Drone flight – what does basic physics say?

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Preamble

This piece is on my meteorology pages mainly because I haven't anywhere else obvious for it. It's true that meteorology is relevant to drone flying in that wind, rain, fog, dust storms and other weather can easily ground a drone and the drone hobbyist needs to be aware of the weather. In the UK at least, hobby drones must be flown no more than 120 m above ground level, which is high enough to give spectacular birds-eye views but not high enough to allow them to be used to explore the meteorology of the vertical structure of the atmosphere.

Introduction

Basic physics such as the definitions of *momentum, work, kinetic and potential energy, power* and *Newton's laws of motion* tells us how drones work, without knowing any aerodynamics. I'm keen on trying to understand things using basic physics. Newton's laws are simply stated; they are correct but interpreting them in context can be the tricky part. Aeronautic engineers have got to get all the detail exactly right. I'd just like to get a feeling for what's going on and estimate some 'ball park' figures for the power needed for different aspects of flight: hovering, climbing and travelling horizontally, though 'estimate' is an important qualification. Over and above these estimates are all sorts of practicalities such as the drag of the rotors and the turbulence produced that all do need aerodynamics to get the answer exactly right. There's also the efficiency of motors and battery, etc., none of which I'll go into. 'Going where angels fear to tread' is a phrase that comes to mind but nonetheless the power figures derived later are at least an estimate of the minimum power needed.

The following is my take on the basics of 1) hovering, 2) rising up and 3) flying horizontally. This at least allows some comparisons of factors that I have little idea of when actually flying. Estimates are given for the minimum power needed to achieve these for an example drone that's sitting in front of me, a Phantom 3 Standard. That was the original plan but I've added some extra paragraphs on why there is an optimum speed to return to home, on wind speed, on balancing propellers and some miscellaneous comments.

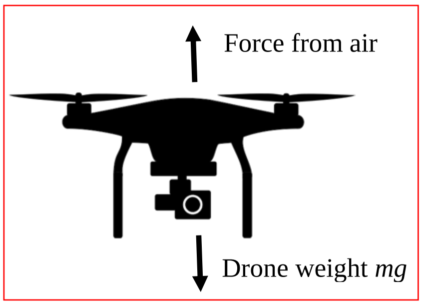
This is version 3, with a better account of air flow. Version 2 removed some approximations made in version 1 and added graphs.

The physics

There's a long-standing joke among physicists and applied mathematicians that you can start a great many estimates by assuming first that the object you're thinking about is spherical. *How many eggs can you pack in a container?* Consider a spherical egg… . *How many cows will fit into a barn?* Lets start by considering a spherical cow… . Well I'm not going to start by assuming a spherical drone but since the same principles apply to all shapes then the actual shape doesn't matter until it comes to getting detailed numbers for the speed achieved, energy needed for flight, etcetera. To get some idea of what is happening I'll work out what the simple physics implies for a Phantom 3 drone, mass 1.22 kg, 4 propellers (aks rotors), each with a length of 240 mm. You can apply the ideas to other drones.

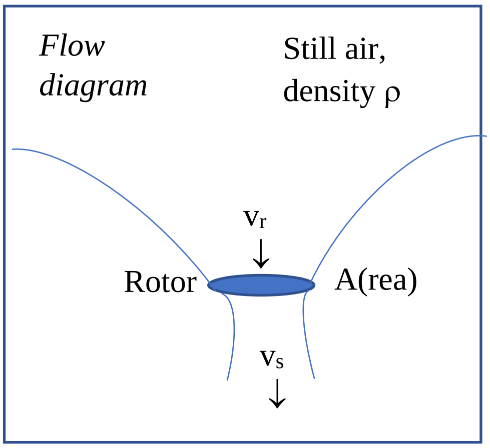
1) **Hovering**

Let the drone have mass *m* (kg). The drone's weight is *mg* (in Newtons), where *g* is the usual gravitational constant (about 10 N kg^{-1}, I'm producing estimates only). I'm standing on the ground and my weight is a force pulling me down but I don't fall downwards because the ground is supplying an equal and opposite upwards force on my shoes. Imagine I'm holding up a drone in still air with rotors switched off. If I let it go it will start to fall to the ground since it has a weight *mg* that acts downwards and there is no balancing upward force. To allow it to hover, the rotors must be switched on at such a speed as to provide an upwards force of *mg*. They do this by pushing air downwards and the air exerts a reaction upwards (Newton's 3rd law). [This 'reaction' is in fact a pressure change across the rotors and is a reminder that much of the force moving a drone is exerted through the rotors. They need to be strong enough to do the job. The rotors pull the drone along and apply their pull through the motor spindles. If any of the threads that holds the plastic rotors onto the metal spindle are not in good enough condition to transmit this pull, then failure will result, and the drone will almost certainly crash. I digress].



Newton's 2nd law says that the force exerted to create motion of the air downwards equals the rate of change of momentum of the air. The air starts at rest (I don't mean by this that there must be no wind, just that there is no significant up-draught or down-draught before the drone is powered up) and the rotor gives the air downward speed. The outer region of the rotor moves faster and is more effective in moving air. To make some amends for this the drone rotor is an odd shape, fatter towards the centre, but narrowed close to the motor where it is ineffective in any case. What happens to that air is largely irrelevant for it is the reaction of the air at the drone that counts.

