Galileo, Newton & Einstein

The birth of physical astronomy

If you look at the ideas Ptolemy wrote about, or the ideas of Copernicus and the theoretical models of Tycho Brahe, they all try to explain the orbits of the planets in mathematical terms. Even Kepler was driven by a search for mathematical connections. Epicycles, the basis of 2000 years of cosmology, were all mathematics and no physics. The numbers that best described the observed motion of the planets were just numbers, with no other significance. Epicycles were circles in space with nothing at their centre. In real life, nothing goes around in circles without something constraining it to do so. The unspoken advantage the heliocentric view had going for it was that with the Sun at the centre there was a potential physical cause for the orbits. What was really needed to explain orbits was a valid knowledge of mechanics and some serious thought about the cause of orbital motion. Galileo and Newton supplied the missing foundations of mechanics. In addition, they emphasised that astronomy should be a physical discipline, a synthesis of understanding based on physical observation and the application of proven laws. The numbers we use today to describe the motion of planets, asteroids and comets now have the significance that they characterise solutions to the equations of motion of bodies orbiting under gravity. Newton, of course, supplied the law of gravity. Galileo and Newton, with the support of others in the 17th century, took astronomy from a quasi mathematical and theological subject and made it into a physical science, where it has stayed ever since. It was a particular struggle for Galileo, as we shall see.

Galileo (1564-1642)

The story of Galileo is one that every educated person should know about, not just those interested in the development of astronomy. Galileo was born in the same year as William Shakespeare. Galileo Galilei ranks among the famous few who are known to posterity simply by their first names. He comes into the story of astronomy in two ways. Most famously, at the birth of one of science's most powerful inventions, the telescope, he made very useful telescopes that were better than others in circulation, sold them on their military advantages and himself pointed them at the heavens. Even today, you might recognise the name 'spyglass' as meaning a telescope. Galileo was an academic who at different times in his life had posts in Pisa and Padua Universities and a personal retainer by the Medicis in Florence. He discovered things rather quickly once he had made a successful telescope in 1609. It was a tube about a metre long with lenses at both ends. He looked though it, wondered, and engaged his brain.

What he saw was not much different from what you would see through a pair of powerful binoculars, except that you will get a wider field of view and see things more sharply than Galileo did. He saw:

- *Mountains and valleys on the Moon*
- *Sunspots*
- *Thousands more stars*
- *4 moons of Jupiter – the Galilean moons*
- *Complete set of phases of Venus* (Look at the animations of the textbook figures to see the significance of this).

Good for him, you may think. *Bravo*; surely his contemporaries were delighted. Not a bit of it. He published the first fruits of his observations in 1610 in a famous tract of 24 pages entitled *Siderius Nuncius*, the Starry Messenger, written in Latin. It was an immediate sell-out. The very day it was published one Sir Henry Wotton bought a copy and sent it to King James VI (Scotland)/I (England), the same James who had made a royal visit to see Tycho Brahe at Hven. News could travel quickly when it was hot!

This tract was followed by letters on the Solar Spots in 1613 - all of which brought him into serious conflict with Catholic and Protestant theology. Why? Listen to how the Bible opens.

Genesis 1.17: (*read from the Bible):*

1.1 "In the beginning God created the Heaven and the Earth.

1.16 And God made two great lights, the greater light to rule the day, and the lesser light to rule the night. He made the stars also.

1.17 And God set them in the firmament of the heaven, to give light upon the earth".

Remember that the words of the Bible were authority that held life and society together. The Church were the guardians of that authority. [In science, the ultimate authority is Nature herself. Nothing is true simply because someone says it or writes it in a book, even if that person is as eminent as Isaac Newton, Erwin Schrödinger, Richard Feynman or any other 'great name'. The arbiter of truth, or as near truth as you get in science, is experimental fact. Remembering what Galileo saw in his telescope, you can see a clash of ideologies was imminent following Galileo's publications. And so it came about].

The Catholic Church pointedly commented:

- Why did God make so many stars that we cannot see with our naked eyes?
- The Earth-centred Ptolemaic model, which was built into standard Christian Theology, could not accommodate moons orbiting Jupiter. All planetary bodies went round the Earth. (Galilean supporters said that here is evidence that Jupiter could carry its moons along with it in its motion; why then shouldn't the Earth be able to carry its moon along with it in motion around the Sun?)
- Sunspots defied the purity of the Sun.
- There is no gibbous phase of Venus in the Ptolemaic model. Galileo claimed he saw the gibbous phase [see lecture discussion and animated slides]. This was pretty convincing evidence for the Copernican model.
- The Church argued that the Sun and Moon behave very similarly to our eyes. Yet the Copernican system has one fixed and the other orbiting the Earth. Surely this difference disagrees with what our senses say?

You can almost imagine the accusations of the cardinals:

We hear that you say there are many stars in the sky that give no useful light on the Earth. This is contrary to Genesis 1.17 where it says that the stars were created to shine light upon the Earth. You, my son, are implying the Bible is a flawed work.

We hear that you say 'the great light in the sky' that rules the day has black spots and blotches on it. You imply, my son, that God's own handiwork is flawed. If we hear correctly, you may be guilty of heresy.

