Toward a More Efficient Generation of Structured Argumentation Graphs

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Abstract. We address the problem of efficiently generating the argumentation graphs from knowledge bases expressed using existential rules and provide a methodology that optimises the generation of argumentation graphs over knowledge bases without rules and filters out a large number of arguments and reduces the number of attacks in the case of argumentation graphs constructed over knowledge bases with rules.

Keywords. Logic Based Argumentation Graphs, Existential Rules

1. Introduction

In this paper we place ourselves in the setting of reasoning with an inconsistent knowledge base [5] expressed using existential rules [12]. Argumentation [15] provides one such reasoning method that has the added value of providing better explanations to users than classical methods [18]. However, one drawback of logic based argumentation frameworks [11, 2, 19] is the large number of arguments generated [23]. Indeed, the argumentative reasoning method relies on the construction of all possible derivations of facts using the rules over the knowledge base (a derivation is called an argument). Usually the only condition one uses to filter out all possible derivations is the condition of minimality and consistency of the facts at the root of the derivation [1] (i.e. all facts are used in the reasoning and these facts will not yield contradictory results). This argument construction step can take a very long time (in the best scenario case when the knowledge base actually allows for this step to finish [6]). The knowledge base inconsistency is represented using the attack relation between arguments. Once the argumentation graph is generated (in this graph the nodes represent the arguments and the attacks the binary attack relation) the reasoning step will compute the extensions using a given semantics (preferred, stable etc.). Please note that even this step is quite expensive from a computational point of view [16]. In the case of knowledge bases expressed using existential rules and allowing for n-ary negative constraints (i.e. n-are logical incompatibilities between the facts) the preferred, stable and semi stable argumentation semantics were proven to coincide with the repairs of the knowledge base (i.e. the maximally with respect to set inclusion subsets of consistent facts) [14]. This result was of practical importance as it allowed to compare the intuitiveness of logical argumentation techniques [18] and to apply them for food science applications [3, 4].

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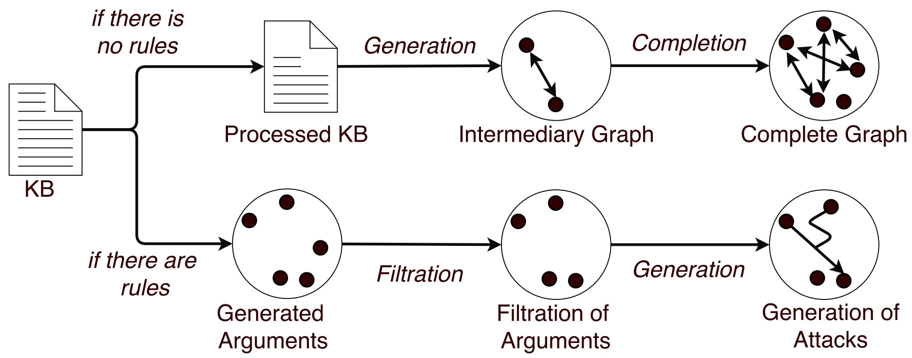

Figure 1. Approach workflow for optimising the argument generation phase.

The main drawback of using argumentation as a reasoning method over inconsistent knowledge bases relies in the large number of arguments generated. For instance, even for a modest knowledge base composed of 7 facts, 3 rules and 1 binary negative constraint one gets an argumentation graph of 383 arguments and 32768 attacks [23]. In this paper we address this drawback and ask the following research question: "*How can one filter out the arguments generated over the knowledge base without compromising the semantic outcome of the corresponding argumentation graph?*". We answer this question by providing a methodology adapted for knowledge bases without rules or knowledge bases with rules. In the first case of knowledge bases without rules, we use the observation that free facts (i.e. facts that are not touched by any negative constraints) induce an exponential growth on the argumentation graph without any impact on its underlying structure [23]. Therefore, we will first generate the argumentation graph corresponding to the knowledge base without the free facts and then redo the whole graph including the arguments of the free facts in an efficient manner. In the second case, of the knowledge bases with rules, we introduce a new structure for the arguments and the attacks. In this new structure, we have less arguments (up to 73% filtered arguments in our experiments). We show that this new framework is semantically equivalent to the framework introduced in [14]. The above methodology is depicted in Figure 1.

The paper is organised as follows. After introducing the background notions needed to formally understand the paper we present the two methodologies explained above. We then provide an empirical evaluation of our work in which we benchmark our approach on the knowledge bases introduced in [23] and show that in most case the number of arguments and attacks of the argumentation graphs corresponding to knowledge bases with rules is reduced (at least by 25 % for the arguments and at least 14 % for the attacks).

2. Background notions

We first introduce some notions of the existential rules language. A *fact* is a ground atom of the form $p(t_1,\ldots,t_k)$ where p is a predicate of arity k and t_i, with $i \in [1,\ldots,k]$, constants. An existential *rule* is of the form $\forall \overrightarrow{X}, \overrightarrow{Y}\ B[\overrightarrow{X}, \overrightarrow{Y}] \rightarrow \exists \overrightarrow{Z} H[\overrightarrow{Z}, \overrightarrow{X}]$ where B (called the body) and H (called the head) are existentially closed atoms or conjunctions of existentially closed atoms and $\overrightarrow{X}, \overrightarrow{Y}, \overrightarrow{Z}$ their respective vectors of variables. We use the notation B_r and H_r to respectively denote the body and head of the rule r. A *rule is applicable* on a set of facts $\mathscr{F}$ if and only if there exists a homomorphism from the body of the rule to $\mathscr{F}$. Applying a rule to a set of facts (also called *chase*) consists of adding the set of atoms of the conclusion of the rule to the facts according to the application homomorphism. The saturation $\mathscr{S}at_{\mathscr{R}}(X)$ of a set of facts X is the set of atoms obtained after successively applying the set of rules until a fixed point. Different *chase* mechanisms use different simplifications that prevent infinite redundancies [9]. We use recognisable classes of existential rules where the chase is guaranteed to stop [9]. A *negative constraint* is a rule of the form $\forall \overrightarrow{X}, \overrightarrow{Y}\ B[\overrightarrow{X}, \overrightarrow{Y}] \rightarrow \bot$ where B is an existentially closed atom or conjunctions of existentially closed atoms, $\overrightarrow{X}, \overrightarrow{Y}$, their respective vectors of variables and $\bot$ is *absurdum*. Negative constraints can be of any arity (i.e. the number of atoms in B is not bounded). A subset X of $\mathscr{F}$ is $\mathscr{R}$-inconsistent if and only if there is a negative constraint that is applicable to the saturation of X, otherwise X is $\mathscr{R}$-consistent.

