

## Switching off the heating when you leave the house - is it worth it?

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If you turn off the heating when you leave your house then the house cools due to continuing loss of energy and you have to make up for this cooling when you come back. Is it worth turning off the heating? It's all a matter of the numbers involved. This piece provides some figures.

Because of heat leakage from the house (walls, windows, roof, floors) power  $P$  needs to be provided to maintain the temperature within at a value of  $T$  degrees when the outside temperature is  $T_{out}$ . In practice the power is usually provided in bursts by a boiler system capable of delivering power at a greater rate so that  $P$  represents the average power needed. If the temperature is maintained pretty steady at  $T$ , then  $P$  also represents the rate of leakage of heat from the house. For moderate values of  $(T - T_{out})$ , the power needed will be proportional to  $(T - T_{out})$ , i.e.  $P = \alpha(T - T_{out})$  where  $\alpha$  is a constant of proportionality. For example  $\alpha$  might be 1 kW per  $C^\circ$ , though as we'll see there is no need to know its value. If the heating is turned off, the house will lose energy at the rate of  $P$ , which decreases as the house cools.

The temperature drop in the house when energy is lost depends on the heat capacity of the house,  $C$ . If the house loses a little energy  $dE$  then the temperature will drop by  $CdT$ . In any short time  $dt$ , the energy lost will be  $Pdt$  and this will produce a temperature drop  $CdT = Cd(T - T_{out})$ . Hence  $\alpha(T - T_{out})dt = Cd(T - T_{out})$ , showing that the rate of change of  $(T - T_{out})$  depends on  $(T - T_{out})$  itself. This results in an exponential decay of excess temperature of the house over the environment once the heating is turned off, i.e.  $(T - T_{out}) = (T_0 - T_{out})e^{-(\alpha/C)t}$ , where  $T_0$  is the temperature inside when the heating is switched off. The 'time constant' of the decay in temperature can be called  $\tau = C/\alpha$ . The constant  $\tau$  controls how quickly the house cools:  $(T - T_{out}) = (T_0 - T_{out})e^{-t/\tau}$ .

- a) If the house is well insulated and the heating is switched off for a short time the temperature inside won't fall much but during the time that the heating is off the house will lose energy at almost the same rate it was being supplied. To make good that lost energy when the heating is switched back on the same amount of energy will be used as would have been used to maintain the temperature. i.e. there is little energy saving in switching off the heating.
- b) If the house is not so well insulated and the temperature drops significantly then because the average temperature when the heating is off is less than when the heating is on, then the house loses less energy to the environment with the heating off. However, that energy which has been lost must be made good when the heating is switched back on again in order to raise the temperature to its former level. A measure of the energy that must be made good is the length of time the normally operating heating system would need to run to deliver this amount of energy.

For example, if the house has been allowed to cool to the environmental temperature  $T_{out}$  then the total energy required to warm it up again is  $C(T_0 - T_{out})$ . The boiler normally provides energy at the rate  $\alpha(T_0 - T_{out})$  and hence the time  $t'$  needed for the boiler working at its normal rate to supply an amount of energy that equals the lost energy is given by  $\alpha(T_0 - T_{out})t' = C(T_0 - T_{out})$  and hence  $t' = C/\alpha = \tau$ . This is the same as the time constant of the exponential decay when the heating is turned off, i.e. the time for the

excess temperature of the interior over that of the outside to fall to  $1/e$  of its value, or 0.368 of its value. [For those who aren't following the mathematics,  $e$  is the well-known mathematical constant whose value is 2.71828]. For example to take typical winter figures for our climate, if  $(T_0 - T_{out})$  is  $15^\circ C$  then the time  $t'$  is the time taken for  $(T - T_{out})$  to fall to  $5.5^\circ C$ . If this time is 16 hours (an example figure) then the same energy would be taken to heat a completely cold house as would normally be used to keep the house warm for 16 hours, or a day with the heating left on. In fact the house will heat up much more quickly than this because the boiler will be on most of the time and not on intermittently as it normally is.