This was no idle accusation. You can go to the very spot in Rome where Giordano Bruno had been burnt alive at the stake only a few years earlier, in Feb 1600, for the heresy of propagating an anti-Aristotelian, pro-Copernican, philosophy of the universe. It wasn't just the Catholic Church that sanctioned the smell of burning flesh. The Calvinists, who ran Geneva, had had Michael Servetus burned at the stake there in 1553 for heresies "against the true foundation of the Christian religion". The issues in his case were theological rather than astronomical. The punishments today for not agreeing with authority aren't usually as severe but it's a lesson many of us still learn the hard way that people in authority don't like to be shown to be wrong, or even hear suggestions that they may be wrong. Parents and bosses can be just as guilty as Church or State.

Much of the astronomy that the Church was so keen to defend was in fact an adaptation of Aristotle's philosophy, formulated around 350 BC. His philosophy asserted the purity of heavenly bodies. Christoph Scheiner was a Jesuit priest and contemporary of Galileo who observed sunspots at about the same time as Galileo. When he asked his superior for permission to publish his observations it is recorded that he was refused with the words "*I have read Aristotle's writings from end to end many times and I can assure you that I have nowhere found in them anything similar to what you mention. Go, my son, and tranquilize yourself; be assured that what you take for spots in the Sun are faults in your glasses or your eyes.*" That was the response from the intellectual and cultural wing of the Catholic Church. Galileo didn't get off with so mild a rebuff.

The combination of Copernican, heliocentric, philosophy and Galileo's descriptions of his telescopic observations taken together were too much for the Catholic Church. In 1616 Galileo, when he was 52 years old, was officially admonished by order of the Pope not to *hold, teach or defend* his condemned version of the Copernican doctrine as true. He was, though, allowed to consider its merits as a mathematical model. Galileo was lucky to escape so lightly.

He retired to a villa outside Florence and led a studious life. Gradually the intellectual climate seemed to become more open and, prompted by others, he wrote over the 6 years from 1624 - 1630 and published in Italian in 1632 *Dialogo sopra i due Massimi Sisteme del Mondo*, a dialogue in which 3 participants argue for and against the Ptolemaic and Copernican world systems. The old system was defended by Simplicio, a well meaning but rather simple character. All Galileo's works had passed the censor's scrutiny before being printed but when the book appeared his enemies said that Simplicio was a representation of the Pope. Urban VIII had been a personal friend of Galileo for many years but that did not stop the full wrath of the establishment coming down on Galileo.

He was summoned before the Inquisition at the personal request of Urban VIII for a 'rigorous examination', not a trivial matter. Over the month before his 69^{th} birthday he was taken to Rome in a litter, because he was too ill to travel otherwise, to face a charge of publishing material contrary to Papal decree of 1616. This was a clever move, for he could hardly deny the charge since the evidence was plain for all to see. Old age and infirmity were no excuse. After a drawn-out interrogation by a Council of 10 inquisitors, with sessions stretching over more than 3 months, Galileo was forced to recant his beliefs, notably that the Earth moved around the Sun.

[In the large hall of a Dominican Convent, in front of an assembly of cardinals and prelates, Galileo heard the pronouncement which, with a few omissions, ran along these lines: *The proposition that the sun is the centre of the world and does not move from its place is absurd, and false philosophically, and formally heretical, because it is expressly contrary to the Holy Scripture. The proposition that the earth is not the centre of the world and immovable, but that it moves, and also with a diurnal motion, is equally absurd and false philosophically; and theologically considered, at least, erroneous in faith…..Invoking, therefore, the most holy name of our Lord Jesus Christ and His most glorious mother and ever Virgin Mary…. we say, pronounce, sentence, declare that you, the said Galileo, by reason of the matters adduced in process, and by you confessed as above, have rendered yourself, in the judgement of this Holy Office, vehemently suspected of heresy, - namely, of having believed and held the doctrine, which is false and contrary to the sacred and divine Scriptures,……We condemn you to the formal prison of this Holy Office during our pleasure, and, by way of salutary penance, we enjoin that for three years to come you repeat once a week the seven Penitential Psalms.* Galileo was required to read out before the conclave of the inquisition and sign the document stating: "Ego Galilæus, filius Vincentii Galilæi Florentinus, ætatis meæ annorum 70, constitutus personaliter in judicio, et genuflexus coram vobis … corde sincero et fide non ficta, Abjuro, Maledico, et Detestor supradictos Errores et Hæreses" *I Galileo, son of Vincenzo Galilei, a Florentine, aged 70 years, brought personally into judgement, and now kneeling before you … with a sincere heart and an unfeigned faith, I abjure, I curse, and I detest the afore-mentioned errors and heresies.* It is not surprising that with the alternative being a fate worse than a quick death he did so.]

There is an apocryphal story that after this recantation and forced agreement with the Earth-centred view, he was heard to say *sotto voce* "E pur si muove" (nonetheless, it moves). It's unlikely to be true that he said this immediately after his trial but it's a good story. If you read the quotation above you'll see that Galileo was condemned as *vehemently suspected of heresy* and sentenced to incarceration in the Inquisition's prison. The *Dialogues* was put on the proscribed list of banned books. However, after only a few days Galileo was allowed to leave for Tuscany to spend the rest of his life under house arrest, first with the Bishop of Sienna and then at his home just outside Florence, where he was *forbidden to entertain friends nor allow the assembly of many at a time*. Galileo himself wrote to his daughter that *my name is erased from the book of the living,* but far from erasing his name this story has rightly been repeated over the succeeding four centuries as an Awful Lesson on what happens when dogma deprives people of both wisdom and humanity. It is a lesson that still needs to be repeated in the twenty-first century.

