

Assessing the Impact of Agents in Weighted Bipolar Argumentation Frameworks

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Abstract. Argumentation provides a formalism consisting of arguments and attacks/supports between these arguments and can be used to rank or deduce justified conclusions. In multi-agent settings, where several agents can advance arguments at the same time, understanding which agent has the most influence on a particular argument can improve an agent’s decision about which argument to advance next. In this paper, we introduce an argumentation framework with authorship and define new semantics to account for the impact of the agents on the arguments. We propose a set of desirable principles that such a semantics should satisfy, instantiate such semantics from two popular graded based semantics, and study to which extent these principles are satisfied. These semantics will allow an observer to identify the most influential agents in a debate.

Keywords: Argumentation · Graded semantics · Authorship

1 Introduction

The Abstract Argumentation Framework (AAF), as introduced in Dung’s seminal paper [8], is a powerful knowledge representation and reasoning paradigm which represents argumentation debates using directed graph where the nodes represent arguments and arcs represent attacks between the arguments. The weighted Bipolar Argumentation Framework (wBAF) [2] was later introduced as a generalization of AAF where arguments have an associated weight and another binary relation between arguments, called *supports*, is added alongside attacks. This particular framework has received much attention in the literature and most of the existing work have focused on defining semantics to reason with wBAFs [10, 1, 11]. One class of semantics, graded semantics, provides an acceptability degree for each argument of the graph, i.e. quantifying the “strength” of based on its initial weight and how much it is attacked and/or supported.

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Let us consider the following situation where three systems of John’s smart home, temperature sensor (p_1), general knowledge system (p_2) and user preference system (p_3), are communicating by exchanging arguments in real-time. The arguments are listed below:

- p_1 : The heater needs to be turned on (a_0).
- p_2 : Low temperature is acceptable during the night (a_1).
- p_1 : The inside temperature is 18 degrees Celsius which is undesirable (a_2).
- p_2 : The residents are sleeping and low temperatures are beneficial (a_3).
- p_3 : John has specified that he is sensitive to cold (a_4).
- p_3 : In John’s history, he has previously set the inside temperature to 18 degree Celsius (a_5).
- p_2 : It is unlikely that the inside temperature is 18 degrees Celsius as the temperature of the area is 23 degrees (a_6).

The relationship between these arguments and their initial strengths (representing the system confidence) is shown in Fig. 1.

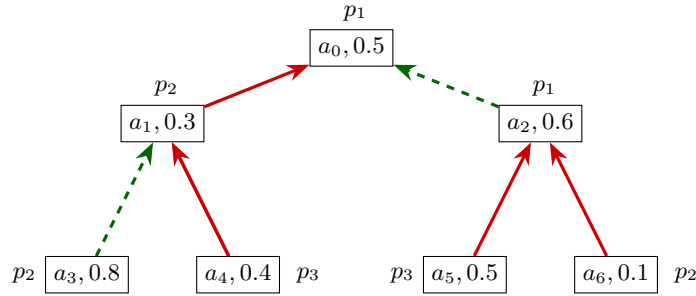


Fig. 1. Graph representation of the smart home example.

The question we are interested in here is: “Which system will be decisive in deciding whether the heater should be turned on?”. More generally, in this paper, we turn our attention to the study of the impact of an agent on the final acceptability degree of an argument.

To illustrate the significance of our contribution, let us consider two motivating examples. For argument-based decision tool, it is useful to see the impact of an agent on the final result. On one hand, this study allows detecting agents that are the most influential. On the other hand, it may lead to a better identification of mischievous behavior that has a real impact on the final result. In addition, for educational purposes, formerly evaluate the impact of an agent on the final result makes it possible to advise a student who wants to learn how to argue. Moreover, it is particularly suited for automated remote training.

This paper is structured as follows. First, in Section 2, we recall the necessary definitions of weighted bipolar framework and graded semantics. Then,

we present the contribution of this paper: (1) a novel bipolar argumentation framework with authorship (Section 3.1), (2) the definition of agent-based impact semantics and their desirable principles (Section 3.2), (3) concrete agent-based impact semantics instantiated using two popular graded semantics, namely Euler-based and DF-Quad based semantics, (Section 4.1) and (4) the analysis of the principles satisfied by the two aforementioned agent-based impact semantics (Section 4.2).

2 Background

We recall the standard weighted Bipolar Argumentation Framework (wBAF) introduced by Amgoud et al. [2, 11]. We start by introducing a weighting on a set of elements as a function that associates to each element of this set, a number between 0 and 1 called its weight.

Definition 1 (Weighting). *Let X be a set of elements, a function $w : X \rightarrow [0, 1]$ is called a weighting on X .*

A weighted bipolar argumentation framework is triple composed of a set of arguments, two binary relations on arguments (attacks and supports) and a weighting on the set of arguments.

Definition 2 (wBAF). *A weighted bipolar argumentation framework (wBAF) is a tuple $\mathbf{F} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ where \mathcal{A} is a finite set of arguments, $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a set of binary attacks, $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a set of binary supports, and w is a weighting on \mathcal{A} .*

As it is common in the literature, we restrict ourselves to acyclic and non-maximal wBAFs, i.e. graphs without cycles nor arguments with a weight of 1. Note that this restriction allows for most of the usual graded semantics defined in the literature to converge.