Definition 1. A *knowledge base* (KB) $\mathscr{K}$ is a tuple $\mathscr{K} = (\mathscr{F}, \mathscr{R}, \mathscr{N})$ where $\mathscr{F}$ is a finite set of facts, $\mathscr{R}$ a set of existential rules and $\mathscr{N}$ a set of negative constraints.

A conflict of a KB $\mathscr{K} = (\mathscr{F}, \mathscr{R}, \mathscr{N})$ is a minimal subset of $\mathscr{F}$ that is $\mathscr{R}$-inconsistent.

Definition 2. Let $\mathscr{K} = (\mathscr{F}, \mathscr{R}, \mathscr{N})$ be a KB, we say that $X \subseteq \mathscr{F}$ is a conflict of $\mathscr{K}$ if and only if X is $\mathscr{R}$-inconsistent and for all $X' \subset X, X'$ is $\mathscr{R}$-consistent. If X is a conflict, $|X|$ is the size of the conflict.

The set of all conflicts of $\mathscr{K}$ is denoted by $MI(\mathscr{K})$. The notion of conflict is the dual of the notion maximal consistent set (also called repair) since removing one element of each minimal inconsistent set restores the consistency.

Let us now introduce the structure of arguments and attacks defined in [14, 13, 22]. In this framework, the arguments are composed of a support (or hypothesis) and a conclusion. The attack relation is a particular undermining where arguments attack the support of other arguments.

Definition 3. Let $\mathscr{K}$ be a KB, the argumentation framework (AF) instantiated from $\mathscr{K}$ is the pair $\mathbb{AS}_{\mathscr{K}} = (\mathscr{A}, \mathscr{C})$ where $\mathscr{A}$ is a set of arguments and $\mathscr{C}$ a set of attacks defined as follows.An *argument* is a tuple (H,C) with H a non-empty $\mathscr{R}$-consistent subset of $\mathscr{F}$ and C a set of facts such that : $H \subseteq \mathscr{F}$ and H is $\mathscr{R}$-consistent *(consistency)*; $C \subseteq \mathscr{S}at_{\mathscr{R}}(H)$ *(entailment)*; $\nexists H' \subset H$ s.t. $C \subseteq \mathscr{S}at_{\mathscr{R}}(H')$ *(minimality)*. We say that $a = (H,C)$ attacks $b = (H',C')$ denoted by $(a,b) \in \mathscr{C}$ iff there exists $\phi \in H'$ such that $C \cup \{\phi\}$ is $\mathscr{R}$-inconsistent.

Let $a = (H,C)$ be an argument of $\mathbb{AS}_{\mathscr{K}}$, we denote by $Supp(a) = H$ the support of a and $Conc(a) = C$ its conclusion. Let E be a set of arguments, the base of E is defined as the union of the supports of the arguments in E, namely $Base(E) = \bigcup_{a \in E} Supp(a)$. Let Y be a set of facts and E be a set of arguments, the arguments of E constructed upon Y is defined as $Arg(Y,E) = \{a \in E \mid Supp(a) \subseteq Y\}$.

Once the argumentation framework is constructed, one can use some of the argumentation semantics [15, 10] to obtain sets of arguments called extensions. We quickly recall the some of the argumentation semantics defined by Dung [15]. Let $E \subseteq \mathscr{A}$ and $a \in \mathscr{A}$. We say that E is *conflict free* iff there exists no arguments $a,b \in E$ such that $(a,b) \in \mathscr{C}$. E *defends* a iff for every argument $b \in \mathscr{A}$, if we have $(b,a) \in \mathscr{C}$ then there exists $c \in E$ such that $(c,b) \in \mathscr{C}$. E is *admissible* iff it is conflict-free and defends all its arguments. E is a *preferred extension* iff it is maximal (with respect to set inclusion) admissible set. E is a *stable extension* iff it is conflict-free and for all $a \in \mathscr{A} \setminus E$, there exists an argument $b \in E$ such that $(b,a) \in \mathscr{C}$. For an argumentation framework $\mathbb{AS}_{\mathscr{K}} = (\mathscr{A},\mathscr{C})$, we denote by $Ext_p(\mathbb{AS}_{\mathscr{K}})$ (resp. $Ext_s(\mathbb{AS}_{\mathscr{K}})$) the set of its preferred extensions (resp. stable extensions).

This argumentation framework possesses many desirable properties [14] such as direct/indirect consistency, closure and the one to one equivalence between preferred/stable extensions and the repairs. However, it has been shown in [23] that the number of arguments can be exponential with respect to the number of facts and even a KB with eight facts, six rules and two negative constraints can lead to an argumentation framework with 111775 arguments.