Remarkably, Galileo was not even now a finished man and in 1638 he published another set of dialogues *Discorsi e dimostrazioni matematiche, intorno a due nuove scienze* in which he mulled over the results and implications of his experiments on Mechanics that he had carried out many years earlier. These he published in Holland, to give the Holy Office as little excuse as he could to make his plight worse.

Galileo led by example in showing that astronomy would develop through observation, not through philosophy. He pioneered the use of astronomy's greatest tool, the telescope. He did not, though, make much use of the telescope for quantitative observations, for example to map the heavens and find the distribution of stars, or to measure the brightness of stars. In following him, scientists of the 17th century didn't see the need to do things any differently and it wasn't really until the 18th century that the telescope was turned from a descriptive instrument into a quantitative one.

[Galileo's finger, by the way, shown on one slide and displayed in an ornamental glass casket in the Institute & Museum of the History of Science in Florence is the real, bony thing. It was removed from his skeleton about a century after his death when his remains were re-interred in the ornate mausoleum in Santa Croce Church shown on the last Galileo slide. The story is that it was meant to be the finger that Galileo had pointed to the heavens when defending himself, as he had done on many occasions. However, by mistake his middle finger was taken, and this is it. Galileo's finger has acquired a symbolic status, a posthumous wag at those who won't believe what observation has revealed. Let me recommend the highly-praised book *Galileo's Finger*: *the ten great ideas of science* by Peter Atkins [OUP, 2005 ISBN 0-19-860941-8]. The book covers ideas in Biology, Physics, Cosmology & Mathematics. It's a good read. If you are ever in Florence, don't miss a visit to one of the world's top science museums: the Institute & Museum of the History of Science. It's just around the corner from the Uffizi gallery, beside the river Arno. They have a fantastic collection of early scientific instruments.]

Isaac Newton (1642 - 1727)

Newton was born in the year that Galileo died (according to the calendar then in use). Our story continues by picking up Galileo's last work, on Mechanics. It was a development of Galileo's ideas that led Isaac Newton to lay down his famous three laws of motion. These, plus Newton's own 'law of gravity', would provide the ultimate explanation for Kepler's elliptical orbits and really tell us what was going on. The success of Newton's laws of motion in explaining how the planets moved made a tremendous impact in society at large. Planetary motion could be understood and modelled, both mathematically and physically. There was no need for complex mechanisms involving 30 or more epicycles. All you needed was the Sun, the planets, their moons and the one word "gravity".

Orreries

Clockwork models showing the planets, with their periods to scale and sometimes their orbits, were made for the nobility and also taken around in the 1700s by travelling lecturers who gave public explanations. With these models you could make predictions of the positions of the planets in the sky, actually not highly accurate predictions because that needed the mathematics of ellipses, but predictions that were good enough. In the 18th century, **orreries**, as these models were called, were the red-hot gadgets to have: the planets in your drawing room, or library; the mystery of the universe at your fingertips.

Newton's laws

What were these laws of Newton that predicted planetary motion, indeed all motion? They are fundamental to astronomy and all of science. Like Kepler, Newton had 3 laws, which are loosely stated as:

1) An object will move at constant speed in a straight line unless acted on by an outside force
2) Force produces an acceleration whose size depends on the mass of the body: "F=ma", where acceleration, *a*, is measured in ms^{-2}; mass, *m*, in kg and force, *F*, in Newtons.
3) If a force is exerted on a body, then the body exerts an equal force back.

You can read in the textbook why these laws were so different from what went before. Let me add a few words. If you see a hay-cart moving between the fields, then you know that there must be a force pulling it, don't you? A bullock, perhaps, hidden behind a hedge. If you see a boat in the distance moving between two islands, then you know that a rower must provide a force to move it, even though you can't see him, or nowadays an engine even though you can't hear it. There is a natural inference from everyday experience that motion needs a force to produce it and this idea was embodied in the old, pre-Newtonian physics. Newton said this was wrong. Motion itself requires no force. Only a change in motion requires force. Acceleration is the measure of change of motion and that's what the second law says. Take away all forces acting on body and it will not stop but it will travel in a straight line at constant speed.

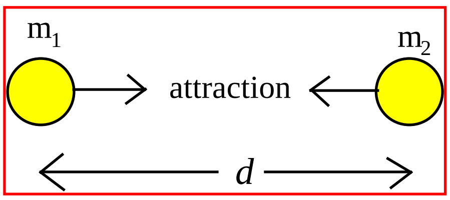
Newton's laws of motion were not a refinement of previous ideas but a completely new way of looking at the world, indeed at the Universe. They are absolutely central to physical science and all that springs from it, such as most mechanical and civil engineering, because all of what we see happening around us and much of what we do ourselves involves change. Change involves the motion of something and Newton's laws are, quite simply, the laws that govern motion in everyday life.

Illustration of Newton's 1st law

See animation in the lectures.