Definition 3 (Acyclic and non-maximal). *A wBAF $\mathbf{F} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ is acyclic iff for any non-empty finite sequence $\langle a_1, a_2, \dots, a_n \rangle$ of arguments in \mathcal{A} , if for every $i \in \{1, 2, \dots, n-1\}$, $(a_i, a_{i+1}) \in \mathcal{S} \cup \mathcal{R}$, then $(a_n, a_1) \notin \mathcal{S} \cup \mathcal{R}$. A wBAF \mathbf{F} is non-maximal iff for every $a \in \mathcal{A}$, $w(a) < 1$.*

A graded semantics is a function assigning a value in $[0, 1]$ to each argument of a wBAF such that arguments with higher values are considered more acceptable, i.e. less attacked.

Definition 4 (Graded semantics). *A semantics σ is a function mapping any wBAF $\mathbf{F} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ into a weighting $Deg_{\mathbf{F}}^{\sigma}$ from \mathcal{A} to $[0, 1]$. For any argument $a \in \mathcal{A}$, $Deg_{\mathbf{F}}^{\sigma}(a)$ is called the acceptability degree of a .*

There are multiple graded semantics for wBAFs defined in the literature. In this paper, we restrict ourselves to two well-known graded semantics for (acyclic and non-maximal) wBAFs, namely the Euler-based [1] and DF-Quad semantics [15]. Of course, without loss of generality, our approach can be extended to other graded semantics.

Definition 5 (Euler-based semantics). *The Euler-based semantics σ_{EBS} is the function that maps any acyclic and non-maximal wBAF $\mathbf{F} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ to the weighting $Deg_{\mathbf{A}}^{\sigma_{EBS}} : \mathcal{A} \rightarrow [0, 1]$, defined as follow:*

$$\forall a \in \mathcal{A}, Deg_{\mathbf{F}}^{\sigma_{EBS}}(a) = 1 - \frac{1 - w(a)^2}{1 + w(a)e^E}$$

$$\text{where } E = \sum_{s|(s,a) \in \mathcal{S}} Deg_{\mathbf{F}}^{\sigma_{EBS}}(s) - \sum_{r|(r,a) \in \mathcal{R}} Deg_{\mathbf{F}}^{\sigma_{EBS}}(r)$$

Please note that if a does not have any attackers nor supporters, $E = 0$.

Definition 6 (DF-Quad semantics). *The DF-Quad semantics σ_{DF} is the function that maps any acyclic and non-maximal wBAF $\mathbf{F} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ to the weighting $Deg_{\mathbf{F}}^{\sigma_{DF}} : \mathcal{A} \rightarrow [0, 1]$ such that for every $a \in \mathcal{A}$, we have: if $v_s(a) = v_a(a)$, $Deg_{\mathbf{F}}^{\sigma_{DF}}(a) = w(a)$; else $Deg_{\mathbf{F}}^{\sigma_{DF}}(a) =$*

$$w(a) + (0.5 + \frac{v_s(a) - v_a(a)}{2 \cdot |v_s(a) - v_a(a)|} - w(a)) \cdot |v_s(a) - v_a(a)|$$

where:

$$- v_a(a) = 1 - \prod_{(b,a) \in \mathcal{R}} (1 - Deg_{\mathbf{F}}^{\sigma_{DF}}(b))$$

$$- v_s(a) = 1 - \prod_{(b,a) \in \mathcal{S}} (1 - Deg_{\mathbf{F}}^{\sigma_{DF}}(b)).$$

Please note that if a does not have any attackers $v_a(a) = 0$. Similarly, if a does not have any supporters $v_s(a) = 0$.

3 A Framework for Agent-based Impact

In this section, we will introduce the framework allowing to study the impact of arguments and agents, i.e. authored wBAF (Section 3.1) and impact semantics (Section 3.2).

3.1 Authored wBAF

We extend the wBAF [2, 13] framework by adding an additional label to each argument representing its author, i.e. the agent that owns it. The intuition of this label is that the agent that first states an argument in the debate is the one that “owns” it. For simplicity, our new framework only accommodates one author per argument but the approach of this paper can be easily extended to multiple authors per arguments by considering that each agent owns only one part of each argument.

Definition 7 (awBAF). *An authored wBAF (awBAF) is a tuple $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ where:*

- \mathcal{A} is a finite set of arguments
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a set of binary attacks
- $\mathcal{S} \subseteq \mathcal{A} \times \mathcal{A}$ is a set of binary supports
- w is a weighting on \mathcal{A}
- P is a finite set of agents such that $\mathcal{A} \cap P = \emptyset$
- $\mathcal{Y} : \mathcal{A} \rightarrow P$ is a function that associates to each argument, the agent that owns it.