The time constant of the house can be estimated without having to leave the heating off for long enough for the temperature difference between inside and outside reduce to  $1/e$  of its normal value. E.g. suppose the house temperature cools from  $20^\circ C$  to  $15^\circ C$  in 4 hours when the outside temperate is  $5^\circ C$ . Putting these numbers into the relationship on page 1 that  $(T - T_{out}) = (T_0 - T_{out})e^{-t/\tau}$  gives  $10 = 15e^{-4/\tau}$ . Hence  $\ln(10/15) = -4/\tau$ . Rearranging this gives  $\tau = 9.9$  hours, say 10 hours in round figures. The same method can be used for any other temperatures and times.

Suppose you don't switch off the heating for long enough for the house to cool down completely. Let's say you go out for  $t = 4$  hours. The excess temperature in the house falls to  $(T - T_{out}) = (T_0 - T_{out})e^{-4/\tau}$ , as in the example in the previous paragraph, where the time constant  $\tau$  is measured in hours. Using the earlier example of  $\tau = 16$  and  $(T_0 - T_{out}) = 15^\circ C$  gives the new inside temperature of the house as  $(T - T_{out}) = 15e^{-4/16} = 11.7^\circ C$ . If  $T_{out}$  is  $5^\circ C$ , as before, then the house temperature has fallen to  $16.7^\circ C$ . Remembering that at any given temperature the energy leakage rate is  $\alpha(T - T_{out})$ , the total energy that has leaked out is

$$\begin{aligned} \int_0^4 \alpha(T - T_{out}) dt &= \alpha \int_0^4 (T_0 - T_{out}) e^{-t/\tau} dt = C(T_0 - T_{out})[1 - e^{-4/\tau}] \\ &= C(T_0 - T_{out}) \times 0.221. \end{aligned}$$

This energy would normally be provided at the rate  $\alpha(T - T_{out})$  and hence the time  $t''$  of normal boiler activity that would supply this energy is  $\alpha(T_0 - T_{out})t'' = C(T_0 - T_{out}) \times 0.221$  giving  $t'' = 0.221\tau = 0.221 \times 16 = 3.5$  h.

What this means with these example figures is that if you go out for 4 hours and turn the heating off then to make good the heat lost by the house the boiler needs to run for as much as it would do for 3.5 hours if you'd left the heat on. It may reheat the house in an hour-and-a-half or even an hour, depending on its power, the water temperature in the radiators and their area, but for saving half-an-hour of normal running you have the inconvenience of living in a colder house until it does so. Maybe it's worth it, maybe not, but the energy saved is not great.

The working above shows how you can get the answer for a different switch-off period.

The energy leak-out with the heating off depends on  $\int_0^t e^{-t/\tau} dt \propto [1 - e^{-t/\tau}]$  and hence the

normal boiler time saved is  $t - \tau[1 - e^{-t/\tau}]$ . If the heating were off for 8 hours, say 8 am to 4 pm, then  $[1 - e^{-8/\tau}] = 0.393$  and the time  $t''$  of normal boiler activity to deliver the equivalent heat lost is  $6.3$  h ( $0.693 \times 16$ ), so  $1.7$  h of normal use time is saved. In that 'normal use' the boiler is going to be off for part of the time, perhaps half the time to take

an example figure, so switching the heating off all day will in fact save some 40 minutes of active boiler time if that is the case.

I've used example figures above but the general conclusion is that if you switch off the heating for  $t'$  hours then the heating needs to be switched on for  $\tau(1-e^{-t'/\tau})$  hours to make up for the lost heat. So you think the heating's been off for  $t'$  hours but you've actually only saved it being on for  $t' - \tau(1-e^{-t'/\tau})$  hours. Even with long  $t'$ , the saving is pretty small.