Universal Gravitation

A planetary orbit is not a straight line. You need a force to bend the orbit into a circle or ellipse. Previously people had imagined that some force (perhaps angels?) behind the planets pushed them around in their orbits. Newton agreed that a force was indeed needed to keep them in their orbits but one that was in the same direction as the change in velocity in each small interval of time. Now velocity has both magnitude and direction. The change in velocity at each bit in the orbit comes about because the direction of the velocity is changing. Newton was an expert at geometry and he quickly showed that this change in velocity is always directed towards the centre of the circular path of a body going around at constant speed. Hence the force causing this change in velocity must be directed towards the centre of the orbit. The situation is similar to the tension in the string needed to keep the stone in orbit as you swing it around you, as in the previous slide. That force is what Newton and we, following in his footsteps, call **gravity** and it was Newton who discovered the law of gravity. Indeed, before Newton there was no concept of gravity as a universal phenomenon of attraction.



$$F = Gm_1m_2/d^2$$

With $G = 6.67\times10^{-11}\ m^3\ kg^{-1}\ s^{-2}$
$= 6.67\times10^{-11}\ N\ m^2\ kg^{-2}$

The force F is measured in Newtons, mass in kg and distance in metres.

To find the total force of attraction of one body on another, you add up the attraction between every constituent of the two bodies. For a spherical planet, the net result of doing this is that the force is just the same as if all the planet's mass were located at its centre. This a convenient result in astronomy and also a convenient result that we take for granted in 'everyday life'. One consequence is that for practical purposes plumb lines that define the local vertical anywhere on Earth are always perpendicular to the ideal horizon, the horizon there would be if the ground were a plain. On a non-spherical moon or asteroid, this isn't the case. Plumb lines everywhere on Earth point to the centre of the Earth, or they would do if it wasn't for the minor inconvenience in this context that the Earth is spinning around on its axis. The Earth's spin has a small effect in throwing plumb lines off the vertical, but not by enough for most builders to worry about it.

There is no shielding from the influence of one mass on another in the Newtonian scheme of things. For example, if you interpose a third body between the two, it doesn't in any way alter the attraction of the two bodies, the one for the other. All you have to do to find the effect of the third body on the first body is to add the separate effect of the attraction of the third body on the first body. An example of this I'll give shortly is to work out the attraction of the hemisphere of the Earth that is on the other side from where we are. To do this you can completely ignore the hemisphere next to us that lies between us and the far hemisphere. The diagram in a later slide may help to make this clearer. There is no hiding from the influence of gravity.

On Gravity

I've quite a lot to say on gravity, and rightly so because Newtonian gravity is considered one of the four fundamental forces of nature.

- Pause for a moment and look at your neighbour. There is an attractive force between you. I, following in the footsteps of Isaac Newton, can work it out. The attractive force - it is always attractive, never repulsive - depends on your masses and the distance between you. Guessing your mass, no personal privacy invasion intended, this force of gravitational attraction is about 1μN if you are sitting close together

$$F = G\frac{m_1 m_2}{r^2} = 6.67x10^{-11} \times \frac{50\times80}{0.5^2} = 1.07\times10^{-6}\ \text{N}$$

The bigger your mass, the bigger the attractive force. Throw away that diet yoghurt.

- Now look down at the ground. The inanimate Earth is also attracting you and you are attracted to it. I, in the footsteps of Isaac Newton, can work out this force, too. Remember the result above for the spherical Earth. I just need to know the mass of the Earth and the distance to the centre of the Earth. The force is between 500 and 1000 N. You are more attracted to the Earth than anything else around you, by millions of times.

- Although the direction of the attractive force to the Earth is towards the Earth's centre, the net force is itself the sum of the attractions between you and every small part of the Earth.

It's not just the part of the Earth under your feet that is involved, but all of it. For example, a not-so-easy calculation will show that about 1/5th of this force is provided by the hemisphere of Earth opposite to where we are – Australia, New Zealand and all that side of the Earth. To make this calculation you can ignore the nearer hemisphere, as I said above, because it doesn't shield us from the influence of the more distant hemisphere.

- If the Earth is massive, the Sun is more so. The Sun has almost 1/3rd million times the Earth's mass. Is its force of attraction that much bigger? Not from here, because it is about 25,000 times further away than the centre of the Earth and the distance away affects the force as its square - the product of 25,000 x 25,000 = 625 million. So the Sun's force on each of us is only a few 1/10^{ths} of a Newton. Still, it's enough to keep us individually in orbit around the Sun, travelling at about 30 km s^{-1}. We're not hitching a free ride on the Earth but each of us is pulling our own 'weight' towards the Sun, or rather the Sun is pulling us. Take away the Earth and we'd still orbit around the Sun nearly once per year. Take away the gravity of the Sun and it is goodbye Sun, goodbye heat, goodbye life, hello stone-cold interstellar space. The force of gravity gives us life. [*Own doodle*: approximate circumference of Earth's orbit = $2\pi \times 1.5 \times 10^8$ km $\approx 10^9$ km. 1 yr ~ 3×10^7 sec $\therefore$ speed ~ 30 km s^{-1}].

- An example of the Sun's independent influence on us can be seen when a satellite is launched away from the Earth. The satellite also receives the Earth's orbital velocity at the time of launch. This is true but because the Sun's gravity continues to act on the satellite just as on the Earth, the satellite will keep up with the changing speed and direction of the Earth in its own orbit because the satellite shares the same orbit around the Sun. In other words, satellites and moons don't feel the effects of any orbital motion of the object they are circulating around. On a larger scale, this is one reason why it took astronomers until the 20th century to realise that the whole solar system and our surrounding stars are orbiting around the galactic centre.