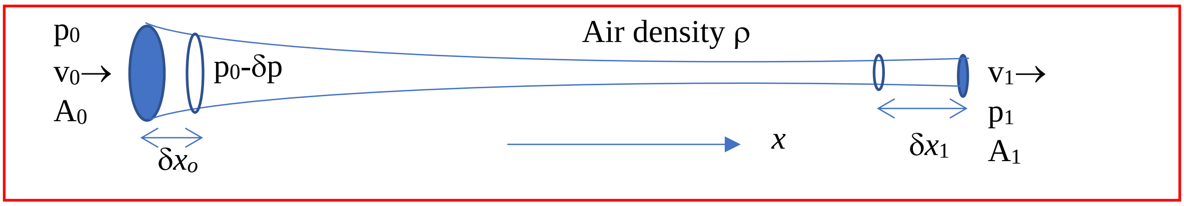
In an earlier version of this piece I took it that the rotors took air more or less at rest and gave it momentum downwards. It had been a long time since I'd thought about fluid flow and this was one simplification too far. The air some distance away from the drone is at rest but by the time it reaches the rotor it has acquired significant speed. One needs to think in terms of fluid flow. Since I'm only looking for approximate figures, it's valid to assume streamline flow with negligible friction and constant density. The diagram here represents a funnel of air streaming through the rotor, area A, from still air some distance away to the slipstream below the drone where it has speed v_S. The pressure of air at the rotor is lower than the still air pressure, which is why the air flows to the rotor, but the air is not significantly compressed. The rotor increases the air pressure above ambient pressure and the resulting air travels down the pressure gradient to ambient pressure, achieving a speed v_s, the streaming speed.. The rotor is the only place on that funnel where the area is well known. If v_r is the speed of the air as it reaches the rotor, then the mass of air passing per second into the rotor will be $\rho A v_r$. Since this same amount



of air is travelling faster as it leaves the drone, it will occupy a narrower area, a result of what is called 'the continuity condition'. This and other detail is discussed in the following box.

Flow

I remember learning about tubes of flow at school and it seemed obvious at the time. It's only obvious, though, if you have the right concepts. Daniel Bernoulli (there was more than one mathematical Bernoulli in the family) figured it out almost 300 years ago, which was quite remarkable because it would be well over a century later before 19th century physicists were comfortable with the relationship between work and energy, which is at the heart of Bernoulli's argument.



Suppose we have a pressure gradient down a tube of air flow that varies in cross-section from A_0 to A_1. In the approximation here, there are no forces from the side of the tube and no frictional loss. The pressure starts at p_0; a short distance δx_0 along the tube it has dropped to p_o - δp and so on down the tube until the pressure is p_1 at the end. The pressure gradient does some work moving a small packet of air through a distance δx_0 given by the force on it times the distance moved, namely $\delta p A_0 \delta x_0$. At the far end of the flow tube the same packet of air occupies a longer length δx_1 since the cross section is smaller. What is grandly called the 'continuity condition' says that no air is lost from the flow tube so $A_0\delta x_0 = A_1\delta x_1 = V$, the volume of the packet.

What is this work doing? It is creating kinetic energy of the air. At the beginning the packet has kinetic energy $\frac{1}{2}\rho V v_0^2$ and at the end more energy, $\frac{1}{2}\rho V v_1^2$. Bernoulli realised that the work done by the pressure gradient must equal the change in the kinetic energy of the air. The total amount of work done by the pressure is $V(p_0 - p_1) = \frac{1}{2}\rho V(v_1^2 - v_0^2)$. The packet volume V is the same on both sides, allowing the relation to be written **$p + \frac{1}{2}\rho v^2$ as a constant** down the flow tube. This is Bernoulli's result as it's relevant to drones, where we can ignore the effect of changes in height of the air flow. Height changes can be easily catered for but the kinetic energy of the air passing through a drone is much greater than the changes in potential energy of the air due to height changes.

There is a counter-intuitive aspect to Bernoulli's result, at least it's counter intuitive to me. The air that has passed the rotor progressively increases its speed as if flows down the pressure gradient towards ambient atmospheric pressure below the drone. It is not fastest at the rotor.

One way of looking at the situation is that the difference in pressure across the rotor provides the force holding the drone up, a force equal to *mg* when the drone is hovering. The second way of looking at the issue is in terms of flow. Over the whole streamline, the air begins at rest and ends up with velocity v_s, thanks to the work of the rotor. The total change in momentum of the air per second is therefore $\rho A v_r v_s$ per rotor. This is the force the rotor exerts on the air and, by Newton's 3rd law, is the force exerted back on the drone to hold it up. Hence, with *n* rotors,

$\boldsymbol{n\rho A v_r v_s = mg}$ is our first drone relationship. (1)

The variables are summarised in the flow diagram on page 2.

Setting all this air in motion creates kinetic energy (KE = "½mv^2") of the air, energy that ultimately the battery must supply. The kinetic energy of the air created per second by n rotors equals ½($n\rho A v_r$)×v_s^2. Applying the first relationship, gives KE = ½mg×v_s. This is the mechanical power needed to hover. It can be calculated if we can work out the streaming speed v_s. We can do that from the relationship (1) if we can work out how v_r is related to v_s.