Please note that $p \in P$ is the author of $a \in \mathcal{A}$ iff $\mathcal{Y}(a) = p$. Similarly, the set of arguments of an agent p is $\mathcal{A}_p = \{a \in \mathcal{A} \mid \mathcal{Y}(a) = p\}$. $P(\mathbf{A})$ is the set of agents owning arguments in \mathbf{A} , i.e. $P(\mathbf{A}) = \{p \in P \mid \text{there exists } a \in \mathcal{A} \text{ s.t. } \mathcal{Y}(a) = p\}$. Given $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w', P', \mathcal{Y}' \rangle$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, $\mathbf{A} \oplus \mathbf{A}'$ is $\langle \mathcal{A}'', \mathcal{R}'', \mathcal{S}'', w'', P'', \mathcal{Y}'' \rangle$ such that $\mathcal{A}'' = \mathcal{A}' \cup \mathcal{A}$, $\mathcal{R}'' = \mathcal{R} \cup \mathcal{R}'$, $\mathcal{S}'' = \mathcal{S} \cup \mathcal{S}'$, $P'' = P \cup P'$ and for all $a \in \mathcal{A}' \cup \mathcal{A}$, the following holds $w''(a) = w(a)$ if $a \in \mathcal{A}$ or $w''(a) = w'(a)$ if $a \in \mathcal{A}'$ and $\mathcal{Y}''(a) = \mathcal{Y}(a)$ if $a \in \mathcal{A}$ or $\mathcal{Y}''(a) = \mathcal{Y}'(a)$ if $a \in \mathcal{A}'$.

Quite naturally, disregarding the authors of an awBAF allows to obtain what we call the *induced wBAF*.

Definition 8 (Induced wBAF). Given a awBAF $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, we call induced wBAF of \mathbf{A} the wBAF $\mathbf{F}_{\mathbf{A}} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$.

Example 1. The awBAF corresponding to the example in introduction, and represented in Fig. 1 $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ such that $\mathcal{A} = \{a_0, a_1, \dots, a_6\}$, $\mathcal{R} = \{(a_1, a_0), (a_4, a_1), (a_6, a_2)\}$, $\mathcal{S} = \{(a_3, a_1), (a_2, a_0), (a_5, a_2)\}$, $P = \{p_1, p_2, p_3\}$.

Every square node represents an argument with its weight. Next to each square node, the corresponding author is represented, e.g. the author of a_0 is p_1 . A dashed green arrow represents a support and a solid red arrow represents an attack. In this example, agent p_1 is trying to increase the acceptability of his own argument a_0 by adding the supporting argument a_2 . On the contrary, p_2 is trying to decrease the acceptability of a_0 by using a_1, a_3 and a_6 . Lastly, p_3 both decreases and increases the acceptability of a_0 with a_5 and a_4 respectively.

Definition 9 (Isomorphism). Given two awBAFs $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w', P', \mathcal{Y}' \rangle$, we say that f is an isomorphism from \mathbf{A} to \mathbf{A}' iff there are two isomorphisms f_1 (from \mathcal{A} to \mathcal{A}') and f_2 (from P to P') such that all the following items are satisfied:

- for every $a, a' \in \mathcal{A}$, $(a, a') \in \mathcal{R}$ iff $(f_1(a), f_1(a')) \in \mathcal{R}'$
- for every $a, a' \in \mathcal{A}$, $(a, a') \in \mathcal{S}$ iff $(f_1(a), f_1(a')) \in \mathcal{S}'$
- for every $a \in \mathcal{A}$, $w(a) = w'(f_1(a))$
- for every $a \in \mathcal{A}$, $f_2(\mathcal{Y}(a)) = \mathcal{Y}'(f_1(a))$

In the next section, we will provide the general definition of an *agent-based impact semantics* and some desirable principles to assess the “quality” of such semantics, i.e. how accurate they are in depicting the attack and support relations of the awBAF.

3.2 Agent-based impact semantics

As shown by Example 1 and Fig. 1, quantifying the impact that an agent has on the acceptability degree of an argument is not straightforward, especially for complex argumentation graphs. In this section, we define the notion of *agent-based impact semantics* and provide some desirable principles for it. Finally, we provide the first agent-based impact semantics.

Definition 10 (Agent-based impact semantics). *An (agent-based) impact semantics is a function δ that associates to each awBAF $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, a in \mathcal{A} , and $p \in P$, a positive real number $\delta(\mathbf{A}, p, a)$. $\delta(\mathbf{A}, p_2, a) \geq \delta(\mathbf{A}, p_1, a)$ means that p_2 impacts at least as much as p_1 on the acceptability of a .*

In the rest of this subsection, we propose the first set of desirable principles for an (agent-based) impact semantics inspired by the principles for graded semantics in existing work on *wBAF* [2].

The *Anonymity* principle states that the agent-based impact semantics should not be defined based on the names of the arguments or the agents.