- The force of gravity between planets and the Sun is large enough to keep the planets in their orbits. Imagine swinging a big stone around you on a rope. It takes a lot of pull. You may have tried hammer throwing in athletics and even if you haven't you know that serious hammer throwers tend to be beefy people. How much more force is it going to take to swing the whole Earth around in its orbit, approximately a circle of radius 1.5 x 10^8 km. A lot! What's the total force exerted by the Sun or the Earth, and vice versa?

$$F = \frac{GM_{earth}M_{sun}}{r^2_{orbit}} = M_{sun}g\left(\frac{r_{earth}}{r_{orbit}}\right)^2 = 3.6\times10^{22}\,\text{N}$$

This is a gigantic force. A similar calculation shows that the Sun attracts the Moon with about twice the force that the Earth attracts it. This seems strange but you can reflect that the Moon orbits the Sun at a faster speed than it orbits the Earth.

With what force is 1 kg mass at the distance of Earth attracted to Sun? The answer is 1/170 N. That's where the earlier figure of a force of about 0.5 N exerted by the Sun on an average person comes from. The effect of this force is to swing us round in our orbit around the Sun.

One conclusion from the numbers above is that gravity is only an important force between bodies when at least one of the bodies is astronomical in size. You need a delicate instrument indeed to measure the force that one object in front of you exerts on another beside it. It can be done. The gravitational force that one atom exerts on another is utterly negligible and beyond measurement. The strength that bodies have on an atomic level is not due to gravity but to electrical forces. Among the fundamental forces of nature, gravity is by far and away the most puny. The entire pull of the Earth on any one of us is barely enough to stop us jumping over an object a metre high. In fact, if gravity were much stronger we wouldn't be here at all because stronger gravity would have pulled all the stars and galaxies together over the time-span for which the universe has existed. It's not obvious, but there is a connection between how high we can jump and the longevity of the universe. Why gravity should be so puny compared with the other fundamental forces of nature is a problem that is still causing some of the world's best theoretical physicists to scratch their heads. No-one yet knows. That, though, is a bit of a diversion.

Gravity gives mass its weight

On the surface of a planet or moon, the gravitational force on a body is simply called its weight. Weight is measured in Newtons, as are all forces.

- You want to loose weight? You need the rocket to the Moon, featured in popular song. Gravity is weaker there because the mass of Moon is 1.2% of the Earth's mass and the radius^2 is $(0.27)^2$, giving a reduction in 'g' by a factor of 6. A 60 kg person, weighs only ~100 N or 1/6th their weight of about 600 N on Earth. 100 N is what 10 kg weighs on Earth. Your mass and strength remain the same. On the Moon most of you could hold up your neighbour above your head with one arm. That's the difference between mass and weight. Console yourself that if aliens land here, they may not be able to stand up properly. Why?

- If you would like an immediate example of the difference between mass and weight, a simple pendulum will swing with a period that is independent of its mass but depends on the local gravity that determines its weight. Take a pendulum a metre long. Whatever it mass, it will swing with a period of about 2 seconds on Earth, one second from side to side. Take the same pendulum and hang it up on the Moon, though the mass is the same it will now have a period of about 12 seconds. Hang it up on Mars' moon Phobos, pull it aide and start it from rest and even though it had a mass of several kg it will take almost an hour to complete one swing, a motion so slow that it would hardly be seen. The difference in behaviour is all due to the change in weight. The mass hasn't changed.

- We have evolved on the largest rocky planet in our solar system, the one with the greatest surface gravity. This has given us physical and mechanical strength we mightn't otherwise have had. Drop an object from 2 m height and it is travelling at over 6 ms^{-1} by the time it lands on the floor. $0 \rightarrow 100$ km per hour in 3 seconds, in the absence of air resistance. Earth's gravity is impressive. So much so that I strongly suspect that anyone who leaves the Earth on a space mission lasting several years will never be able to return. In your mission you may spend time every day on exercise machines but muscles will waste, including your heart. Being asked to live again where gravity is as strong as it is here will be asking too much. Gravity may well be our silent shield against invasion from intelligent beings. It's no defence, though, against invasion from alien microbes.

- You can never truly escape from gravity - it just gets weaker and weaker at longer distances. Gravity governs the large-scale structure of the Universe. It is the only long-range force between ordinary bodies. Gravity not only controls the solar system motion but the motion of the galaxy as a whole and, indeed, how our galaxy moves relative to other galaxies in our local cluster.

Orbiting a common centre of mass

Everyone is familiar with the idea of 'centre of gravity'. It's a balance point when you hold something up - the point where the twist of gravity of the bits to the left are balanced by the twist of bits to the right, to put it loosely; and the same for the front and back. Interestingly enough, that balance point doesn't actually depend on the strength of gravity. It would be the same if you had the object in weaker or stronger gravity, on the Moon for instance for weaker gravity. The illustration on the slide shows two balls on a light rod suspended about their centre of gravity. The animation shows them twisting and rotating about the suspension line. If you think about it, they will still twist about the same line even in the complete absence of gravity. The idea of centre of gravity, then, doesn't actually depend on gravity at all. The more general idea is called **centre of mass**. The illustration is therefore labelled 'centre of mass'.

See the animation in the lecture.