At this stage, I didn't see how v_r must be related to v_s. Recourse to a textbook revealed that the answer can be found by applying the flow relationship p + ½ρv^2 before and after the rotor. The result gives the answer v_r = ½v_s. In other words, half the change of speed of the air from rest occurs before the rotors. The way forward is now clear. Relationship (1) gives the streaming velocity as v_s =sqrt(2mg/($n\rho$A)) and hence the (minimum) energy the battery must supply for hovering as E = sqrt($(mg)^3$/(2$n\rho$A)).

Notice that the air density 'ρ' is on the bottom line of both expressions. To allow a drone to hover, the air flowing down must move faster if the air pressure is low (since atmospheric density follows pressure at a given temperature). The air must also move faster if one takes a drone up above sea level. At 3000 m high, air pressure has decreased by about 30% of its sea-level value so the air must be moved about 15% faster to provide the same upwards force. Faster air implies more power is needed.

- In my example, one rotor has a radius 0.12 m. 'nA' (= 4A) = 0.18 m^2. Finally, we get v_s = $(2* 1.22×10/(1.22×0.18))^{½}$ = 10.5 m s^{-1}.
- Energy needed for hovering = $((1.22*10)^{3}/(2*1.22×0.18))^{½}$ = 64.1 J(oules). Since this energy is created per second, this is equivalent to 64.1 W(atts, since one watt is the same as a joule per second).

How does this compare with the energy needed to make the drone climb or travel horizontally? The figures in the next two sections suggest that this less than half the maximum power needed for these activities. The energy needed for hovering depends on $m^{3/2}$. Could the drone lift its own weight? Probably not quite, for according to the relationship above that would take 181 W. The prediction that could be tested except that what the drone can and can't do is circumscribed by the control electronics, effectively a mysterious black box, or at least a mysterious circuit board.

[Anyway, one could still test this in ways that don't interfere much with the airflow. For example, one could wrap some lead sheeting around the drone's feet to give added weight without changing the drag much. One could attach a rope that lay on the ground beneath and find out how much of the rope the drone could lift and then weigh that. [Actually, I did try this last one but with about 10 metres of dangling rope the drone became increasingly unstable and I decided it was prudent to stop]. The Youtube video https://www.youtube.com/watch?v=E6zZWsWhC8s shows a Phantom 3 lifting a water bottle and it managed up to 1.1 kg, which is in good accord with the calculation and the available power figures given later. Another side-line is to try the formula on a human powered drone, say weighing 100 times as much, with pilot. For the same v_s, you need rotors 10 times as large and the pilot must supply 100 times the power, more than anyone can sustain. However, add electric car technology and a single-seater drone could carry a human without the human needing to do any work. Another digression.]

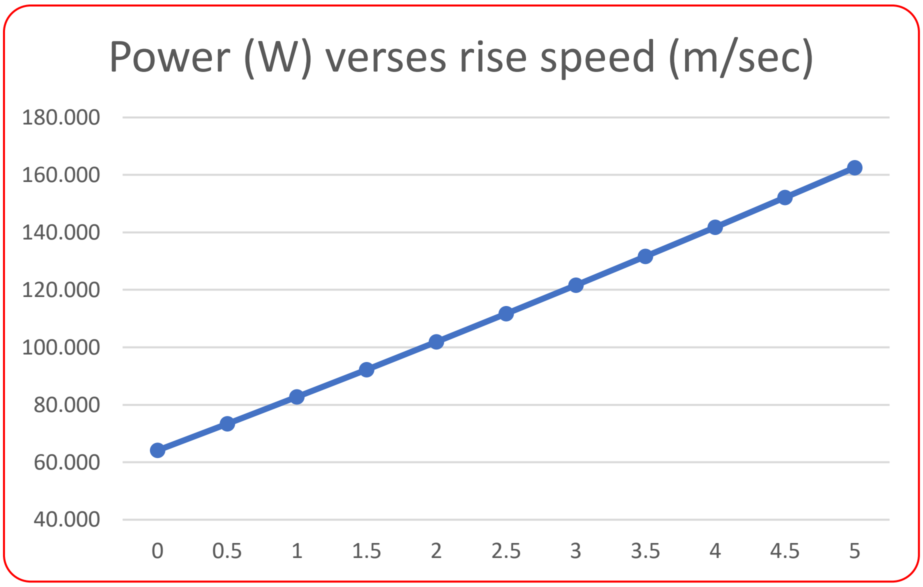
2) **Climbing**

Climbing a height *h* gives the drone potential energy *mgh*. The power needed for this is the potential energy created in 1 second. If the drone is to climb at 1 m s^{-1} then this is just *mg*. Twice this power is needed to climb at 2 m s^{-1}, and so on. The power needed to generate potential energy at speed *v* is *mgv*.

However, there is another effect when climbing. The air arriving at the rotors has speed v_r that will depend on the climbing speed *v*. The streaming speed will be v_s, which needs calculating anew. In unit time the mass of air passing the rotors is ρnAv_r. The change in momentum of the air per second is now $\rho nAv_r(v_s – v) = F = mg$, at constant rising speed. As before we need to know how v_r and v_s are related and by a similar argument (not given in detail) $v_r = (v_s + v)/2$. Hence the axial air speed v_s is determined from the solution to the quadratic equation $½\rho nA(v_s + v)(v_s – v) = mg$. The energy of the air per second given by the drone rotors is $\mathbf{P} = ½\rho nA\, v_r(v_s^2 – v^2) + mgv = ½mgv_r + mgv = \mathbf{½mgv_s + {}^3/_2mgv}$. The first term is a hovering term (with a different v_s), the second term one and a half times that needed to overcome the potential energy gain. This is the 2nd drone relationship. (2)

There is also the power needed to overcome the drag experienced when moving upwards, a point I'll come to below.