For example, using the figures for my house that has  $\tau = 30$  hours, switching off the heating for 8 hours actually saves only 1 hour of having the heating switched on. Switching off for 4 hours saves 15 minutes. It is hardly worth it. The saving in hours doesn't depend on the outside temperature. I now have a thermostat that records the actual temperature and for a typical winter's day the temperature falls at about 0.5 degree Celsius per hour when the heating is switched off and rises at 1 degree per hour when it is switched on again.

You may think I've gone on long enough on this topic but if you've skipped some of the previous paragraphs then this one may make it clearer. In short, the total energy used during a day is proportional to the average excess temperature of the house over a day. If you switch off the heating for a few hours and the house starts to cool, then it may end up a few degrees cooler but that won't change the average excess temperature over a day by much. Hence it won't change your heating bill by much.

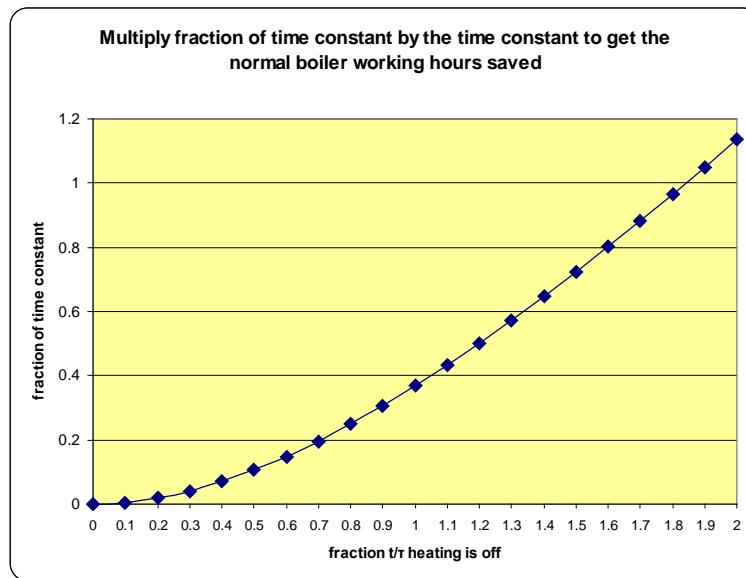
The statement above is borne out by the mathematics. The energy used in a short time  $dt$  is  $P(t)dt$  where I've made it clear that  $P$  may vary with time. Over a length of time  $L$  (say 24 hours), then the total energy used is  $\int_0^L P(t)dt$ . Remembering that  $P(t) = \alpha(T(t) - T_{out})$ ,

and the average temperature  $\bar{T}$  over a period  $L$  is  $\int_0^L T(t)dt / L$ , then the total energy used in length of time  $L$  is  $\int_0^L \alpha(T(t) - T_{out})dt = \alpha L \left( \bar{T} - T_{out} \right)$ .