Principle 1 (Anonymity) *An impact semantics δ satisfies Anonymity iff for any two awBAFs $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and \mathbf{A}' such that there exists an isomorphism from \mathbf{A} to \mathbf{A}' (and the corresponding isomorphisms f_1 and f_2 between the arguments and agents respectively), for any $a \in \mathcal{A}$, we have that for all $p_1, p_2 \in P$, $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ iff $\delta(\mathbf{A}', f_2(p_1), f_1(a)) \leq \delta(\mathbf{A}', f_2(p_2), f_1(a))$.*

The following principle states that adding a dummy argument to an agent (an argument not involved in any attacks or supports) should keep the order of the agent impacts unchanged. Please note that a stricter variant of the dummy principle would imply that adding dummy arguments would keep the (agent-based) impact values unchanged.

Principle 2 (Dummy) *An impact semantics δ satisfies Dummy iff for any awBAF $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, for any $p \in P$, for any $a \in \mathcal{A}$ and for any $r \notin \mathcal{A}$ such that $\mathbf{A}' = \langle \mathcal{A} \cup \{r\}, \mathcal{R}, \mathcal{S}, w', P, \mathcal{Y}' \rangle$, where for all $b \in \mathcal{A}$, $w'(b) = w(b)$, $\mathcal{Y}'(b) = \mathcal{Y}(b)$ and $\mathcal{Y}'(r) = p$, for all $p_1, p_2 \in P$, if $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ then $\delta(\mathbf{A}', p_1, a) \leq \delta(\mathbf{A}', p_2, a)$.*

The *Silent Authorship* principle states that an agent without arguments should have less impact than other agents. This is important as it highlights that only the agents that own arguments can affect the (agent-based) impact semantics.

Principle 3 (Silent Authorship) *An impact semantics δ satisfies Silent Authorship iff for any awBAF $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, for any $a \in \mathcal{A}$, for any $p \notin P(\mathbf{A})$ and for any $p' \in P$, it holds that $\delta(\mathbf{A}, p, a) \leq \delta(\mathbf{A}, p', a)$*

Please note that silent authorship also implies that all the agents without arguments will always have the same amount of impact, and this amount will always be minimal.

Directionality states that the order of the agent impacts on a particular argument should only be based on its incoming attacks and supports.

Principle 4 (Directionality) *An impact semantics δ satisfies Directionality iff for any awBAFs $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w, P, \mathcal{Y} \rangle$ with $a, b, x \in \mathcal{A}$, $p_1, p_2 \in P$ such that:*

- $\delta(\mathbf{A}, p_1, x) \leq \delta(\mathbf{A}, p_2, x)$
- $\mathcal{R} \subseteq \mathcal{R}'$, $\mathcal{S} \subseteq \mathcal{S}'$ and $\mathcal{R}' \cup \mathcal{S}' = \mathcal{R} \cup \mathcal{S} \cup \{(a, b)\}$,
- *there is no path from b to x*

then $\delta(\mathbf{A}', p_1, x) \leq \delta(\mathbf{A}', p_2, x)$

The independence principle states that the impact of an agent on an argument a should be independent of any arguments (and thus agents) that are not connected to a .

Principle 5 (Independence) *An impact semantics δ satisfies Independence iff for any awBAFs $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w', P', \mathcal{Y}' \rangle$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$, it holds that for every $p_1, p_2 \in P$ such that $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ then $\delta(\mathbf{A} \oplus \mathbf{A}', p_1, a) \leq \delta(\mathbf{A} \oplus \mathbf{A}', p_2, a)$.*

4 Instantiating Impact Semantics

4.1 Degree-based Argument Impact

In this section, we provide the first instantiations of (agent-based) impact semantics for awBAF by using graded semantics. We start by defining the notion of *degree-based impact semantics* of an argument in a wBAF. Intuitively, the degree-based *impact* of an argument a on another argument r is the difference between acceptability degrees of r with or without a .

Definition 11 (Degree-based Argument Impact). *Let $\mathbf{F} = \langle \mathcal{A}, w, \mathcal{R}, \mathcal{S} \rangle$ be a wBAF with $r, a \in \mathcal{A}$. Let σ be a graded semantics for wBAF. The impact of a on r (w.r.t. \mathbf{F}) is $\text{imp}_{\mathbf{F}}^{\sigma}(a, r) = |\text{Deg}_{\mathbf{F}}^{\sigma}(r) - \text{Deg}_{\mathbf{F} \setminus \{a\}}^{\sigma}(r)|$.*

Please note that for every $r \in \mathcal{A}$, the impact of r on itself is 0.

We can now define the way to aggregate multiple argument impacts to represent the impact of an agent's arguments.

Definition 12 (Aggregation function). *An aggregation function is a function $\text{agg} : [0, 1]^n \rightarrow \mathbb{R}$ for $n \in \mathbb{N}$.*

In this paper, we use the standard *average*, *median*, *sum* and *maximum* aggregation functions and introduce the *product* aggregation as follows. Let $X, X' \in [0, 1]^n$ such that $X' = (x_1, x_2, \dots, x_n)$ is the sequence resulting from sorting X in ascending order. We have (*average*) $ave(X) = (\sum_{i=1}^n x_i)/n$, (*median*) $med(X) = \frac{1}{2}(x_{\lfloor (n+1)/2 \rfloor} + x_{\lceil (n+1)/2 \rceil})$, (*sum*) $sum(X) = \sum_{i=1}^n x_i$, (*maximum*) $max(X) = x_n$, and (*product*) $prod(X) = (\prod_{i=1}^n 1 + x_i) - 1$.

