R}_{BBS})} = \mathscr{O}^{\mathsf{L}_{PST}^{\succeq}}_{(cf_{max}^{card},\mathsf{R}_{BBS})} = \{\{c,d\}\}$

Example 8. Let us consider the AF $\mathbb{AS} = (\mathscr{A},\mathscr{C})$ with $\mathscr{A} = \{a,b,c,d\}$ and $\mathscr{C} = \{(b,a),(a,b),\ (a,c),\ (c,a),\ (b,d),\ (d,b),\ (c,d),\ (d,c)\}$. Suppose that R is $b,c,d \succ a$:

- $\mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(cf_{max}^{card},\mathsf{R})} = \mathscr{O}^{\mathsf{L}_{leximax}^{\succeq}}_{(st,\mathsf{R})} = \{\{b,c\}\}$
- $\mathscr{O}^{\mathsf{L}_{PST}^{\succeq}}_{(cf_{max}^{card},\mathsf{R})} = \mathscr{O}^{\mathsf{L}_{PST}^{\succeq}}_{(st,\mathsf{R})} = \{\{b,c\},\{d,a\}\}$

4. Framework Postulates

In the previous section, we showed how various outputs of the proposed framework relate to each other. Now, let us introduce a set of postulates that ensure the sensible behaviour of $\mathbb{RSF}$: *anonymity* (i.e. the output of a $\mathbb{RSF}$ should be defined only on the basis of the attacks between arguments), *dummy* (i.e. adding dummy [2] arguments should slightly change the output of the $\mathbb{RSF}$ by adding the dummy arguments into the sets of the output) and *compositionality* (i.e. the output of an $\mathbb{RSF}$ for an AF should be determined by joining the output of the same $\mathbb{RSF}$ on its connected components). Formally, let $\mathbb{AS} = (\mathscr{A},\mathscr{C}),\mathbb{AS}'$ be two AFs and $\mathbb{RSF} = (\mathsf{S},\mathsf{R},\mathsf{L})$:

- **Anonymity:** $\mathbb{RSF}$ satisfies Anonymity if and only if for any isomorphism γ s.t. $\mathbb{AS}' = \gamma(\mathbb{AS})$, we have $E \in \mathscr{O}^{(\mathsf{L},\mathbb{AS})}_{(\mathsf{S},\mathsf{R})}$ iff $\gamma(E) \in \mathscr{O}^{(\mathsf{L},\mathbb{AS}')}_{(\mathsf{S},\mathsf{R})}$ where isomorphism is defined in the standard way.
- **Dummy:** $\mathbb{RSF}$ satisfies Dummy if and only if for any $a \notin \mathscr{A}$, we have $\mathscr{O}^{(\mathsf{L},(\mathscr{A}\cup\{a\},\mathscr{C}))}_{(\mathsf{S},\mathsf{R})} = \{X \cup \{a\} \mid X \in \mathscr{O}^{(\mathsf{L},\mathbb{AS})}_{(\mathsf{S},\mathsf{R})}\}$.
- **Compositionality:** $\mathbb{RSF}$ satisfies Compositionality if and only if we have $\mathscr{O}^{(\mathsf{L},\mathbb{AS})}_{(\mathsf{S},\mathsf{R})} = \{\bigcup_{\mathbb{AS}'\in cc(\mathbb{AS})} x_{\mathbb{AS}'} \mid$ for every $\mathbb{AS}' \in cc(\mathbb{AS}), x_{\mathbb{AS}'} \in \mathscr{O}^{(\mathsf{L},\mathbb{AS}')}_{(\mathsf{S},\mathsf{R})}\}$ where $cc(\mathbb{AS})$ is the set of all connected components of $\mathbb{AS}$; a connected component is a maximal set such that there is a path from each two argument of that set with respect to $\mathscr{C} \cup \mathscr{C}^{-1}$.

In the remainder of the section, we identify broad classes of instantiations of our framework that satisfy the above postulates. In order to show that postulates are satisfied by those large classes, we do not base our result on particular selections, rankings or liftings. Instead, we introduce the principles on selections, rankings and liftings that are sufficient so that the whole framework behavest in a rational way.

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[2] Arguments that are not attacked and do not attack other arguments.

In what follows, we introduce: **abstraction-S** (i.e. the set of sets of arguments returned by the selection function should be defined only on the basis of the attacks between arguments), **dummy-S** (i.e. adding dummy arguments should update the selected sets by adding the dummy arguments to every previously selected sets) and **compositionality-S** (i.e. the result of the selection function on an AF should be determined by joining the output of the same selection function on its connected components). Last, we can consider for the lifting the corresponding **abstraction-L** (i.e. a lifting function should only be defined on the basis of the attacks between arguments) and **dummy-L** (i.e. a lifting function should conserves its ranking after the addition of a dummy argument). Formally, let $\mathbb{AS} = (\mathscr{A}, \mathscr{C}), \mathbb{AS}'$ be two AFs, $\mathtt{S}$ a selection function and $\mathtt{L}$ a lifting function:

- **Abstraction-S:** $\mathtt{S}$ satisfies Abstraction-S if and only if for any isomorphism γ s.t. $\mathbb{AS}' = \gamma(\mathbb{AS})$, we have $E \in \mathtt{S}(\mathbb{AS})$ iff $\gamma(E) \in \mathtt{S}(\mathbb{AS}')$
- **Dummy-S:** $\mathtt{S}$ satisfies Dummy-S if and only if for any $a \notin \mathscr{A}$, we have $\mathtt{S}((\mathscr{A} \cup \{a\}, \mathscr{C})) = \{X \cup \{a\} \mid X \in \mathtt{S}(\mathbb{AS})\}$
- **Compositionality-S:** $\mathtt{S}$ satisfies Compositionality-S if and only if $\mathtt{S}(\mathbb{AS}) = \{\bigcup_{\mathbb{AS}' \in cc(\mathbb{AS})} s_{\mathbb{AS}'} \mid \text{for every } \mathbb{AS}' \in cc(\mathbb{AS}), s_{\mathbb{AS}'} \in \mathtt{S}(\mathbb{AS}')\}$
- **Abstract-L:** $\mathtt{L}$ satisfies Abstract-L if and only if for any isomorphism γ s.t. $\mathbb{AS}' = \gamma(\mathbb{AS})$ and any order $\succeq$, we have $(E, E') \in \mathtt{L}^{\succeq}(\mathbb{AS})$ iff $(\gamma(E), \gamma(E')) \in \mathtt{L}^{\succeq}(\mathbb{AS}')$
- **Dummy-L:** $\mathtt{L}$ satisfies Dummy-L if and only if for any $a \notin \mathscr{A}$ and any order $\succeq$, we have $(E, E') \in \mathtt{L}^{\succeq}(\mathbb{AS})$ iff $(E \cup \{a\}, E' \cup \{a\}) \in \mathtt{L}^{\succeq}((\mathscr{A} \cup \{a\}, \mathscr{C}))$

We can now show that our framework satisfies the postulates for a large class of instantiations.

Proposition 3. Let $\mathbb{RSF} = (\mathtt{S}, \mathtt{R}, \mathtt{L})$ and $\mathbb{AS}$ be an AF:

- if $\mathtt{L}$ satisfies *Abstraction-L*, $\mathtt{S}$ satisfies *Abstraction-S* then $\mathbb{RSF}$ satisfies Anonymity.
- if $\mathtt{L}$ satisfies *Dummy-L*, $\mathtt{S}$ satisfies *Dummy-S* then $\mathbb{RSF}$ satisfies Dummy.

For the last result we recall the Independence postulate for ranking-based semantics, introduced by [3].

- **Independence:** Let $\mathbb{AS} = (\mathscr{A}, \mathscr{C})$, if $\mathtt{R}$ satisfies Independence if and only if for every $\mathbb{AS}' = (\mathscr{A}', \mathscr{C}') \in cc(\mathbb{AS})$ and for every $a, b \in \mathscr{A}', (a, b) \in \mathtt{R}(\mathbb{AS}')$ implies $(a, b) \in \mathtt{R}(\mathbb{AS})$

Proposition 4. Let $\mathbb{RSF} = (\mathtt{S}, \mathtt{R}, \mathtt{L}_{leximax}^{\succeq})$ and $\mathbb{AS}$ be an AF. If $\mathtt{S}$ satisfies *Compositionality-S* and $\mathtt{R}$ satisfies Independence then $\mathbb{RSF}$ satisfies Compositionality.

5. Discussion

In this paper we gave the first principled approach to compute viewpoint(s) from a ranking semantics. The proposed framework $\mathbb{RSF}$ was introduced formally and analysed with respect to its modularity and to the principles it abides by. We provided a representation of the landscape of the different outputs that the $\mathbb{RSF}$ can generate for general argumentation graphs and identified some broad classes of instantiations of the $\mathbb{RSF}$ that respect the several postulates.

Our framework generalises several notions from the state of the art. *Global evaluation* of [13] is a special case of a $\mathbb{RSF}$ with $\mathbb{S}$ being a given Dung's semantics, $\mathbb{R}$ being a graded semantics that attaches to each argument a the number of sets in $\mathbb{S}$ that a belongs to, and $\mathbb{L}$ being a specific lifting operator. *Candidate sets* [13] are obtained as another case of a $\mathbb{RSF}$ with $\mathbb{S}$ the set of conflict-free sets, $\mathbb{R}$ a graded semantics that attaches to each argument a the number of extensions w.r.t. a given Dung's semantics that a belongs to, and $\mathbb{L} = \mathbb{L}_{PST}^{\succeq}$.

There are many possible choices for selection functions, ranking semantics and lifting operations. Some might not be suitable; some others might be appropriate for some but not all applications. This opens another research question which is to study the behaviour and the outputs of the framework depending on the selection / ranking / lifting used. Since this is the first paper that opens the possibility of using this general framework, those questions will be part of our future work.

Acknowledgments

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