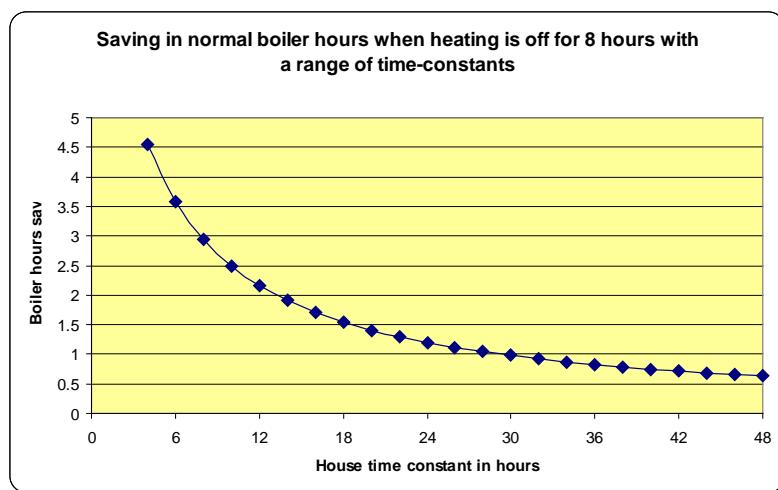
### Conclusions

- 1) The important factor in determining any fuel saving from switching off house heating when you are out is the cooling time-constant of the house (symbol  $\tau$  in the working above). This is determined by the ratio of the specific heat of the house (how much heat energy it takes to change the house temperature by 1° C) and the heat leakage constant (the rate of loss of energy per degree Celsius difference between inside and outside temperatures). The cooling time-constant is equal to the time it takes for the excess temperature of the inside compared with the outside to drop to 0.368 (1/e) of its initial value.
- 2) If your house is well insulated (the temperature decay time  $\tau$  when you switch the boiler off is large) then you save very little energy by turning off the heating when you go out for less than a day.

- 3) The graph here lets you read off the saving in boiler hours when you switch off for a given time, once you know the time constant of your house. The x-axis is the fraction of the time constant that your heating is off for. The y axis the time you save expressed as a fraction of the time constant. For example, if you have switched off for 0.4 time constants, then the fraction of boiler time saved will be 0.07 of the time constant.



- 4) The second graph shows for a range of time constants how much boiler-on-time will be saved if you switch off for 8 hours. If your house is not so well insulated ( $\tau$  is smaller), then you save more energy. If your house is very well insulated, then you save only a little. For example, with a short time constant of 6 hours you will save 3.6 hours of boiler-on-time



but with a time constant of 30 hours you will save barely 1 hour. In short, you only save fuel if your house cools significantly when the heating is off. The shop owner of a medium or large sized premises who needs to have the shop up to temperature at opening time will save little fuel by turning the heating off at night. They can't afford to let the temperature drop much overnight since the time taken to get it back is likely to be hours and the fuel used almost as much as they 'saved' by turning the heating off.

- 5) I'll add another conclusion that all the above does not need to be applied to the house as a whole. I'll give two examples applying the ideas to a well-lagged hot water tank. Suppose the insulation is such that the cooling time constant was  $\tau = 50$  hours. Is it worth switching off the water heating when you are out of the house for 10 hours every weekday? Suppose the thermostat is set to  $60^\circ C$  and the temperature outside the tank that is stored in a cupboard is  $20^\circ C$ . In 10 hours the water temperature has fallen to about  $53^\circ C$ , still very usable. The energy needed to reheat the water is the same as the energy supplied to the boiler over 9 hours (with no water being used), so you have saved an hour of keeping the water hot. That said, in most houses the hot water tank is located well within the house so the energy leaking from the tank is not wasted but goes into the house. The tank acts like a small radiator. If the tank holds 120 litres of water whose

temperature has dropped by  $7^\circ\text{C}$  then it has lost  $7*120*4.12*1000 \approx 3.5 \text{ MJ}$  of energy in 36000 seconds. That's equivalent to a 96 W radiator, a very modest addition of heat energy to the house.

Taking another example, my mother-in-law had her insulated hot water tank in the garage. The thermostat was normally set to  $60^\circ\text{C}$ . Suppose the cooling time constant was  $\tau = 50$  hours. She goes away for a week. Will she save much energy by resetting the thermostat to  $40^\circ\text{C}$  when the outside temperature is  $10^\circ\text{C}$ ? In this scenario, the adjacent oil-fired boiler maintains the set constant temperature. For simplicity, assume the water starts the week at its target temperature.

From the paragraphs above, the tank normally loses energy at the rate  $\alpha(60 - 10) = 50\alpha$ . With the reduced thermostat, the rate of loss is  $\alpha(40 - 10) = 30\alpha$ . Hence the saving is at a rate of  $20\alpha$ . How big is  $\alpha$ ? From paragraph 2,  $\alpha = C/\tau$ . The heat capacity  $C$  mainly comes from the water, say 120 litres. Hence  $C \approx 0.5 \text{ MJ K}^{-1}$ .  $\tau = 50 \times 3600 = 180000 \text{ s}$ . Hence  $\alpha \approx 3 \text{ W}$ . Therefore the power saved,  $20\alpha$ , is about 60 W. This is not much, coming directly from the adjacent boiler, but over a week adds up to over 36 MJ (there are 604800 seconds in a week). Converting into litres of central heating fuel and allowing some inefficiency, the saving is less than 2 litres of fuel. A saving of 60W, though, would be significant if her energy came from solar power. You will probably have spotted that although the energy needed to maintain the water temperature depends on the outside temperature, the energy saved by reducing the thermostat from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  does not, which is slightly unexpected.