An aggregation function satisfies *fairness aggregation* iff the order of the sequence has no effect on the output value, i.e. for all $X, Y, Z \in [0, 1]^n$ such that Z is the sequence resulting from sorting X or Y in ascending order, then $agg(X) = agg(Y)$. All the aforementioned aggregation functions satisfy fairness aggregation.

We now define the notion of aggregated (agent-based) impact semantics as the aggregated impact of an agent's arguments.

Definition 13 (Aggregated agent-based impact semantics). *Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ be a awBAF, $\mathbf{F}_{\mathbf{A}} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w \rangle$ the induced wBAF, σ a graded semantics, agg an aggregation function and $r \in \mathcal{A}$ an argument. The (aggregated agent-based) impact semantics w.r.t. σ and agg is δ_{agg}^{σ} , s.t. for any $p \in P$, $\delta_{agg}^{\sigma}(\mathbf{A}, p, r) = agg((imp_{\mathbf{F}_{\mathbf{A}}}^{\sigma}(a_1, r), \dots, imp_{\mathbf{F}_{\mathbf{A}}}^{\sigma}(a_n, r)))$, where $\forall a_i, \mathcal{Y}(a_i) = p$.*

4.2 Formal Analysis of the Principles

In this section, we study the similarities and differences of the Euler-based and DF-Quad-based aggregated impact semantics. First, we show, in Example 2, that these two semantics provide quite significant differences in terms of results.

Example 2. Let us consider the awBAF $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, represented in Fig. 2, inspired from [3], where $\mathcal{A} = \{a, b, c, d, \dots, j\}$, $\mathcal{R} = \{(d, a), (d, b), (e, b), (e, c), (f, c), (g, e), (h, f)\}$, $\mathcal{S} = \{(j, i), (i, a), (i, b), (i, c)\}$, $w(a) = w(b) = w(c) = 0.6$, $w(d) = 0.22$, $w(e) = w(f) = 0.4$, $w(g) = 0$, $w(h) = w(j) = 0.99$, $w(i) = 0.1$, $P = \{p_0, p_1, p_2\}$, and \mathcal{Y} is defined as $\mathcal{Y}(a) = \mathcal{Y}(b) = \mathcal{Y}(c) = \mathcal{Y}(i) = \mathcal{Y}(j) = p_0$, $\mathcal{Y}(e) = \mathcal{Y}(f) = p_2$, and $\mathcal{Y}(d) = \mathcal{Y}(g) = \mathcal{Y}(h) = p_1$.

In Table 1, we show the impact of the agents on the acceptability of b , e.g. the value 0.22 means that $\delta_{med}^{\sigma_{DF}}(\mathbf{A}, p_0, b) = 0.22$. From the table, we can see that in the case of the Euler-based aggregated impact semantics, the agent p_2 has the most impact on argument b (for all aggregation functions), whereas in the case of the DF-Quad aggregated impact semantics, the agent p_0 is the one with the most impact. This is caused by the *big jump problem* [3] as the acceptability degree of i will be 0.991 with the DF-Quad semantics whereas with the Euler-based semantics it is only of 0.22. Given that i is connected to b , since it is a supporter, i has a huge impact in the case of $\delta^{\sigma_{DF}}$ compared to $\delta^{\sigma_{EBS}}$. The author of i , p_0 , will hence have a much bigger impact on b in the former case.

In the second part of this section, we analyse the principles satisfied by the two aggregated agent-based impact semantics for each aggregation function. The

	$\delta_x^{\sigma_{EBS}}(\mathbf{A}, p_i, b)$					$\delta_x^{\sigma_{DF}}(\mathbf{A}, p_i, b)$				
	Average	Median	Sum	Product	Max	Average	Median	Sum	Product	Max
p_0	0.01	0.01	0.04	0.04	0.03	0.24	0.22	0.95	1.17	0.5
p_1	0.01	0	0.03	0.03	0.03	0.02	0	0.05	0.05	0.05
p_2	0.03	0.03	0.06	0.06	0.06	0.06	0.06	0.12	0.12	0.12

Table 1. Impact of agents on b for the Euler-based and DF-Quad-based aggregated impact semantics w.r.t. different aggregation functions. The number 1.17 in column $\delta_x^{\sigma_{DF}}(\mathbf{A}, p_i, b)$, sub-column *Product* and row p_0 means that $\delta_{prod}^{\sigma_{DF}}(\mathbf{A}, p_0, b) = 1.17$.

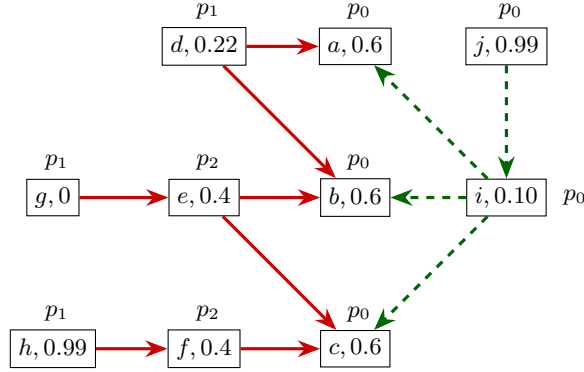


Fig. 2. An awBAF for which Euler-based and DF-Quad-based aggregated impact semantics give different results.

results are summarised in Table 2. The remainder of this section provides the proofs and counter-examples.