With the example drone, the constants as before are $nA = 0.18$, $\rho = 1.22$ and $mg = 12.2$. To rise at the maximum speed quoted of 5 m s^{-1} the axial air speed must rise to over 11.6 m s^{-1}. This requires a power of 162 W. The power needed increases approximately linearly with rise speed, as shown here.



Rising at 5 m s^{-1} the drone will also experience drag that must also be overcome. The drag is expected to add only a modest power requirement, for 5 m s^{-1} is a lot less than the maximum (horizontal) drone speed quoted as 16 m s^{-1}. Moreover, drag increases as the square of the speed so the drag at 5 m s^{-1} is only about a tenth of the drag at maximum speed and the power needed a third of that. Without any detailed calculation, about 5 W should be enough to account for the drag. Adding up the power requirements, this makes a total of nearly 170 W of motive power that needs to be available, over and above internal losses and the drag of the rotors against their turning. This works out at over 40 W per motor, which seems to me an impressive amount of power for a small motor.

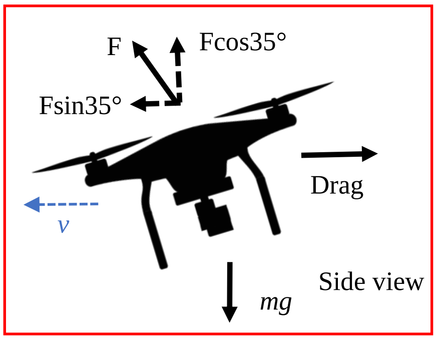
I'll add a comment on descending. A drone descending vertically at a few metres per second is falling into the turbulent and downward flowing air it has just created. The rotors have to work even harder to hold up the drone since they have to give the air an additional downward momentum to support the drone's weight. The drone is now falling into even faster flowing air. It doesn't take a very rapid descent before the situation is unsustainable and the drone can't support its own weight even with the rotors turning at full power. The result is a crash even though no part of the drone has failed. The controller ought to prevent a rapid vertical descent but the moral is: either descend slowly or descend at an angle like an incoming plane.

3) **Forward motion**

This is the hardest motion to estimate. With a propeller plane moving forward, the airscrew is at right angles to the direction of motion, which makes looking at the flow of air past the propeller simpler. With the drone, the rotors are steeply angled to the direction of travel. I think we can avoid looking at airflow. First, we'll look at things from the point of view of someone moving with the wind. If there is no wind this will be the same as the being on the ground. Otherwise the ground will be moving past but that won't affect the physics of what's happening at the drone. Moving the drone horizontally at a steady speed relative to the air, power is needed to work against the frictional drag of pushing the air out of the way and past the drone. The force of air resistance can be represented by $\frac{1}{2}c_d\rho Av^2$ where c_d is a drag coefficient that depends on the shape of the object moving through the air, A is the cross-sectional area (m^2) of the object in the direction of motion and v ($m\ s^{-1}$) is the drone speed relative to the surrounding air.

When travelling at constant speed the drone has to provide an equal and opposite force to the drag (Newton's 1st law). It does so by tilting so that the reaction force the air exerts on it now has a horizontal component. Of course, the vertical component is now reduced a bit so the rotors must work faster to make good the loss of vertical component if the drone is not to fall. At least that would happen in the absence of any lift being produced by the forward motion of the drone. If you look at the profile of a drone it is curved on top and less so underneath, a bit like the wing of a plane, though with a very broken profile. I suspect there is some added lift from the forward motion but I also suspect it's not much. A model glider shows how effective aircraft wings are. Throw the glider forward and the lift supports its weight, or almost so. Throw a tilted drone forward and … . It has no useful gliding capability and hence no useful wing lift. In the spirit of making approximations, I'll ignore any lift on the body induced by moving the tilted drone through the air.

Returning to the basics, the faster the horizontal speed needed, the more the drone tilts to increase the forward force. The rotors are a moving surface that is angled up into the wind and pushes the air backwards (relative to the drone). Tilting into the wind, though, reduces the angle of attack of the rotors, reducing their capacity to produce thrust. The faster the drone goes, the less thrust is available from the rotors so there is a speed limit from this effect as well as from the drag of the drone through the air.



The power needed to move the drone is the work that is done against the retarding drag taken over the distance travelled in one second. 'Work' in physics is force times the distance over which it acts. This distance is *v*. So the power needed is $\frac{1}{2}c_d\rho A v^3$. Just as with moving a car or bicycle through the air, the power needed increases as the cube of the (drone) speed through the air.

A drone doesn't have a nice simple shape like a sphere or cylinder but has arms, legs, a dangling camera, etc. so it pretty well needs a wind-tunnel test to see what the drag actually is. Also, the cross-sectional area presented by the drone isn't clearly defined, since the legs, arms and rotors have gaps between them. There are therefore two poorly known quantities in the formula above, namely c_d and *A*. Fortunately, it is only the product of the two that we need and this can be found from the maximum speed that the manufacturer quotes.