### *Further considerations*

Is the time constant  $\tau$  of a house always the same, independent of the weather conditions? In my experience, wind makes a significant difference to the rate at which a house cools. Walls, roof spaces and under-floor spaces need ventilation to reduce the risk of condensation and rot. Strong winds, though, can remove warm air from a house faster than the design rate and quickly bring cool outside air to parts of the house that normally provide some thermal insulation. In short, the time constant for cooling can be noticeably reduced by strong wind, experience suggests by as much as a factor of 2. As I write this, snow is piled on the roof of our house, undoubtedly changing the rate at which heat escapes from the roof, reducing it from what it would be if the sub-zero outside temperature was in direct contact with the roof. However, there is no snow on the walls or under the house and I suspect that the added insulation provided by the snow does not make much overall difference. Likewise, there is no obvious change in the time constant when the house is wet with rain. The effect of changing outside temperature is built into the analysis so apart from leaving doors and windows wide open, wind is likely to be the biggest variable influence on the cooling time constant of the house.

The most conspicuous approximation to reality in the forgoing analysis is the assumption of a constant outside temperature. The assumption is more nearly valid in winter, when heating is more of an issue, than in summer. In this part of the world at a coastal location, typical daily temperature fluctuations in winter may be 4 degrees Celsius, in summer over 10 degrees. With no internal heating, the heat loss will now depend on when in the cycle of daily temperature change the heat is switched off. Consider the case of switching off the heating overnight, say for 8 hours, during which the temperature drops. Some numerical simulations with example numbers show that the heat loss increases over that under the assumption of

constant outside temperature, as expected, but so does the energy that the boiler needs to put into the house to maintain the temperature if it is left on. Both increase by similar percentages and hence the number of boiler-on hours saved by turning off the heating is not much different from the earlier figures with constant outside temperature. What appears to be an unrealistic simplifying assumption does not have much effect on the final answer and makes the calculation much simpler.

In short, the previous working and figures seem to me a useful guide on how much energy one can save by turning off the heating and it's not as much as most people would expect.

The same idea of exponential drop in temperature applies to individual objects when the heating is turned off, such as the hot water pipes under the floor or within the walls. Their time constant will depend on the lagging, with good lagging increasing the time constant considerably. Hot water pipes start off at a much higher temperature than the house, perhaps 60° C or 70° C and cool quickly but if the outside temperature is only just below freezing they will take a long time to cool to 0° C and even longer to freeze because the latent heat of formation of ice, the heat that must be extracted from water at 0° C to form ice, is as much as the heat that has to be removed from water at 80° C to cool it to 0° C. The problem with pipes is usually an overlooked unlagged section that may be cooled by a sub-zero wind, reducing the cooling time constant to less than an hour. If there is no water flow, conduction of heat along the pipe from the warmer neighbouring sections is a very slow process and won't save this one section from freezing overnight. Freezing water expands, creating a significant risk of cracking the pipe or opening a joint and creating a leak when the ice eventually melts.

#### *Appendix 1: on heat loss - U & R values, etc.*

Heating a house is like filling a bath with the plug out. As heat is put in, it leaks out through walls, floor and ceiling. With no plug, the bath drains at a rate that depends on the depth of water. Likewise, a house loses heat at a rate that depends on how much the inside temperature is above the outside temperature. The U value is a measure of the rate of loss of heat energy for every square metre and for each degree Celsius of excess internal temperature. U is measured in  $\text{W m}^{-2} \text{ K}^{-1}$  (1 degree K interval being the same as 1° C). U values can be different for walls, ceiling and floors. What counts is the total heat loss from the house as a whole, adding over all the floors, walls and ceilings.