Theorem 1. *The anonymity principle is satisfied by the aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and agg is an aggregation function that satisfies fairness aggregation.*

Proof. Let $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$, $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w', P', \mathcal{Y}' \rangle$ be two awBAF such that there exists an isomorphism from \mathbf{A} to \mathbf{A}' (and the corresponding isomorphisms f_1 and f_2 between the arguments and agents respectively).

We show the theorem by contradiction. We assume that there exists $a \in \mathcal{A}$ and $p_1, p_2 \in P(\mathbf{A})$ with $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ but $\delta(\mathbf{A}', f_2(p_2), f_1(a)) > \delta(\mathbf{A}', f_2(p_1), f_1(a))$.

As Euler-based and DF-Quad are only based on the structure of the graph, we have that for every $a' \in \mathcal{A}$, $Deg_{\mathbf{A}}^{\delta}(a') = Deg_{\mathbf{A}'}^{\delta}(f_1(a'))$. Thus, for all $a, r \in \mathcal{A}$ we have that $imp_{\mathbf{A}}^{\sigma}(a, r) = imp_{\mathbf{A}'}^{\sigma}(f_1(a), f_1(r))$. Consequently, for all $p \in P(\mathbf{A})$,

	σ^{EBS}					σ^{DF}				
	Average	Median	Sum	Product	Max	Average	Median	Sum	Product	Max
Anonymity	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Dummy	✗	✗	✓	✓	✓	✗	✗	✓	✓	✓
Silent Auth.	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Directionality	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Independence	✗	✗	✓	✓	✓	✗	✗	✓	✓	✓

Table 2. Principles satisfied by the Euler-based and DF-Quad-based aggregated (agent-based) semantics; ✓ (resp. ✗) indicates that the principle is satisfied (resp. not satisfied).

we have $X_p = \{imp_{\mathcal{A}}^{\sigma}(r, a) \mid \forall r \in \mathcal{A}, \mathcal{Y}(r) = p\}$, $X'_p = \{imp_{\mathcal{A}'}^{\sigma}(r', f_1(a)) \mid \forall r' \in \mathcal{A}', \mathcal{Y}'(r') = f_2(p)\}$ such that $X_p = X'_p$. Thus, since agg satisfies fairness aggregation, for any sequences X_p^1 and X_p^2 obtained on X_p and X'_p respectively, we have $agg(X_p^1) = agg(X_p^2)$. By definition, we have that $\delta(\mathcal{A}', f_2(p_1), f_1(a)) \leq \delta(\mathcal{A}', f_2(p_2), f_1(a))$, contradiction. \square

Theorem 2. *The dummy principle is satisfied by the aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{sum, max, prod\}$.*

Proof. Let $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$, $\mathcal{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, for any $p \in P$, for any $a \in \mathcal{A}$ and for any $r \notin \mathcal{A}$ such that $\mathcal{A}' = \langle \mathcal{A} \cup \{r\}, \mathcal{R}, \mathcal{S}, w', P, \mathcal{Y}' \rangle$, where for all $b \in \mathcal{A}$, $w'(b) = w(b)$, $\mathcal{Y}'(b) = \mathcal{Y}(b)$ and $\mathcal{Y}'(r) = p$.

Let $p_1, p_2 \in P$ such that $\delta(\mathcal{A}, p_1, a) \leq \delta(\mathcal{A}, p_2, a)$, we show that $\delta(\mathcal{A}', p_1, a) \leq \delta(\mathcal{A}', p_2, a)$. Since r does not interact with a , we have that for all $a' \in \mathcal{A}$, $Deg_{\mathcal{A}}^{\delta}(a') = Deg_{\mathcal{A}'}^{\delta}(a')$. Thus, we have that for every $a' \in \mathcal{A}$, $imp_{\mathcal{A}}^{\sigma}(a', a) = imp_{\mathcal{A}'}^{\sigma}(a', a)$ and $imp_{\mathcal{A}'}^{\sigma}(r, a) = 0$ (because σ satisfies independence for graded semantics) [6]. Hence, for $p \in \{p_1, p_2\}$, we have $agg(\langle imp_{\mathcal{A}}^{\sigma}(a, r) \mid \forall a, \mathcal{Y}(a) = p \rangle) = agg(\langle imp_{\mathcal{A}'}^{\sigma}(a, r) \mid \forall a, \mathcal{Y}(a) = p \rangle)$. Hence, $\delta(\mathcal{A}', p_1, a) \leq \delta(\mathcal{A}', p_2, a)$. \square

Please note that the dummy principle is not satisfied by a aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{ave, med\}$. We show the counter-examples below.