3. Graph Generation With No Rules

In this section, we propose an optimisation for the generation of the aforementioned argumentation framework in the case where KBs contain no rules. The idea is to process the KB before generating the argumentation graph and recreate the whole argumentation graph from this reduced graph.

We first introduce the notion of free fact and dummy arguments.

Definition 4. Let $\mathscr{K} = (\mathscr{F},\mathscr{R},\mathscr{N})$ be a KB, a fact $f \in \mathscr{F}$ is a free fact if and only if for every minimal inconsistent set $m \in MI(\mathscr{K}), f \notin m$.

We denote by $Free(\mathscr{K})$, the set of free facts of $\mathscr{K}$. In the case where the KB does not contain any rules, it has been proven that the number of arguments that are not attacked and do not attack other arguments (dummy arguments) is exponential with respect to the number of free facts.

Proposition 1. Let $\mathscr{K} = (\mathscr{F},\mathscr{R},\mathscr{N})$ be KB such that $\mathscr{R} = \emptyset$ and $|\mathscr{F}| = n$. There are exactly $2^k - 1$ dummy arguments a in $\mathbb{AS}_{\mathscr{K}} = (\mathscr{A},\mathscr{C})$ where $k = |Free(\mathscr{K})|$.

As the reader can notice, as the number of free facts increases, the number of dummy arguments grows exponentially. However, a further result of [23] is that if one removes the free facts from the KB before generating the argumentation graph, this argumentation graph possesses "exponentially less arguments" than the original argumentation graph with respect to the number of free facts. Hence, we propose a four step approach for generating the original argumentation graph faster and without any losses:

1. We identify the set $Free(\mathscr{K})$. This step can be done by finding the minimal inconsistent sets using existing algorithms [17, 21]
2. We create the graph $\mathbb{AS}_{\mathscr{K}'}$ where $\mathscr{K}' = (\mathscr{F} \setminus Free(\mathscr{K}),\mathscr{R},\mathscr{N})$ following Definition 3. Please note that this step can be achieved using the argumentation graph generator proposed by [23].
3. Then, we grow the generated graph to its original size. This can be done by copying each arguments 2^k times where $k = |Free(\mathscr{K})|$ and adding attacks following the two principles: (1) if a attacks b then a attacks all the copies of b and (2) if b is a copy of a in then b has the same attackers and attacks the same arguments than a.

4. Last, we add $2^k - 1$ dummy arguments to the generated graph.

Example 1. Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a KB such that $\mathcal{F} = \{a(m), b(m), c(m)\}$, $\mathcal{R} = \emptyset$ and $\mathcal{N} = \{\forall x(a(x) \wedge b(x) \rightarrow \bot)\}$. We have that $Free(\mathcal{K}) = \{c(a)\}$. Hence, we generate the argumentation graph $\mathbb{AS}_{\mathcal{K}'}$ where $\mathcal{K}' = (\{\{a(m), b(m)\}, \mathcal{R}, \mathcal{N})$ (nodes in purples in Figure 2). Then, from the graph of $\mathbb{AS}_{\mathcal{K}'}$ (in purple), one can construct the graph (in gray). Finally, the dummy arguments are added (in orange). The nodes encircled in red correspond to the arguments of $\mathbb{AS}_{\mathcal{K}}$.

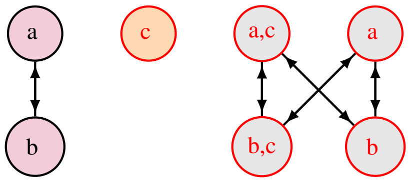

Figure 2. Nodes in red represent arguments of $\mathbb{AS}_{\mathcal{K}}$, nodes in purple represent arguments of $\mathbb{AS}_{\mathcal{K}'}$ and gray nodes represent the subgraph of $\mathbb{AS}_{\mathcal{K}}$ that is a 2 copy graph of $\mathbb{AS}_{\mathcal{K}'}$.

4. Graph Generation With Rules

We now present a novel argumentation framework that is aimed at reducing the number of arguments and the number of attacks in the case where the set of rules is not empty. We show several desirable results such as the equivalence between the preferred/stable extensions of the aforementioned framework and the new one and some basic properties regarding attacks in the new framework. The idea behind this new framework is to remove, amongst the arguments with the same support, those that have conclusions that can be "decomposed". Let us illustrate the idea with the following example.

Example 2. Let us consider the KB $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ with $\mathcal{F} = \{pitbull(Tom), cat(Tom)\}$, $\mathcal{R} = \{\forall x(pitbull(x) \rightarrow dog(x))\}$ and $\mathcal{N} = \{\forall x(dog(x) \wedge cat(x) \rightarrow \bot)\}$. We have 3 arguments: $a_1 = (\{pitbull(Tom)\}, \{pitbull(Tom)\})$, $a_2 = (\{pitbull(Tom)\}, \{dog(Tom)\})$, $a_3 = (\{pitbull(Tom)\}, \{dog(Tom), pitbull(Tom)\})$ and $a_4 = (\{cat(Tom)\}, \{cat(Tom)\})$. Our approach will delete the argument a_3 because we can easily reconstruct the argument a_3 from a_1 and a_2.

Why do we only filter the arguments those with the same support? Let us illustrate this by an example. Suppose that we also have the fact $adult(Tom)$ in $\mathcal{K}$. Amongst many other, we would have the following two arguments: $a_5 = (\{adult(Tom)\}, \{adult(Tom)\})$ and $a_6 = (\{pitbul(Tom), adult(Tom)\}, \{pitbul(Tom), adult(Tom)\})$. Could we remove the argument a_6 and reconstruct it from a_1 and a_5? We chose not to do this because it is not obvious that a_1 and a_5 are compatible with respect to the ontology and the negative constraints. That is the reason why we keep the argument a_6. On the contrary, note that in the case of a_1 and a_2 that have the same support they will always be compatible together.