- For the example drone, the maximum tilt is 35° (from the manufacturer's data) so if the force F the air exerts back on the drone is angled at 35° to the vertical, then Fcos35° (= 0.82 F) is the vertical component that holds the drone up and Fsin35° (= 0.57 F) the horizontal component that makes it go forward. This gives 0.82 F = *mg* = 12.2 N(ewtons) and hence F = 14.9 N. The horizontal force is therefore 8.54 N [= *mg*tan(35°)]. If the drone can maintain 16 m s^{-1} with this driving force (manufacturer's maximum speed) then, $0.5\times1.22\times c_d A\times 16^2 = 8.54$, giving $c_d A = 2*8.54/(1.22*16^2) = 0.0547$. [A guess at c_d from the values for simple shapes might be that $c_d \approx 0.6$. This would give the effective area *A* as about 0.1 m^2, which is a plausible figure for our tilted drone (say 40 cm wide by 25 cm high). The value found for $c_d A$ is plausible].

  Now we have the key term $\frac{1}{2}\rho c_d A$ as $mg\tan\theta/v^2 = 1.22*10*\tan(35°)/16^2 = 0.03337$ for our example drone, giving air resistance as $0.03337v^2$. You will have spotted that the effective cross section depends on the drone tilt so strictly speaking $c_d A$ depends on the drone speed. This must be true but I think I'm not going to introduce a big mistake by taking it as constant.

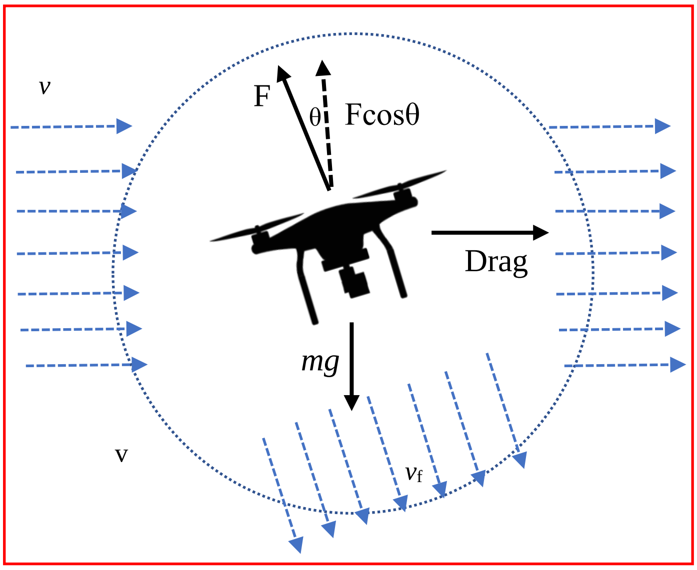
In summary, the 3rd drone relationship is

$$\frac{1}{2}c_d\rho A v^2 = mg\tan\theta, \qquad (3)$$

giving the relationship between tilt and horizontal velocity.

Having discussed how to quantify the drag, it's now necessary to work out what the rotors need to do to **both hold the drone aloft and move it horizontally at speed *v***. [As an aside, notice one big difference between a drone and an aircraft. The engines in a passenger aircraft do not produce enough thrust to support the weight of the craft. Typical figures are that the maximum thrust might equal about a quarter of the craft's weight. In flight the weight is supported by the lift produced by the wings as the craft speeds forward. It's a nice trick. For example, the largest Airbus has a maximum weight of 575 tonnes, kept in the air at take-off by the lift from about 850 m^2 of wing area. The forward speed is created by 4 engines that can provide up to 140 tonnes of thrust altogether. Not all of this thrust is needed to support the weight. The surplus allows the plane to climb and increase its speed.]

Returning to the drone, it's perhaps easiest to look at the forces from the drone's viewpoint. Imagine the drone in a wind tunnel supplying horizontal air at speed v. The drone is tilted so its axis makes an angle θ to the vertical. With no rotors turning, the oncoming air has an axial flow velocity of $v\sin\theta$.



Power the rotors so they produce a thrust F (mainly upwards from the drone) along the axis so that the drone does not fall. The thrust must support the drone, so $F\cos\theta = mg$. (4)

The horizontal component of the thrust, $F\sin\theta$, will only match the drag for one speed v. At this speed the drone will remain stationary. At other speeds it will drift backwards or forwards if it is held at the tilt angle θ. The stationary configuration is equivalent to the drone flying at speed v over the ground in still air. The 3rd drone relation (3) gives this speed from $v^2 = 2\tan\theta \times mg/\rho c_d A$, with A (italics) the cross-sectional area of the drone in the direction of motion, not the area of a rotor sweep.