Two bodies orbiting each other under the influence of their mutual gravitational attraction, like a planet and its moon, or even two stars swinging around each other, will orbit around their centre of mass. The centre of mass will stay fixed and the bodies will lie at opposite ends of a line that passes through it. Therefore both bodies will take exactly the same length of time to go round once. In astronomy, the mutual attraction of the bodies stops them flying apart, in the same way that the light the rod holding the balls does in the picture. The centre of mass can be calculated from the distribution of mass that both bodies have and it doesn't need any consideration of Newton's law of gravity. Centre of mass is a very useful idea in astronomy, for it tells us precisely which point two bodies rotate about. The Earth and Moon, for example, don't rotate about the centre of the Earth but rotate about a point that lies on a line joining the centres of the Earth and Moon and that is within the Earth, and not too far from the surface

The remaining part of this chapter involves some quite hard concepts. You'll need your thinking cap on. I'm going to talk about tides, in particular, saying more than is in the course textbook.

Tides

Gravity is responsible for tides. Tides are important for sailors and those who travel by sea. Tides also influence astronomical bodies; not the tides on Earth but other tides. The variation in the total gravitational force on Jupiter's closest moon, Io, as it travels around its orbit is responsible for heating that moon red-hot. Tides between close binary stars can seriously disrupt the stars. Tides close to black holes can rip falling matter apart. The sketch on the slide illustrates the tidal force of a small white dwarf pulling matter off a nearby red giant to form an accretion disc around itself. This process can't be seen (the stars are too far away) but it can be inferred as happening by analysing the radiation emitted and received on Earth. Tides are responsible for more, as we'll see.

Underlying cause of our tides

The tides are caused by the gradient of the gravitational force over the Earth, i.e. the variation of the gravitational force with increasing distance from the Moon (and Sun). Put another way, tides are caused by the difference between the *average* gravitational force that acts on the Earth and the local force in different places. This difference would have no effect if the Earth were completely rigid but in practice the Earth is not rigid, for two reasons. First, a lot of it is covered with water, which very clearly isn't rigid. Secondly the forces are so great that even the solid Earth itself deforms.

The result is a tidal bulge, or rather two tidal bulges, one oriented more or less towards the direction of the Moon and one on the far side. These tidal bulges move relative to the Earth, mainly because the Earth is spinning beneath them but also because the Moon is orbiting the Earth.

Variation of attraction of the Moon

The Moon exerts its attraction towards its own centre. Remember that each point on the Earth is independently attracted. This means that the Moon's attraction varies across the Earth, because different parts of the Earth are at somewhat different distances from the Moon and in slightly different directions.

The average attraction is the attraction at the centre of the Earth, point O on the diagram. The attraction at the point P on the slide is greater than that at O and hence there is a difference at P represented by an outward force. The differences at other points around the Earth are shown in the second diagram. For example, the difference between the actual attraction at Q and that at O is a force that is inclined, as shown.

Tidal forces caused by the Moon

The tidal forces are the differences we've just been looking at. These differences generally have vertical and horizontal components. The vertical components just add or subtract to the vertical force experienced at a place, namely the weight of a body. The horizontal components of the forces cause the tides we experience. These are shown in the diagram. You can see how they induce a tidal bulge both on the side of the Earth where the Moon is, and on the opposite side. The result is a distortion of both the oceans and the solid Earth itself.

The tide not only affects the sea, but also the very rocks of the Earth itself. There is an Earth tide that results in the whole Earth becoming slightly ellipsoidal and we rise up and down by about 300 mm, twice a day. Stretching the Earth and letting it go down again can't be done without frictional loss. Friction causes heat and the energy must come from the mechanical energy possessed by the Earth-Moon system. I'll come back to this point shortly.

Consequences of the double tidal bulge

Because there are 2 tidal bulges, there are 2 high tides per day, or strictly speaking 2 tides every 24 hours 50 minutes.

The tidal bulge is slightly elliptical and, because of the Earth's rotation, the maximum of the bulge is ahead of a direct line under the Moon. To make matters a bit harder to visualise, the Sun's bulge is located under the Sun somewhere on the ecliptic, depending on the time of year, and the Moon's bulge is located under the Moon, whose orbit may differ by about 5° from the ecliptic. The Earth spins about its axis, inclined at 23.5° to the ecliptic. It gets rather messy to work out exactly what is happening, which is why mariners use pre-prepared daily tide tables or, nowadays, daily predictions calculated by computer. However, it is not too hard to see overall what is happening. The water goes around with the Earth but the main tidal bulge moves round with the Moon, taking about 27 days to go around once. To us on Earth the high tide bulge seems to be sweeping around the Earth but in fact is moving comparatively slowly in space.

See the animation on the next slide.

Earth's rotation affecting the tidal bulge

Animation from the textbook, showing the tidal bulges rotating in space following the rotation of the Moon.

Combined effects of Sun & Moon

The Sun also produces a tidal bulge, about 45% of the Moon's bulge.

When Sun and Moon are in line this adds to the Moon's bulge and the enhanced tide is called a **spring tide**. This occurs on days near new moon and full moon, roughly once a fortnight. Any sailors reading this might like to know that the spring tides at full moon and new moon are generally a different height. In the broad summer season, the new moon spring tides are largest; in the broad winter half of the year the full moon spring tides are highest. I'll leave this as an exercise for the interested to work out why this might be so. In addition, tides in the sea are not just about changing height of water but also about tidal flow. There are plenty of passages in the West Coast of Scotland where the tidal flow can reach 6 to 8 knots when the height of the water is changing quickly. The water often swirls around too, and becomes very choppy, the net effect being that small boats and sailing boats have to avoid these places for several hours each day, solely on account of the tides.