Example 3. Let $\mathcal{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ be a awBAF such that $\mathcal{A} = \{a_1, r, a_2\}$, $\mathcal{R} = \{(a_1, r)\}$, $\mathcal{S} = \{(a_2, r)\}$, $w(a_1) = w(a_2) = w(r) = 0.5$, $P = \{p_0, p_1, p_2\}$, $\mathcal{Y}(a_1) = p_1$, $\mathcal{Y}(r) = p_0$ and $\mathcal{Y}(a_2) = p_2$. We define $\mathcal{A}' = \langle \mathcal{A} \cup \{a_3\}, \mathcal{R}, \mathcal{S}, w', P, \mathcal{Y}' \rangle$, where for all $b \in \mathcal{A}$, $w'(b) = w(b)$, $\mathcal{Y}'(b) = \mathcal{Y}(b)$ and $\mathcal{Y}'(a_3) = p_1$ (see Fig. 3).

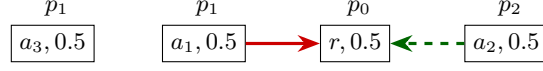


Fig. 3. Counter-example for the satisfaction of the dummy principle when $agg \in \{sum, med\}$ (A' is represented).

$Deg_{\mathbf{A}}^{\sigma_{EBS}}(r) = Deg_{\mathbf{A}'}^{\sigma_{EBS}}(r) = 0.5$, $Deg_{\mathbf{A} \setminus \{a_1\}}^{\sigma_{EBS}}(r) = Deg_{\mathbf{A}' \setminus \{a_1\}}^{\sigma_{EBS}}(r) \simeq 0.589$ and $Deg_{\mathbf{A} \setminus \{a_2\}}^{\sigma_{EBS}}(r) = Deg_{\mathbf{A}' \setminus \{a_2\}}^{\sigma_{EBS}}(r) \simeq 0.425$. Thus, $imp_{\mathbf{A}}^{\sigma_{EBS}}(a_1, r) = imp_{\mathbf{A}'}^{\sigma_{EBS}}(a_1, r) = 0.089$, $imp_{\mathbf{A}}^{\sigma_{EBS}}(a_2, r) = imp_{\mathbf{A}'}^{\sigma_{EBS}}(a_2, r) = 0.075$ and $imp_{\mathbf{A}}^{\sigma_{EBS}}(a_3, r) = 0$. Consequently, $med(\langle 0, 0.089 \rangle) = ave(\langle 0, 0.089 \rangle) = 0.0445$ and $med(\langle 0.075 \rangle) = ave(\langle 0.075 \rangle) = 0.075$. As a result, we have that $\delta_{agg}^{\sigma_{EBS}}(\mathbf{A}, p_2, r) \leq \delta_{agg}^{\sigma_{EBS}}(\mathbf{A}, p_1, r)$ but $\delta_{agg}^{\sigma_{EBS}}(\mathbf{A}', p_2, r) > \delta_{agg}^{\sigma_{EBS}}(\mathbf{A}', p_1, r)$ for $agg \in \{med, ave\}$.

This counter-example also holds for σ_{DF} .

Theorem 3. *The silent authorship principle is satisfied by the aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{ave, med, sum, max, prod\}$*

Proof. This is trivially true, by definition, since an agent with no argument will have an impact of 0, formally for all $p \notin P(\mathbf{A})$ and for all $a \in \mathcal{A}$, $\delta_x^{\sigma}(\mathbf{A}, p, a) = 0$, where $x \in \{ave, med, sum, max, prod\}$ and $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$. \square

Theorem 4. *The directionality principle is satisfied by the aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{ave, med, sum, max, prod\}$*

Proof. Let $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$ and $\mathbf{A}' = \langle \mathcal{A}, \mathcal{R}', \mathcal{S}', w, P, \mathcal{Y} \rangle$ with $a, b, x \in \mathcal{A}$, $p_1, p_2 \in P$ such that $\delta_{agg}^{\sigma_{EBS}}(\mathbf{A}, p_1, x) \leq \delta_{agg}^{\sigma_{EBS}}(\mathbf{A}, p_2, x)$, $\mathcal{R} \subseteq \mathcal{R}'$, $\mathcal{S} \subseteq \mathcal{S}'$ and $\mathcal{R}' \cup \mathcal{S}' = \mathcal{R} \cup \mathcal{S} \cup \{(a, b)\}$ and there is no path from b to x .

We know that σ_{EBS} satisfies directionality for graded semantics [2], thus the acceptability degree of x depends only on the arguments linked to it via a path. This means that for every $u \in \mathcal{A}$ such that u is not linked to x with a path, $imp_{\mathbf{A}'}^{\sigma_{EBS}}(u, x) = 0$ and $imp_{\mathbf{A}}^{\sigma_{EBS}}(u, x) = 0$. Similarly, for every $v \in \mathcal{A}$ such that v is linked to x with a path, $imp_{\mathbf{A}'}^{\sigma_{EBS}}(v, x) = imp_{\mathbf{A}}^{\sigma_{EBS}}(v, x)$. Hence, adding the interaction from a to b in \mathbf{A}' does not change the impact of any argument on x . We conclude that $\delta_{agg}^{\sigma_{EBS}}(\mathbf{A}', p_1, x) \leq \delta_{agg}^{\sigma_{EBS}}(\mathbf{A}', p_2, x)$ for $agg \in \{ave, med, sum, max, prod\}$.