Look at the axial flow of air. Let the axial flow of streaming air have speed v_s. As earlier, the mass of air passing along the axis in one second is $\rho n \mathrm{A} v_r$ when there are n rotors (sorry 'A', not italics, is the area of the rotor sweep of the air pushed by a rotor, as earlier). The change in momentum of this air due to the rotors is $\rho n \mathrm{A} v_r(v_s - v\sin\theta) = F = mg/\cos\theta$. The term $v\sin\theta$ is the component of the forward velocity perpendicular to the rotors. We now have $v_r = (v_s + v\sin\theta)/2$. Hence the axial air speed v_s is determined by the positive root of the quadratic equation

$$\tfrac{1}{2}\rho n \mathrm{A}(v_s + v\sin\theta)(v_s - v\sin\theta) = mg/\cos\theta \, . \qquad (5)$$

$$\text{i.e. } v_s^2 = 2mg/(\rho n \mathrm{A}\cos\theta) + v^2\sin^2\theta$$

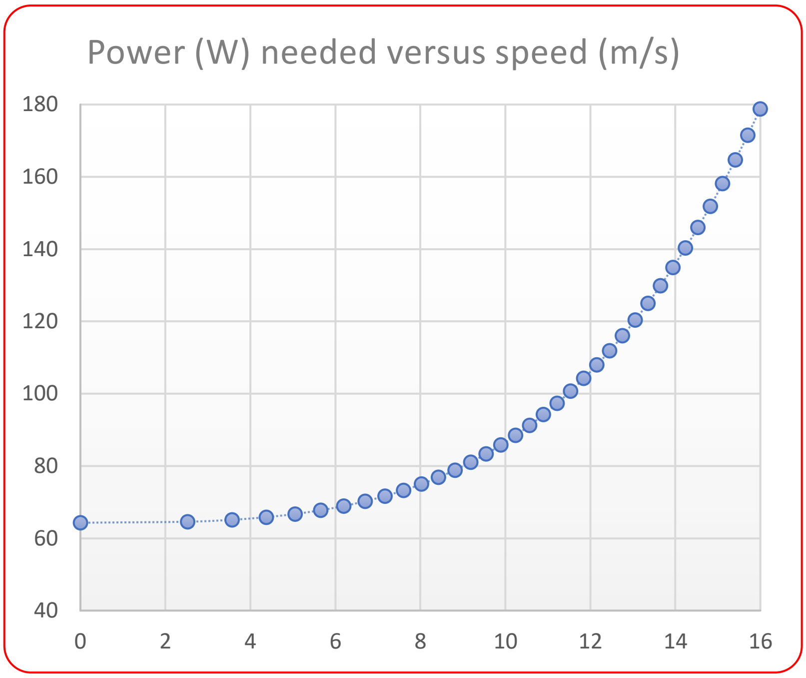
The power P needed is the extra axial kinetic energy created by the rotors per second.

$$P = \tfrac{1}{2}\rho n \mathrm{A} v_r(v_s^2 - v^2\sin^2\theta) = \tfrac{1}{2}\, mg(v_s + v\sin\theta)/\cos\theta \, . \qquad (6)$$

The earlier discussion gave all the constants needed to evaluate the power needed for flight at a given tilt θ. The corresponding drone speed v is found from the 3rd drone relationship; the axial airspeed v_s from equation (5) and the total power needed to generate that speed from equation (6). The result for the Phantom 3 is shown in the next graph. The power needed for maximum speed comes out at 179 W.

4) **Discussion**

I guess you could call the calculations above the minimum figures to achieve motion – the motive power needed. They do at least show how a drone works and what it has to do to hover, rise and travel forwards, or backwards by tilting the other way. Actual energy



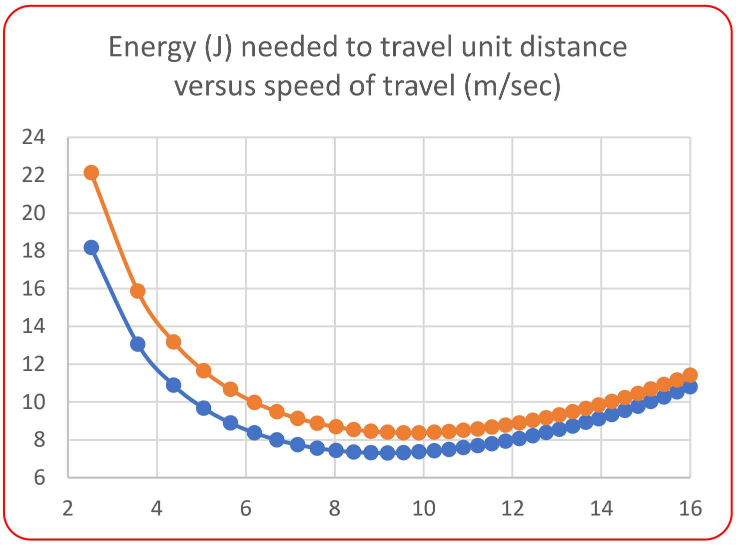
requirements will be higher, for the rotors generate turbulent air not nice cylinders of smoothly flowing air; the rotors themselves experience significant drag as they whirl around, needing more energy to turn them than just the kinetic energy given to the air; the battery drives the motors and these aren't a hundred percent efficient at converting electrical energy to mechanical energy. Indeed, the battery itself isn't a hundred percent efficient either in that it generates internal heat that comes from its own stored energy. All the figures above will be underestimates of the battery power needed.