When Sun and Moon are at right angles, approximately at first quarter or third quarter of the Moon's phases, we get **neap tides**, which have the least height. At these times the minimum of the Sun's bulge adds to the maximum of the Moon's bulge and vice-versa. This reduces the height of the high tides and increases the height of the low tides, reducing the whole effect of the tidal experience. At neap tides the speeds of the tidal flows are less and it is generally easier to navigate in waters where the tidal flow has to be taken into account.

A digression

This is an appropriate point for a digression. I remember not that long ago sitting on the rocks by the North West coast of Portugal. It was a dull day. The Atlantic Ocean 100 metres away tumbled and cascaded towards me, sweeping over some of the outlying granite and around the larger rocks that poked above the sand. The low roar of the sea appeared continuous but if you listened more carefully you heard it rise and fall as the white foam tumbled over itself or an especially big wave crashed down on the outliers. This symphony of the sea, with ever changing visual accompaniment, was driven by the elements, the sun and the wind.

As I looked around, gradually the nearby rock pools began to fill with water. Little rivulets ran over their sea-facing ledges. Sometimes water ran out again over other ledges but slowly the little rivulets in a dozen nearby pools ran more towards me than away and the rock pools became deeper. It struck me then that the wind and the waves were creating the eye-catching and noisy spectacle before me but beneath them both was a silent and invisible force, more powerful by far, that was lifting up the whole sea, to the horizon and far beyond. This force was the combined tidal force of Moon and Sun. The tides are a stunning demonstration of the strength of gravity. A whole sea can be lifted up. We individuals are too puny to lift a bath full of water. You can experience the power of the Moon to create tides without going to northern Portugal, for the rock pools south of Aberdeen make ideal spots for contemplation.

If the Moon is physically that strong, what else can it do, how else can it affect our lives? It shines some light on us at nights, but that doesn't involve any significant force. Apart from raising the tides, the Moon has one more physical effect on the Earth of stunning power, an effect that has influenced every one of us today and will affect mankind far into the future. The Moon twists the Earth's spin axis around in space. I'll say more about this in a few minutes time.

Effects of the tides

Coming back to the frictional loss of energy that is a consequence of all this tidal motion, where is the energy coming from? Tidal energy isn't a free gift of nature. The energy comes largely from the rotational energy of the Earth. In the past the Moon has contributed so much of its own rotational energy about its axis that it scarcely rotates at all now. In short:

- The Earth's rotation period is slowing up - in future there will be fewer pay-days in the year and more of today's seconds in the day.
- The Moon's rotation period has already slowed up - it now rotates exactly once in each orbital period and hence the Moon always shows the same face to the Earth. (We'll meet this later).
- The Moon gains rotational energy around the Earth (from the Earth's rotational energy) and hence is slowly receding from the Earth. This is because the asymmetry of the tidal bulge produces a net accelerating torque on the Moon, slowly giving it more energy, which means it is escaping from the Earth's gravity very gradually. More advanced students in the class might like to test their physical insight and reflect that something rather odd seems happening here. The torque is giving an acceleration to the Moon, resulting in the Moon having more energy and yet it will travel with a slower speed in its orbit, for by Kepler's second law a satellite further from the Earth travels more slowly in its path. How is it possible to have more energy yet travel slower? [The answer is quite straightforward, once you see what's happening: mechanical energy has two components].

Precession of the Earth's axis

One final effect of gravity is that the elliptical shape of the spinning Earth in the gravitational field of the Moon (and the Sun, to a lesser but not insignificant extent) means that the net gravitational torque on the Earth is giving the Earth a slight twist. As a result, the Earth's rotation axis precesses, or moves around in a circle in space. This takes about 26,000 years. At the moment, the axis through the Earth's North Pole points close to Polaris, the Pole Star. In about 13,000 years time it will point to another bright star some considerable distance away, called Vega. The half-angle of the precession cone is 23.5°, though there are some complications here, too.

Over the centuries this affects the appearance of the night sky. For example, stars that are on the celestial equator now rise due East and set due West. They won't do so in several centuries' time because they will no longer be on the celestial equator. This is an important effect in 'archeoastronomy', the branch of archaeology that investigates astronomical alignments of ancient monuments. There are plenty of ancient monuments in Western Europe and the Middle East that are at least 4000 years old. Stonehenge and some of the pyramids are examples. Any stellar alignments of these monumental stones set up by our distant ancestors will no longer work. To investigate such alignments, allowance has to be made for the precession of the Earth's axis.

This precession has an important influence on our climate. At the moment our winter occurs in the northern hemisphere when the Earth is nearest the Sun in its elliptical orbit. This makes the winters shorter and warmer than they would otherwise be. The winter is when the Sun is lowest because the Earth's axis is tilted away from the Sun in the northern hemisphere. In 13,000 years' time, when the Earth's axis points to Vega, our winter will occur when the Earth is farthest from the Sun in its orbit. Winters will become colder and longer. Patterns of climate will change, probably by enough to cause the icecap in the Northern hemisphere to extend again over a fair bit of Europe and N America. Another ice age awaits us – unless of course we can induce global warming!