This reasoning is valid for any graded semantics that satisfies directionality, hence it covers σ_{DF} as well. \square

Theorem 5. *The independence principle is satisfied by the aggregated impact semantics δ_{agg}^{σ} where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{sum, max, prod\}$*

Proof. We first show, by contradiction, that if a aggregated impact semantics δ satisfies dummy and directionality then it satisfies independence. Assume that δ satisfies dummy and directionality but not independence. This means that there exists two awBAFs $\mathbf{A} = \langle \mathcal{A}, \mathcal{R}, \mathcal{S}, w, P, \mathcal{Y} \rangle$, $\mathbf{A}' = \langle \mathcal{A}', \mathcal{R}', \mathcal{S}', w', P', \mathcal{Y}' \rangle$ such that $\mathcal{A} \cap \mathcal{A}' = \emptyset$ and there exists $p_1, p_2 \in P$ such that $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ and $\delta(\mathbf{A} \oplus \mathbf{A}', p_1, a) > \delta(\mathbf{A} \oplus \mathbf{A}', p_2, a)$.

We know that by adding an argument from \mathcal{A}' to \mathbf{A} (without attacks nor supports), we have $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ (by dummy). Thus, we can add all arguments from \mathcal{A}' to \mathbf{A} without any changes on the impact. Then, we know that by adding all attacks from \mathcal{R}' and all supports from \mathcal{S}' to \mathbf{A} , we have $\delta(\mathbf{A}, p_1, a) \leq \delta(\mathbf{A}, p_2, a)$ (by directionality since there are no paths from arguments in \mathcal{A}' to $a \in \mathcal{A}$). The resulting graph is $\mathbf{A} \oplus \mathbf{A}'$ and $\delta(\mathbf{A} \oplus \mathbf{A}', p_1, a) \leq \delta(\mathbf{A} \oplus \mathbf{A}', p_2, a)$, contradiction.

Thus, from Theorems 2 and 4, δ_{agg}^σ satisfies independence, for $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{sum, max, prod\}$.

Please refer to Example 3 for a counter-example for δ_{agg}^σ , where $\sigma \in \{\sigma_{EBS}, \sigma_{DF}\}$ and $agg \in \{ave, med\}$, such that $\mathbf{A} = \langle \{r, a_1, a_2\}, \{(a_1, r)\}, \{a_2, r\}, w, \{p_1, p_2\}, \mathcal{Y} \rangle$, with $\mathcal{Y}(r) = p_0$, $\mathcal{Y}(a_1) = p_1$ and $\mathcal{Y}(a_2) = p_2$, and $\mathbf{A}' = \langle \{a_3\}, \emptyset, \emptyset, w, \{p_1\}, \mathcal{Y}' \rangle$ with $\mathcal{Y}'(a_3) = p_1$. \square

5 Discussion

In this paper, we have presented a novel way to rank agents with respect to their impact on the argumentation debate. Formally, we generalised the weighted bipolar framework, for the multi-agent context, by labelling each argument with an *author* and defined the notion of agent-based impact semantics. Those semantics allow to rank the agents from the most impactful to the least for a particular argument. We introduced a new framework, called *aggregated* agent-based impact semantics, to instantiate such impact semantics by using an aggregation function as well as a graded semantics (for bipolar argumentation frameworks). To illustrate, we used two classical graded semantics to instantiate this framework, the DF-Quad [15] and Euler-based semantics [1]. Finally, in order to assess the desirability of the instantiated impact semantics, we defined intuitive principles for such impact semantics, and assessed which principles were satisfied.

As far as we know, the only work that is similar to our approach is the work of Todd Robinson [16]. In this paper, the author introduces the notion of information value in Argumentation to identify the most “important” arguments. He uses two functions called *value of observed* and *value of observation* to represent respectively the value of arguments currently in the framework and the value of adding a new argument to the framework. His framework does not take into account the notion of Authorship which is essential in multi-agent contexts. Moreover, this framework is based on utility functions defined on extensions rather than graded semantics. This however could open up interesting avenues of research by combining the two approaches.

There are multiple possible future research avenues to extend our approach. First, we can study how the aggregated agent-based impact semantics behaves when it is instantiated with other graded semantics such as Potyka’s *continuous modular semantics* [14] or quadratic energy model [12]. Second, we can broaden up this research by considering more general argumentation frameworks with additional features. For example, using a temporal argumentation framework [4, 7], one can determine avant-gardist leaders that have an early influence on a specific argument, or using argumentation frameworks with sets of attacking arguments (SETAFs) to add expressivity to the attack/support relation [18, 9]. Links can also be drawn from previous research in Argumentation Dynamics [5, 17] to determine the effect of, say, a particular expansion (i.e. the addition of some arguments) on the impact of agents on particular arguments. This would allow to assess the interest, in terms of impact on the discussion, for an agent to enunciate some arguments.

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