- The Phantom 3 battery has a capacity of 68 W h when new and fully charged so I guess must be able to provide motive power over 150 W since the maximum flight time is quoted as 23 minutes. If all 68 W h were spent in 23 minutes the average use would be 177 W. This is close to the figures calculated earlier. However, I have ignored inefficiencies and the battery also has to power the control electronics, the camera and the communications signalling between drone and the controller, including sending real-time images on a microwave link, all amounting to a few more watts. In addition, it has to provide constant corrections of rotor speeds to match very local changes in wind speed and air turbulence that would otherwise rock the drone, so the picture of the drone flying steadily through constantly flowing air doesn't usually match reality.
- The graph show that the power needed for maximum horizontal speed is similar to the power needed to rise at the maximum rate. The manufacturer's figures are consistent. The graph also shows why it is difficult to control the drone at slow speeds. There is only a 2.5 W difference in the power needed to hover and that needed to go forward at 5 m s^{-1}, with a tilt angle of only 4 degrees. The power range over all speeds covers 100 W, so 2.5 W is a small fraction of the controller movement.

I didn't say this earlier but what set me off writing this was wanting to know how the power needed to move compared with the power expended in just holding up the drone. Now I have some idea.

Return to home

There's one more question the figures above can answer. If the battery is running low, is it better to travel back at a modest speed or at the fastest speed? This raises the issue of the energy used to travel a given distance. Since the drone is travelling at constant speed, any distance can be used for comparison. Let's choose 1 metre. At speed *v* the time taken is 1/*v* and hence the energy used is the power expended multiplied by the time, namely P/*v*. For a



low speed, the energy is high, for it spends almost all its energy holding itself aloft and little getting along. For very high speeds the drone is battling against high air resistance (that depends on the square of the speed) so again this takes a lot of energy even though the time taken is short. Using the results from the previous section, the minimum energy is achieved for a speed of about 8.6 m s^{-1}, which is therefore the optimum speed to bring the example drone back under all the assumptions above. Plot here shows the energy in joules needed to travel unit distance at a range of speeds. The blue line is the calculation just given. The brown line is discussed below.

You may think there's a bit of sleight-of-hand in the above since it shows that the mechanical energy needed to travel a given distance is minimised by a speed of about 9 m s^{-1} but the crucial issue is minimising the battery use. In fact there is nothing in it if the battery power needed is proportional to the mechanical energy. Of course the battery has to supply communications power and this energy usage increases with travel time, effectively increasing the baseline energy. This does increase the optimum return to home speed a little, favouring a shorter time. Adding 10 watts for communications, etc. (purely a guess) does increase the minimum energy speed to 9.6 m s^{-1}. The brown curve shows the result. The Phantom has a 'return to home' option, which presumably the manufacturers have arranged at the optimum speed. A test of my drone in fairly calm conditions shows a return to home speed of about 10 m s^{-1}, achieved after a few seconds of acceleration.

Returning home at 15 m s^{-1}, almost 'full gas', instead of the optimal 9 m s^{-1} uses some 30% more energy for the journey (over twice the power but for a reduced time). There's nothing special about the return to home journey (except the battery is probably running low) so the same energy saving may be achieved on any leg of a journey. Travelling a given distance such as 1 km at 9 m s^{-1} relative to the surrounding air rather than 15 m s^{-1} will take longer but it will use less energy and therefore less of the battery. Keep the speed down a bit to lengthen your flight time. Car drivers will recognise the same effect for more or less the same reason. You'll go further on a tank of fuel if you don't keep flooring the accelerator.

On weather

The calculations above show the speed relative to still air so the actual return to home speed depends also on the wind at the time, and its direction. A 5 m s^{-1} headwind speed against a returning drone (less than 20 km h^{-1}) will halve the actual return speed across the ground and hence double the time taken to get back. This raises the issue of how strong is the wind on any given day? This is just the problem that sailors and others faced before the days of wind-speed meters (anemometers). The Beaufort scale gives the answer, relating observations like swaying branches to wind speed. There is a version for land and a version involving the waves for the sea. I fly for fun and wouldn't start if the wind was above force 3. The Beaufort scale gives a figure for the average wind, what appears on weather forecasts. The gustiness of the wind depends on the kind of weather at the time. A ball-park figure for here (the UK) is that quoted gusts may be about twice the average windspeed. If the windspeed is 4 on the Beaufort scale, around 16 mph or 25 km h^{-1}, then in gusty conditions the gusts could be 50 km h^{-1}, more than enough to drift a drone in a short time. Of course there is a whole range of gusts and lulls. The other thing to remember is that the wind aloft, even 10 m up, is stronger than the wind on the ground, the reason being simply that the ground provides friction that slows the ground-level layer. Once the drone is airborne, the wind speed can be tested by switching off the GPS stabilised positioning (P mode) to the attitude only stabilised position (Atti mode). The drone will quickly drift with the wind at a surprising speed, 10 m in a couple of seconds in a seemingly innocuous force 3, the speed showing on the controller readout.

Balancing the rotors

One of the finer points of a propeller driven craft is that there are two desirable balances that each propeller should exhibit: *static balance* and *dynamic balance*. A drone with 4, 6 or even 8 propellers (I've mainly used the word 'rotor' earlier) has plenty of scope for lack of propeller balance and the net result will be unwanted vibration, extra wear on the motors and probably extra noise. Static balance is simple enough to explain. The centre of mass of each propeller should lie on its axis of rotation. If the propeller is balanced on a vertical needle point that coincides with the rotation axis, then it should balance horizontally, both longways and sideways. Each propeller is screwed onto its motor (on the Phantom 3 - other models may have alternative securing means) so to check this effectively is not as easy as it seems, for how can you be certain to get the needle exactly on the rotation axis of the propeller? Securing the propeller by the same means onto a freely rotating horizontal axis and seeing if one side drops down is not a particularly sensitive test and checks only the longways balance. If the propeller is not statically balanced, then the motor is essential swinging around a small mass in a circle at some speed. To do this it must exert a rotating force and from Newton's third law the propeller will exert a rotating force back on the motor. This is not good. I haven't measured the propeller rotation speed but web sources quote 5000 – 10,000 rpm in normal use.