Houses come in many shapes but assuming that the attic is unheated and the basement or sub-floor area isn't heated then the heated living space is probably not much different from a rectangular box. For the sake of some example figures, take a specimen house living space to be 2.5 m high by 10 m long by 7 m wide. The floor and ceiling areas are each 70 m<sup>2</sup> and the wall areas in total 85 m<sup>2</sup>. This is quite a 'boxy' house but notice that the combined areas of floor and ceiling are not far off twice the wall area. Stopping heat escaping through floor and ceiling is therefore even more important than stopping heat going out through the walls. Putting down carpet on top of underlay, and laying thick glass wool above the ceiling is sensible. If you live in a flat, then unless you are on the ground floor or top floor, your neighbours above and below will likely maintain their flats at a similar temperature to yours. There's likely to be some inter-floor ventilation but you'll lose little heat up or down, making a noticeable difference to your heating bill. You may even have a neighbour on at least one side. Flats take less to heat than detached houses.

To keep the house at a constant temperature, heat has to be provided at the same rate it leaks

out (like keeping the level of the bath the same when water is leaking out). The rate of supply of heat energy is often denoted by the symbol  $Q$ , which has the dimensions of power, the same as our  $P$  used much earlier. For example, if the  $U$  value of all the surfaces was  $0.4 \text{ W m}^{-2} \text{ K}^{-1}$ , then for the example house in the previous paragraph whose living quarters have a total surface area of  $225 \text{ m}^2$ , the heat needed to maintain a temperature difference of  $15^\circ \text{C}$  is given by  $Q = U \times 225 \times 15 = 0.4 \times 225 \times 15 = 1350 \text{ W}$ , or about  $1.4 \text{ kW}$ . Such a house would be considered quite well insulated. The parameter  $\alpha$  mentioned at the beginning of this piece is clearly given by  $\alpha = U \times \text{area}$ .

If the floor, walls and ceiling all have different  $U$  values, then  $\alpha = U_{\text{floor}} \times \text{Area}_{\text{floor}} + U_{\text{walls}} \times \text{Area}_{\text{walls}} + U_{\text{ceiling}} \times \text{Area}_{\text{ceiling}}$ . One needs  $U$  values for each part of the house. Windows let heat out more readily than walls, for example, so a wall may need to be divided further, depending on its construction. There are standard methods of calculation that enable architects to work out the  $U$  value for a given design of building. There's much more about this on the web and taking typical values you can estimate the  $U$  value for your house.

$U$  is essentially the thermal *conductance* of a building component. If a wall is made of various layers then the thermal *resistance*,  $R$ , of each layer is added to obtain the thermal resistance of the whole wall. Resistance and conductance are reciprocal concepts so that  $U = 1/R$ . For example, suppose a wall has a layer of plasterboard, a layer of expanded polystyrene, a layer of air and a layer of brick with a concrete finish. The  $R_{\text{wall}} = R_{\text{concrete}} + R_{\text{brick}} + R_{\text{air}} + R_{\text{eps}} + R_{\text{plasterboard}} + R_{\text{boundary layer}}$  and  $U_{\text{wall}} = 1/R_{\text{wall}}$ . The final term is an allowance for a still layer of air that clings to the wall. The method is just like adding electrical resistances in series. In this picture, the outer concrete is at outside air temperature and the inside is at room temperature, with each layer having part of the temperature drop across it. However, if ventilation, wanted or unwanted, lets outside air into the cavity without heating it, then the first 3 items no longer contribute to the temperature drop and the  $R$  value falls significantly. Indeed, if outside air get to the inside of the expanded polystyrene then the only insulation is the plasterboard (and its wooden supports), the  $R$  value tumbles and the  $U$  value can increase several fold. Hence the building cools quickly and much more heat is needed to maintain the internal temperature. This usually happens to part of the house, at least, in conditions of strong wind. More beneficially, if the sun beats down on an outside wall, then the temperature of the outside concrete can rise to at least that of the room inside and after a while there is no loss of internal heat through this wall.