Let us now formalise this intuition.

Definition 5. Let $\mathcal{K}$ be a KB and $\mathbb{AS}_{\mathcal{K}} = (\mathcal{A}, \mathcal{C})$ the argumentation framework constructed from $\mathcal{K}$ using Definition 3. Let $D(\mathbb{AS}_{\mathcal{K}}) = \{a = (H, C) \in \mathcal{A} \mid$ there exists $X \subseteq \mathcal{A} \setminus \{a\}$ such that for every $b \in X, Supp(b) = H$ and $\bigcup_{b \in X} Conc(b) = C\}$. The filtrated set of arguments is $\mathcal{A}^* = \mathcal{A} \setminus D(\mathbb{AS}_{\mathcal{K}})$.

Example 3. In this example, the set $D(\mathbb{AS}_{\mathscr{K}})$ is $\{a_3\}$ and $\mathscr{A}^* = \{a_1, a_2, a_4\}$.

Since we dropped some arguments, the attack relation have to be redesigned in order to keep all the conflicts. In particular, we allow for n-ary attacks where arguments with the same support can jointly attack an argument.

Definition 6. An attack is a pair (X, a) where $X \subseteq \mathscr{A}^*$ and $a \in \mathscr{A}^*$ such that X is minimal for set inclusion such that for every $x_1, x_2 \in X, Supp(x_1) = Supp(x_2)$ and there exists $\phi \in Supp(a)$ such that $(\bigcup_{x \in X} Conc(x)) \cup \{\phi\})$ is $\mathscr{R}$-inconsistent.

The next example shows that this definition of attack is necessary in order to capture some attacks that would be lost otherwise.

Example 4. Let us consider the KB $\mathscr{K} = (\mathscr{F}, \mathscr{R}, \mathscr{N})$ with $\mathscr{F} = \{a(m), b(m), c(m)\}, \mathscr{R} = \{\forall x(a(x) \wedge b(x) \rightarrow d(x) \wedge e(x))\}$ and $\mathscr{N} = \{\forall x(c(x) \wedge d(x) \wedge e(x) \rightarrow \perp)\}$. We have the attack $(\{a2\_3, a2\_7\}, a3\_0)$ where $a2\_3 = (\{a(m), b(m)\}, \{d(m)\}), a2\_7 = (\{a(m), b(m)\}, e(m))$ and $a3\_0 = (\{c(m)\}, \{c(m)\})$. Note that if the classical attack definition was used, $a3\_0$ would not be attacked anymore since we removed its attackers, for instance $(\{a(m), b(m)\}, \{b(m), d(m), e(m)\})$.

Definition 7. Let $\mathscr{K}$ be a KB, the corresponding filtrated argumentation framework $\mathbb{AS}^*_{\mathscr{K}}$ is the pair $(\mathscr{A}^*, \mathscr{C}^*)$ where $\mathscr{A}^*$ is as defined in Definition 5 and $\mathscr{C}^*$ is the set of all possible attacks that can be constructed using Definition 6.

The propose argumentation framework $\mathbb{AS}^*_{\mathscr{K}}$ is an instantiation of the framework proposed by Nielsen and Parsons [20]. For the purpose of the paper being self-contained, we briefly recall the necessary definitions.

Definition 8. Let $\mathbb{AS}^*_{\mathscr{K}} = (\mathscr{A}^*, \mathscr{C}^*)$ be an argumentation framework:

- A set of arguments S is **conflict-free** if and only if there is no argument $a \in S$, such that $(S, a) \in \mathscr{C}^*$.
- A set of arguments of arguments S_1 attacks a set of arguments S_2 if and only if there exists $a \in S_2$ such that $(S_1, a) \in \mathscr{C}^*$.
- An argument a is said to be **acceptable** with respect to a set of arguments S, if S defends a from all attacking sets of arguments in a.
- A set of arguments S_1 defends an argument a if and only if for every set of arguments S_2 such that $(S_2, a) \in \mathscr{C}^*$, we have that S_1 attacks S_2.
- A conflict-free set of arguments S is said to be **admissible** if each argument in S is acceptable with respect to S.
- An admissible set S is called a **preferred extension** if there is no admissible set $S' \subseteq \mathscr{A}^*, S \subset S'$.
- A conflict-free set S is a **stable extension** if S attacks all arguments in $\mathscr{A}^* \setminus S$.

With a slight abuse of notation, we also use the notation $Ext_p(\mathbb{AS}^*_{\mathscr{K}})$ (resp $Ext_s(\mathbb{AS}^*_{\mathscr{K}})$) to refer to the set of all preferred extensions (resp. stable extensions) of $\mathbb{AS}^*_{\mathscr{K}}$.

Proposition 2. Let $\mathbb{AS}^*_{\mathscr{K}} = (\mathscr{A}^*, \mathscr{C}^*)$, it holds that $Ext_x(\mathbb{AS}^*_{\mathscr{K}}) = \{Arg(A', \mathscr{A}^*) \mid A'$ is a repair $\}$ for $i \in \{s, p\}$.

Proof. The proof is split in two parts. Let $\mathbb{AS}^*_{\mathscr{K}} = (\mathscr{A}^*, \mathscr{C}^*)$ and $N = \{Arg(A', \mathscr{A}^*) \mid A'$ is a repair $\}$. We first show that $N \subseteq Ext_s(\mathbb{AS}^*_{\mathscr{K}})$. Then since, the set of stable extensions is included in the set of preferred extensions [20], we have that $N \subseteq Ext_p(\mathbb{AS}^*_{\mathscr{K}})$. In the second part of the proof, we show that $Ext_p(\mathbb{AS}^*_{\mathscr{K}}) \subseteq N$.