Dynamic balance introduces a further complication, but it is one every car owner knows about. The important concept is the moment of inertia. Moments of inertia are calculated from the distribution of mass away from the rotation axis weighted according to the square of the distance from the axis. In comparison, the centre of mass is calculated from the distribution of mass weighted according to the distance from the axis. The two are clearly not the same. If you spin a body about an arbitrary axis, it will wobble if free to do so. If it is fixed to the axis so it can't wobble, the axis must generally exert restraining forces to prevent the wobble and the body will exert forces back on the axis. For any spinning shape there are 3 axes about which it can rotate without wobbling, called the Principal Axes of Inertia. Dynamic balancing entails making the desired rotation axis a principal axis. If it is not, then a force couple will be exerted on the rotation axis at right angles to the axis. This couple will rotate with the body. A propeller should have a 2-fold rotation axis of symmetry about its rotation axis. That would guarantee the rotation axis is a principal axis (in fact is should be the longest principal axis). However, in practice the symmetry may not be quite there if the propeller is slightly warped, or damaged, or perhaps more subtly has blemishes in its manufacture or the composition of the plastic. In which case, the motor axis will experience a perpendicular couple rotating at 5000 – 10,000 rpm.

Dynamically balancing a car wheel is what a garage does on its spinning balance machine, adding small lead weights to the inner rim to correct any dynamic imbalance. A car wheel has typically only a tenth of the rpm of a drone rotor and all off-balance forces increase as the square of the rotation speed. I've not seen an equivalent machine for drone rotors and adding weights is not really an option, nor drilling out or shaving off mass, for that is likely to affect airflow. Correcting static balance is no guarantee of ensuring dynamic balance. For example, a little extra mass on one side can be statically balanced by adding a little mass to the other side but if the added mass is not in the right place then dynamic balance will be off. Drones rely on the manufacturer to get the propeller balanced right. A propeller may look like a cheap piece of moulded plastic but there is much more to it than meets the eye. Get yours from a reliable source and replace if damaged!

On battery size

The speed of a drone depends on the mass of the battery, 350 g for the Phantom 3. If the battery were half the weight then it would take only about 60 W to hover the drone and with the same motors there would be a little more power available for horizontal flight but it would make little difference. Changing in the other direction, if one strapped on and connected up a second battery without much influence on the aerodynamics, an ideal state of affairs, could the drone be made to go much further? The power required to make the drone hover depends on the drone mass $m^{3/2}$, from the first section. Doubling the battery weight to 700 g, increases the power needed to hover to 73 W, leaving about 10 W less for horizontal flight. If the control electronics does not interfere, this will reduce the maximum speed by only about 0.5 m s^{-1}. In practice there is bound to be some extra air resistance so the loss of speed would be greater but with twice the watt-hours available, the drone could go considerably further. The answer to the question above then is 'yes'. Of course, add too much in the way of extra batteries and the drone will have difficulty lifting off and will fly very slowly, so there is a limit to the ability to extend flight distance.

Closing comment

Drones have no ailerons. The angular variables of pitch, roll and yaw determine the orientation of the drone in the air and that fixes its direction of travel. Thereafter the only freedom left is its speed. Four motors are the minimum needed to control these four variables. There is a good bit more physics associated with drone flying, such as how tilt and rotation are produced by controlling the relative speeds of the counter-rotating propellers in different ways. Rotational motion is produced by torque and there are reasonable explanations on the web. It is the need to balance torques as well as forces that dictates the use of pairs of counter-rotating rotors. If one rotor fails while flying a 4-rotor drone such as my example, it's 'game over'. Although the remaining rotors can produce enough force to sustain the drone in the air and in principle you could sacrifice control of yaw so the drone spun freely around like a top, in practice the unbalanced torques aren't controlled enough to allow the drone to maintain a roughly level attitude and descend safely. Anecdotal reports say the drone will crash land with serious damage at least to itself.

I hadn't noticed before writing this piece but looking at the drone I see now that the front pair of motors are tilted very slightly outwards from each other and likewise the rear pair. If the speed of each rotor in a pair are not the same, then a small sideways force is generated and I presume this is used for stability control, holding station when hovering. It doesn't invalidate the earlier argument that significant horizontal motion is produced by tilting the drone.

NASA Ames Research Lab released a video in 2017 showing on a fine scale the detail of the pressure changes created by the rotors of a Phantom 3 – see https://www.youtube.com/watch?v=hywBEaGiO4k. The complex detail is impressive but it's a mistake to think that this fine detail must all be understood to have any idea of how a drone flies. Newton's laws of motion and related concepts give the overall picture. Measured data on drone performance is likely to be in the form of battery voltage and current, the product giving the power the battery supplies. It would be very interesting to see how these figures compare with the 'mechanical' estimates just given. Until then, this is my best estimate to date for what is happening during drone flight.

Version 3 – Jun 2020

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