I'll finish off with some example numbers. You can put in your own numbers. If my house cools from  $20^\circ \text{C}$  to  $16^\circ \text{C}$  in 8 hours when the outside temperature is  $3^\circ \text{C}$ , then:

- What is the decay time constant of the house? From the earlier discussion, the decay constant  $\tau = 8/\ln(17/13) = 30 \text{ hours}$ .
- If I leave the house unheated for 10 hours at night when the outside temperature is  $0^\circ \text{C}$ , what will the temperature have dropped to from its initial value of  $20^\circ \text{C}$ ? From the same discussion,  $T = 20e^{-10/30} = 14.3^\circ \text{C}$ .
- If the area of my outside walls, floor and ceiling is  $700 \text{ m}^2$  (it's a larger than average house) and the average  $U$  value is  $0.4 \text{ W m}^{-2} \text{ K}^{-1}$ , at what rate is the house losing heat energy when it is  $0^\circ \text{C}$  outside and  $20^\circ \text{C}$  inside?  $Q = 0.4 \times 700 \times 20 = 5.6 \text{ kW}$ .
- If the calorific value of my central heating fuel is  $35 \text{ MJ l}^{-1}$  (an estimate) and my heating system is 80% efficient at getting heat energy to the radiators (the boiler may be 90% efficient but there are transmission losses), how much fuel do the above figures suggest my boiler will use if it is on 24 hours a day when the outside temperature is  $0^\circ \text{C}$ ? The boiler must supply  $5.6 \text{ KW}$  for 24 hours, i.e.  $5600 \times 24 \times 3600 \text{ J day}^{-1} = 484 \text{ MJ day}^{-1}$

and hence will use  $484/(0.8 \times 35)$  litres of fuel, i.e. 17.3 litres. This isn't an unreasonable figure. In reality I would turn the thermostat down at night if I left the heating on. It is probably better to reverse the calculation. If I could determine the fuel usage, I could use this to find the U value for the whole house.

- e) If my central heating boiler can deliver 25 kW to the radiators in the house, what fraction of the time must it be on to maintain the temperature at 20 °C? Fraction is  $5.6/25 = 0.22$ , or 22% of the time.
- f) What are the values of  $\alpha$  and C for the house?  $\alpha = U \times \text{area} = 0.4 \times 700 = 280 \text{ W K}^{-1}$ ;  $C = \alpha\tau = 280 \times 30 \times 3600 \text{ J K}^{-1} = 3.02 \times 10^7 \text{ J K}^{-1}$ . Typical specific heat capacities are in the region of  $1 \text{ kJ K}^{-1} \text{ kg}^{-1}$  and hence this value of C implies a mass of  $30 \times 10^3 \text{ kg}$  (i.e. 30 tonnes) of material that heat and cool daily. This is a representative figure for a complex situation.

The story above relates to houses of traditional design in which heat flow is entirely passive. There is a huge and at the moment little tapped market for more advanced building design in which active heat exchange recycles some of the heat that would leak out in a passive design, while still allowing the air changes needed for healthy living. There is also a huge untapped scope for utilising solar energy falling on building walls and roofs.

#### *Appendix 2: freezing pipes*

I'm adding this piece during an exceptionally cold spell in this part of the world with night-time temperatures falling below -10 °C. If the outside temperature is just below freezing, say -1 °C, then pipes exposed to the air are very much less likely to freeze than if the outside temperature is well below freezing, say -10 °C. The reason is the exponentially decreasing rate of cooling of a warm body as its temperature approaches the outside temperature. The following numbers illustrate what is happening.