- We first show $\{Arg(A', \mathscr{A}^*) \mid A'$ is a repair $\} \subseteq Ext_s(\mathbb{AS}^*_{\mathscr{K}})$. Let A' be a repair of $\mathscr{K}$ and let $E = Arg(A', \mathscr{A}^*)$. Let us prove that E is a stable extension of $\mathbb{AS}^*_{\mathscr{K}}$. We first prove that E is conflict-free. By means of contradiction we suppose the contrary, i.e. let $X \subseteq E$, $b \in E$ such that $(X, b) \in \mathscr{C}^*$. From the definition of attack, there exists $\phi \in Supp(b)$ such that $(\bigcup_{a \in X} Conc(a)) \cup \{\phi\}$ is $\mathscr{R}$-inconsistent. Let $x \in X$ then $Supp(x) \cup \{\phi\}$ is $\mathscr{R}$-inconsistent and A' is $\mathscr{R}$-inconsistent, contradiction. Therefore E is conflict-free.
  Let us now prove that E attacks all arguments outside the set E. Let $b \in \mathscr{A}^* \setminus Arg(A', \mathscr{A}^*)$ and let $\phi \in Supp(b)$, such that $\phi \notin A'$. By definition, we know that there is $S' \subseteq A'$ such that $c = (S', A')$ is an argument in $\mathbb{AS}_{\mathscr{K}}$. If $c \in \mathscr{A}^*$, we have that $(\{c\}, b) \in \mathscr{C}^*$. Otherwise, there exists a set of arguments $M = \{a_1, a_2, \ldots, a_n\} \subseteq \mathscr{A}^*$ with $Supp(a_i) = S'$ such that $\bigcup_{a_i \in M} Conc(a_i) = A'$. Hence, $(M, b) \in \mathscr{C}^*$ and E is a stable extension.
- We now show that $Ext_p(\mathbb{AS}^*_{\mathscr{K}}) \subseteq \{Arg(A', \mathscr{A}^*) \mid A'$ is a repair$\}$. Let $E \in Ext_p(\mathbb{AS}^*_{\mathscr{K}})$ and let us prove that there exists a repair A' such that $E = Arg(A', \mathscr{A}^*)$. Let $S = Base(E)$. Let us prove that S is $\mathscr{R}$-consistent. Aiming to a contradiction, suppose that S is $\mathscr{R}$-inconsistent. Let $S' \subseteq S$ be such that:

  1. S' is $\mathscr{R}$-inconsistent
  2. Every proper set of S' is $\mathscr{R}$-consistent. Let us denote $S' = \{\phi_1, \phi_2, \ldots, \phi_n\}$.

  Let $a \in E$ be an argument such that $\phi_n \in Supp(a)$. Let $S'' \subseteq S' \setminus \{\phi_n\}$ such that $c = (S'', S' \setminus \{\phi_n\})$ is an argument of $\mathbb{AS}_{\mathscr{K}}$. Then, if $c \in \mathscr{A}^*$, we have that $(\{c\}, a) \in \mathscr{C}^*$. Otherwise, there exists a set of arguments $M = \{a_1, a_2, \ldots, a_n\} \subseteq \mathscr{A}^*$ with $Supp(a_i) = S''$ such that $\bigcup_{a_i \in M} Conc(a_i) = S' \setminus \{\phi_n\}$. Hence, a is attacked by c or by M. Since E is conflict-free, we have that $c \notin E$ (resp $M \not\subseteq E$). However, since E is admissible, there exists $B = \{b_1, \ldots b_n\}$ such that $B \subseteq E$ and $(B, c) \in \mathscr{C}^*$ (respectively there exists $a_i \in M \setminus E$ and $(B, a_i) \in \mathscr{C}^*$). By definition, this means that there exists $\phi_j \in Supp(c)$ (resp $\phi_j \in Supp(a_i)$) such that $(\bigcup_{b_i \in B} Conc(b_i)) \cup \{\phi_j\}$ is $\mathscr{R}$-inconsistent. Since $\phi_j \in S$, there is an argument $d \in E$ such that $\phi_j \in Supp(d)$. Therefore, $(B, d) \in \mathscr{C}^*$. Contradiction and E is $\mathscr{R}$-consistent.
  Let us now prove that there exists no $S' \subseteq \mathscr{F}$ such that $S \subset S'$ and S' is $\mathscr{R}$-consistent. We use the proof by contradiction. Thus, suppose that S is not a maximal $\mathscr{R}$-consistent subset of $\mathscr{F}$. Then, there exists a repair S' of $\mathscr{K}$, such that $S \subset S'$. We have that $E \subseteq Arg(S, \mathscr{A}^*)$. Denote $E' = Arg(S', \mathscr{A}^*)$. Since $S \subset S'$ then $Arg(S, \mathscr{A}^*) \subset E?$. Thus, $E \subset E'$. From the first part of the proof, $E' \in Ext_s(\mathbb{AS}^*_{\mathscr{K}})$. Consequently, $E' \in Ext_p(\mathbb{AS}^*_{\mathscr{K}})$. We also know that $E \in Ext_p(\mathbb{AS}^*_{\mathscr{K}})$. Contradiction, since no preferred set can be a proper subset of another preferred set. Thus, we conclude that $Base(E)$ is a repair of $\mathscr{K}$.
  Let us show that $E = Arg(Base(E), \mathscr{A}^*)$. It must be that $E \subseteq Arg(S, \mathscr{A}^*)$. Also, we know (from the first part) that $Arg(S, \mathscr{A}^*)$ is a stable and a preferred extension, thus the case $E \not\subseteq Arg(S, \mathscr{A}^*)$ is not possible

□

$a0\_0$	$(\{a(m)\},\{a(m)\})$	$a2\_9$	$(\{a(m),b(m)\},\{b(m),e(m)\})$
$a1\_0$	$(\{b(m)\},\{b(m)\})$	$a2\_10$	$(\{a(m),b(m)\},\{a(m),b(m),e(m)\})$
$a2\_2$	$(\{a(m),b(m)\},\{a(m),b(m)\})$	$a2\_11$	$(\{a(m),b(m)\},\{d(m),e(m)\})$
$a2\_3$	$(\{a(m),b(m)\},\{d(m)\})$	$a2\_12$	$(\{a(m),b(m)\},\{a(m),d(m),e(m)\})$
$a2\_4$	$(\{a(m),b(m)\},\{a(m),d(m)\})$	$a2\_13$	$(\{a(m),b(m)\},\{b(m),d(m),e(m)\})$
$a2\_5$	$(\{a(m),b(m)\},\{b(m),d(m)\})$	$a2\_14$	$(\{a(m),b(m)\},\{a(m),b(m),d(m),e(m)\})$
$a2\_6$	$(\{a(m),b(m)\},\{a(m),b(m),d(m)\})$	$a3\_0$	$(\{c(m)\},\{c(m)\})$
$a2\_7$	$(\{a(m),b(m)\},\{e(m)\})$	$a4\_2$	$(\{a(m),c(m)\},\{a(m),c(m)\})$
$a2\_8$	$(\{a(m),b(m)\},\{a(m),e(m)\})$	$a5\_2$	$(\{b(m),c(m)\},\{b(m),c(m)\})$

Table 1. Arguments in $\mathbb{AS}_{\mathcal{K}}$ obtained from the KB of Example 4

Corollary 1. Let $\mathbb{AS}_{\mathcal{K}}$ be an argumentation framework and $\mathbb{AS}^*_{\mathcal{K}}$ the corresponding filtrated AF. It holds that $Ext_x(\mathbb{AS}^*_{\mathcal{K}}) = \{E \cap \mathscr{A}^* \mid E \in Ext_x(\mathbb{AS}_{\mathcal{K}})$ with $x \in \{s,p\}$.

Example 5. The argumentation framework $\mathbb{AS}_{\mathcal{K}}$ is composed of 18 arguments and 51 attacks. The corresponding filtrated argumentation framework $\mathbb{AS}^*_{\mathcal{K}}$ has 12 arguments and 66 attacks. Note that the list of all the arguments and those filtrated (in grey) are represented in Table 1. There are 3 preferred/stable extensions of $\mathbb{AS}_{\mathcal{K}}$: $\{a0\_0, a1\_0, a2\_2, a2\_3, a2\_4, a2\_5, a2\_6, a2\_7, a2\_8, a2\_9, a2\_10, a2\_11, a2\_12, a2\_13, a2\_14\}$, $\{a0\_0, a3\_0, a4\_0\}$ and $\{a1\_0, a3\_0, a5\_2\}$. The extensions in $\mathbb{AS}^*_{\mathcal{K}}$ are $\{a0\_0, a1\_0, a2\_2, a2\_3, a2\_4, a2\_5, a2\_7, a2\_8, a2\_9\}$, $\{a0\_0, a3\_0, a4\_0\}$ and $\{a1\_0, a3\_0, a5\_2\}$.

Proposition 3. Let $\mathcal{K}$ be a KB, $\mathbb{AS}_{\mathcal{K}} = (\mathscr{A}, \mathscr{C})$ the corresponding argumentation framework and $\mathbb{AS}^*_{\mathcal{K}} = (\mathscr{A}^*, \mathscr{C}^*)$ the filtrated AF. Then it holds that $|\mathscr{A}^*| \leq |\mathscr{A}|$.

Please note that it is not true that $|\mathscr{C}| \leq |\mathscr{C}^*|$ as shown by Example 6.

Example 6. Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a KB with $\mathcal{F} = \{a(m), b(m), c(m)\}$, $\mathcal{R} = \{\forall x(a(x) \rightarrow b(x))\}$ and $\mathcal{N} = \{\forall x(a(x) \wedge c(x) \rightarrow \bot)\}$. The set $\mathscr{C}$ is composed of 10 attacks whereas $\mathscr{C}^*$ has 8 attacks.

Proposition 4. Let $\mathcal{K}$ be a KB, $\mathbb{AS}_{\mathcal{K}} = (\mathscr{A}, \mathscr{C})$ the corresponding argumentation framework and $\mathbb{AS}^*_{\mathcal{K}} = (\mathscr{A}^*, \mathscr{C}^*)$ the filtrated AF. It holds that:

1. if $a \in \mathscr{A}^*$ is not attacked in $\mathbb{AS}_{\mathcal{K}}$ if and only if a is not attacked in $\mathbb{AS}^*_{\mathcal{K}}$
2. if $a \in \mathscr{A}^*$ is attacked in $\mathbb{AS}_{\mathcal{K}}$ then $|Att^-_{\mathbb{AS}_{\mathcal{K}}}(a)| \leq |Att^-_{\mathbb{AS}^*_{\mathcal{K}}}(a)|$

Proof. The proofs are as follows:

1. We show that $a \in \mathscr{A}^*$ is not attacked in $\mathbb{AS}_{\mathcal{K}}$ if and only if a is not attacked in $\mathbb{AS}^*_{\mathcal{K}}$. The proof is split in two parts:

   - $(\Rightarrow)$ Let $a \in \mathscr{A}^*$ and suppose that a is not attacked in $\mathbb{AS}_{\mathcal{K}}$ but a is attacked in $\mathbb{AS}^*_{\mathcal{K}}$. If there exists $b \in \mathscr{A}^*$ such that $(\{b\}, a) \in \mathscr{C}^*$. Then, we have that $(b, a) \in \mathscr{C}$, hence a is attacked in $\mathbb{AS}_{\mathcal{K}}$. Contradiction. Otherwise, there exists $B = \{b_1, \ldots, b_n\} \subseteq \mathscr{A}^*$ such that $(B, a) \in \mathscr{C}^*$. Let $CB = \bigcup_{b_i \in B} Conc(b_i)$. Let $SB = Supp(b_1) = \cdots = Supp(b_n)$. We now show that $b = (SB, CB)$ is an argument in $\mathbb{AS}_{\mathcal{K}}$. Indeed, SB is consistent, $CB \subseteq \mathscr{S}at_{\mathcal{R}}(SB)$. By means of contradiction, suppose that SB is not a minimal set satisfying the previous twi conditions and let $SB' \subset SB$ such that $CB \subseteq \mathcal{S}at_{\mathcal{R}}(SB')$. Then, b_1 is not an argument since $(SB', Conc(b_1))$ is an argument. Thus, b is an argument in $\mathbb{AS}_{\mathcal{K}}$ and we have that $(b,a) \in \mathcal{C}$.

- ($\Leftarrow$) Suppose now that a is attacked in $\mathbb{AS}_{\mathcal{K}}$ but that a is not attacked in $\mathbb{AS}^*_{\mathcal{K}}$. Let $(b,a) \in \mathcal{C}$. The case $b \in \mathcal{A}^*$ is trivial because $(\{b\},a) \in \mathcal{C}^*$. Suppose that $b \notin \mathcal{A}^*$. This means that there exists $B = \{b_1,\ldots,b_n\} \subseteq \mathcal{A}^*$ such that $Supp(b_1) = \cdots = Supp(b_n)$ and $Conc(b) = \bigcup_{b_i \in B} Conc(b_i)$. So either $(B,a) \in \mathcal{C}^*$ or there exists $B' \subset B$ such that $(B',a) \in \mathcal{C}^*$.

2. We show that if $a \in \mathcal{A}^*$ is attacked in $\mathbb{AS}_{\mathcal{K}}$ then $|Att^-_{\mathbb{AS}_{\mathcal{K}}}(a)| \leq |Att^-_{\mathbb{AS}^*_{\mathcal{K}}}(a)|$. Suppose that $a \in \mathcal{A}^*$ is attacked in $\mathbb{AS}_{\mathcal{K}}$. Let $B = \{b_1,\ldots,b_n\}$ be the set of all attackers of a in $\mathbb{AS}_{\mathcal{K}}$. Without loss of generality, let $B \cap \mathcal{A}^* = \{b_k,\ldots,b_n\}$. Let us define a function $f : B \to \{B' \subseteq \mathcal{A}^* \mid (B',a) \in \mathcal{C}^*\}$ as follows:

$$f(b_i) = \begin{cases} \{b_i\} & \text{if } b_i \in \mathcal{A}^* \\ B' = \{b_i^1,\ldots,b_i^k\} \text{ where } B \text{ is an arbitrary set such} & \\ \text{that for every } b_i^j \in B', Supp(b_i^j) = Supp(b_i) \text{ and} & \\ \bigcup_{b_i^j \in B'} Conc(b_i^j) = Conc(b_i) \text{ and } B' \text{ is a minimal} & \\ \text{set satisfying the previous two conditions} & \text{otherwise} \end{cases}$$

We have that f is well defined since such B' exists if $b_i \notin \mathcal{A}^*$, by definition of $\mathcal{A}^*$. Let us prove that f is an injective function. Let $b_i, b_j \in B$. The case $i \geq k$ or $j \geq k$ is obvious. In the remainder of the proof, we suppose that $i < k$ and $j < k$. Denote $B' = f(b_i)$ and $B'' = f(b_j)$. Suppose that $B' = B''$. Thus $Supp(b_i) = Supp(b_j)$. Note that $Conc(b_i) = \bigcup_{b \in B'} Conc(b) = \bigcup_{b \in B''} Conc(b) = Conc(b_j)$. Thus $b_i = b_j$. This shows that f is injective. We conclude that $|Att^-_{\mathbb{AS}_{\mathcal{K}}}(a)| \leq |Att^-_{\mathbb{AS}^*_{\mathcal{K}}}(a)|$.

□

We now show that it is not always possible to find a set of arguments X, with the same support, after the filtration such that the conclusions of X are distinct and the union of their conclusions is equal to a filtered argument.

Example 7. Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a KB such that $\mathcal{F} = \{a(m), c(m)\}$, $\mathcal{R} = \{\forall x(a(x) \to b(x))\}$ and $\mathcal{N} = \emptyset$. As one can see, the argument $d = (\{a(m),c(m)\}, \{a(m), c(m), b(m)\})$ is filtrated because there of the two arguments $x_1 = (\{a(m),c(m)\}, \{a(m),c(m)\})$ and $x_2 = (\{a(m),c(m)\}, \{b(m),c(m)\})$. Note that here, it holds that $X = \{x_1,x_2\}$ satisfies $Supp(x_1) = Supp(x_2) = Supp(d)$ and $\bigcup_{x_i \in X} Conc(x_i) = Conc(d)$ but for all $x_1,x_2 \in X$, $Conc(x_1) \cap Conc(x_2) = \emptyset$ is not true.

We now show that in the case where the set of rules is empty, the set of filtrated arguments is empty and $\mathbb{AS}_{\mathcal{K}}$ is equivalent to $\mathbb{AS}^*_{\mathcal{K}}$.