For example, suppose a poorly insulated stretch of heating pipe cools with a time constant of half-an-hour. Then the page 1 expression for the change in temperature of the pipe with time is  $(T - T_{\text{out}}) = (T_0 - T_{\text{out}})e^{-t/\tau}$  with  $\tau = 0.5$ . Suppose the pipe is at 50 °C when the heating is switched off. Let's take the outside temperature  $T_{\text{out}}$  as -1 °C. The exponential cooling relationship quoted tells us that after half-an-hour the pipe is at 18.8 °C and after only about 2 hours ( $4\tau$ ) its temperature has fallen to 0 °C. This looks like trouble on a cold winter's night, but wait! The latent heat that has to be extracted from water to freeze it is 80 times its heat capacity and the rate of loss of heat ('P' on page 1) has fallen to  $\alpha \times 1$ . Hence the time taken to lose energy  $80 \times C = 80 \times C / (\alpha \times 1) = 80 \times \tau$  (since  $\tau = C/\alpha$ , also from page 1). i.e. it will take 40 hours to freeze all the water in the pipe. I've assumed that most of the heat energy in the pipe is in the water and this is certainly so for 12 mm diameter heating pipes, one of the standard sizes. What saves the pipe is that the rate of transfer of energy between a pipe and its surroundings when there is only a 1 °C temperature difference between them is very small, and the amount of energy that has to be transferred is large. By morning the pipe will contain some ice but will still be mostly water.

Now look at the case when the outside temperature  $T_{\text{out}}$  is -10 °C. The cooling relationship becomes  $(T - (-10)) = (50 - (-10))e^{-t/0.5}$ . i.e. the pipe will be at 0 °C when  $60/10 = e^{t/0.5}$  and hence  $t = 0.5 \times \ln(6) = 0.9$  (hours). The pipe is still 10 °C warmer than the surroundings of -10 °C and hence loses heat at 10 times the rate when the outside temperate is only -1 °C. The latent heat will have gone in  $8\tau = 4$  hours and therefore within 5 hours of switching off the heating the pipe will be frozen. The moral is: keep the heating on overnight when a sharp

frost is expected but you can turn the thermostat down.

*Appendix 3: on turning down the house thermostat*

Standard ‘green advice’ these days is to save energy by turning down the house thermostat. So how much energy is saved, then, in turning down the thermostat by  $1^\circ\text{C}$ ?

Going back to the very first page, the power needed to keep the house at a temperature  $T$  is given by  $P = \alpha(T - T_{\text{out}})$ . Hence the smaller power needed to keep the house at a temperature of  $(T - 1)$  is just  $\alpha(T - 1 - T_{\text{out}})$ . The difference between these two is the power saved by turning down the thermostat  $1^\circ\text{C}$ . This difference is  $\alpha$ , which has units  $\text{W K}^{-1}$ , independent of the outside temperature. This is the same argument as in the water tank example a few pages back. The power saved as a fraction of the power used to maintain the temperature  $T$  is therefore  $\alpha/(\alpha(T - T_{\text{out}})) = 1/(T - T_{\text{out}})$ .

On a typical winter’s day when  $(T - T_{\text{out}}) = 15^\circ\text{C}$ , the fraction of power saved is  $1/15$ , or as a percentage  $100/15 = 6.7\%$ . On a summer’s day when  $(T - T_{\text{out}}) = 5^\circ\text{C}$ , then the fraction saved is  $1/5 \equiv 20\%$ . In a way these figures are misleading for it is the same quantity of energy that is saved in both winter and summer, namely the energy required to raise the temperature of the house by  $1^\circ\text{C}$ . Using the example figures in appendix 1,  $\alpha = U \times \text{area} = 0.4 \times 700 = 280 \text{ W K}^{-1}$ . The saving in power for a  $1^\circ\text{C}$  drop in house temperature is therefore  $280 \text{ W}$ . This applies to the whole day. Over a time  $t$  the energy saved is  $\alpha t$ . In 24 hours, for example the energy saved is  $280 \times 24 \equiv 6.7 \text{ kW h}$ . If this energy were supplied by electricity, then the saving is about £1 per day at current prices. The figures are for a larger than average house and assume the heating is on all the time but even a smaller figure is likely to come to over £100 per year. One has to get used to a slightly cooler environment inside or perhaps get used to wearing another layer of clothes.

*JSR*