Proposition 5. Let $\mathcal{K} = (\mathcal{F}, \mathcal{R}, \mathcal{N})$ be a KB such that $\mathcal{R} = \emptyset$ and $\mathbb{AS}_{\mathcal{K}} = (\mathcal{A}, \mathcal{C})$ is the corresponding AF. It holds that $D(\mathbb{AS}_{\mathcal{K}}) = \emptyset$ and $\mathbb{AS}^*_{\mathcal{K}} = (\mathcal{A}^*, \mathcal{C}^*)$ is such that $\mathcal{A}^* = \mathcal{A}$ and $(b,a) \in \mathcal{C}$ if and only if $(\{b\},a) \in \mathcal{C}^*$.

5. Experimentation and Discussion

In this section, we show the efficiency of the new argumentation framework based on the filtration of arguments for reducing the number of arguments and attacks by comparing the number of arguments in $\mathbb{AS}_{\mathscr{K}}$ and $\mathbb{AS}^*_{\mathscr{K}}$.

We chose to work with a particular subset of 108 KBs (named b_1 to b_{108}) extracted from the study of [23]. These KBs were generated by fixing the size of the set of facts and successively adding negative constraints until saturation. This dataset is composed of KBs with two to seven facts with different characteristics as shown in Table 2.

Name of the KB	Number of facts	Number of rules	Number of NC	Type of NC
b_1 to b_6	2 to 7	$\emptyset$	1	Binary
b_{32}	3	$\emptyset$	2	Binary
b_{33} to b_{35}	4	$\emptyset$	2 to 3	Binary
b_{36} to b_{40}	5	$\emptyset$	2 to 3	Binary
b_{41} to b_{56}	3 to 6	$\emptyset$	1 to 3	Ternary
b_7 to b_{12}	2	1 to 6	1	Binary
b_{13} to b_{18}	2	2,4 or 6	1	Binary
b_{19} to b_{28}	2 to 7	1 or 3	1	Binary
b_{29} to b_{31}	3	2	1	Binary
b_{57} to b_{58}	3	1	2	Binary
b_{59} to b_{82}	4	3	2 to 4	Binary
b_{83} to b_{84}	3	1	1	Ternary
b_{85} to b_{87}	3	2	1	Ternary
b_{88} to b_{108}	4	3	1 to 2	Ternary

Table 2. Characteristics of the KBs

We provide a generator based on the Graal Java Toolkit [8] for directly generating $\mathbb{AS}^*_{\mathscr{K}}$ from an inconsistent existential rules KB expressed in DLGP format [7]. This tool can be downloaded along with the dataset used in this paper at `https://gite.lirmm.fr/yun/paper-comma-generator`.

In Table 3, we present the number of arguments and attacks in $\mathbb{AS}_{\mathscr{K}}$ and $\mathbb{AS}^*_{\mathscr{K}}$ along with the percentage of arguments filtered and the percentage of reduction of attacks. These two percentages are defined as $\%Arg.Filtrated = \frac{|\mathscr{A}|-|\mathscr{A}^*|}{|\mathscr{A}|}$ and $\%Att.Reduction = \frac{|\mathscr{C}|-|\mathscr{C}^*|}{|\mathscr{C}|}$. We can make the following observations. This method does not provide any advantages in the case where the KB is devoid of rules. Second, although the instance with the highest percentage of reduction of attacks is b_{12} with 88% is also the instance with the highest percentage of arguments filtered (73%). This is not always the case. Indeed, the instances b_{10} and b_{13} both have a percentage of arguments filtered of 33% but they have a percentage of attacks filtered of 50% and 33% respectively. Last, in all the instances with rules, there are less arguments and less attacks in $\mathbb{AS}^*_{\mathscr{K}}$ compared to $\mathbb{AS}_{\mathscr{K}}$.

Name of the KB	Number of arg. $\mathbb{AS}_{\mathcal{K}}$	Number of attacks $\mathbb{AS}_{\mathcal{K}}$	Number of arg. $\mathbb{AS}^*_{\mathcal{K}}$	Number of attacks $\mathbb{AS}^*_{\mathcal{K}}$	Percentage of arg. filtrated	Percentage of attacks reduction
b_1 to b_6	2 to 95	2 to 2048	2 to 95	2 to 2048	0	0
b_{32}	4	6	4	6	0	0
b_{33} to b_{35}	7 to 9	24 to 32	7 to 9	24 to 32	0	0
b_{36} to b_{40}	14 to 19	56 to 128	14 to 19	56 to 128	0	0
b_{41} to b_{56}	6 to 55	9 to 752	6 to 55	9 to 752	0	0
b_7 to b_{12}	4 to 30	5 to 240	3 to 8	4 to 30	25 to 73	20 to 88
b_{13} to b_{18}	6 to 30	15 to 450	4 to 8	9 to 120	33 to 73	33 to 78
b_{19} to b_{28}	11 to 383	32 to 32768	8 to 143	24 to 16384	25 to 52	25 to 50
b_{29} to b_{31}	16	27 to 30	8	20 to 22	50	26 to 28
b_{57} to b_{58}	8	13 to 14	6	11 to 12	25	14 to 15
b_{59} to b_{82}	22 to 71	123 to 896	12 to 27	83 to 384	44 to 63	32 to 57
b_{83} to b_{84}	12	29 to 39	9	19 to 29	25	26 to 35
b_{85} to b_{87}	24	93 to 147	12	39 to 75	50	49 to 58
b_{88} to b_{108}	78 to 103	990 to 2496	32 to 39	380 to 928	59 to 63	56 to 71

Table 3. Characteristics of the $\mathbb{AS}_{\mathcal{K}}$ and $\mathbb{AS}^*_{\mathcal{K}}$ generated from the KBs.

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