Precession of the Ecliptic

The rolling of the Earth's axis is easy to visualise from an outside viewpoint. For us on the Earth, who use the Earth's equator and poles as the basis of measuring where stars and planets are, it is the stars and planets that roll around on the timescale of 26,000 years. From home base Earth, the ecliptic rolls around in this time. In brief, the 'precession of the ecliptic' is the same phenomenon as the 'precession of the Earth's axis', the 'precession of the ecliptic' being the description of the effect that we see looking at the sky from our vantage on the Earth. The intersection points between ecliptic and celestial equator, called the nodes of the Sun's orbit, precess around in the 26,000 years. The two nodes are ♈, the first point of Aries i.e. the vernal equinox, and ♎, the first point of Libra, the autumnal equinox. Because of the precession of the ecliptic, ♈ is now in Pisces and ♎ in Virgo. This is another effect of the precession that has gradually come about over several millennia.

The Lagrangian Points

When you look back on the achievements of many well-known names in science, with all the advantages of hindsight you can see a clever person who by good fortune was in the right place at the right time with the right interests and the right skills. Had you been in that position at that time you just might have made the same discovery. No-one, though, has thought that about Newton. Scientists and professional historians of science have been looking at Newton's achievements for 300 years and they still scratch their heads and say 'How did he do that?' His ability to see through a problem to a solution, the depth of his perceptions, the technical mastery he had over his subjects are such that no ordinary person in his position in his time would have come up with the insights that he generated. Indeed, Newton was in contact with a number of highly gifted contemporaries and they too were in awe of his achievements. True, he made mistakes on occasions and he was a curmudgeonly character who certainly couldn't be described as the life and soul of a party. Nonetheless, 300 years has not tarnished his reputation as one of humanity's real geniuses and that is as good a final thought to almost sign off Newton from our story.

Newton left to British science a legacy of his mathematical methods. He is credited as one of the co-inventors of differential calculus but in fact most of his published results were proved geometrically. His legacy was the power of geometry. I have on my shelves a fine leather-bound copy of John Keill's 'Introduction to Natural Philosophy' being a well-known account of the physics of the time delivered in the early 1700s by the author, who was Professor of Astronomy at Oxford. In the preface he despairs to his readers "*that Men ignorant of Geometry presume to philosophize, and to give the Causes of Natural Things. For what can we expect but Mistakes, from such, as having neglected Geometry, the Foundation of all Philosophy, and being unacquainted with the Forces of Nature, which can only be estimated by means of Geometry, do yet attempt to explain its Operations, by a Method not at all agreeing with the Rules of Mechanicks?*" After some further paragraphs he adds "*such is the State of Mechanical Philosophy that there is no Admittance to it but by Geometry*". Newton died in 1727 and for the rest of the 18th century, if not for a century following his death, British science and maths tried to use geometry to solve hard physical problems.

Joseph-Louis Lagrange was one of the great mathematicians of the French school of the second-half of the 18th century who didn't go along with this idea. He was a strong promoter of the use of analytical mechanics – equations to you and me. In the preface to his *Mécanique Analytique* he said with pride that "*the reader will find no diagrams in this work*". Through algebra he developed techniques to solve problems in celestial mechanics that hadn't been solved before, and there were plenty of them.

The crucial problem that everyone has faced since Newton spelt out how gravity works in principle is that a simple situation with just 3 bodies attracting each other under gravity cannot in general be solved. That is, you can't find named mathematical curves that predict the subsequent motion of 3 arbitrarily chosen bodies. Newton at least realised this. You can find solutions in special circumstances. Lagrange found solutions of interest for the special case when one body was very large, one medium sized and one very small. The 5 points that now carry his name that are shown on the slide are all points in the Sun/Earth system in which a 3rd, very small, body will orbit around the Sun at the same rotational speed as the Earth under the influence of the two larger bodies. Put another way, a body at any of these points will stay at a constant distance from the Earth and the Sun as the Earth orbits around the Sun. Two points, L5 and L4 in the diagram, are points of stable equilibrium that support stable orbits of bodies around them, albeit odd shaped orbits. (You find similar points with the Earth/Moon system and with other planets). These points in the Earth/Sun system seem to be occupied by accumulations of interplanetary dust.

L3, on the far side of the Sun, isn't of much interest, unless you're keen on science fiction. It's a hidden point behind the Sun where alien craft could in principle congregate and remain using little fuel. Unfortunately in practice their party would be spoilt by the influence of other planets on their orbit. L2 and L1 are points where there is no stable state but around which a body can orbit. The lack of stability means that if the body starts to drift away from either region either towards the Sun or towards the Earth, then the net forces on it will take it further away. As a consequence, these points tend to be very free of inter-planetary debris. They are now becoming increasingly used to locate space probes. One of the most successful of all is the SOHO probe, that orbits around L1 about 1.5×10^6 km from the Earth, permanently pointed at the Sun. It gives us a continual view of the solar corona, of sunspots, of the solar wind and more besides. SOHO orbits around L1 but as it starts to drift away its orbit needs to be corrected with a firing of its hydrazine (N_2H_4) rockets, which will eventually fail through lack of fuel. Also orbiting around L1 is ACE, whose prime function is to monitor the composition of the solar wind.

L2, behind the Earth, is an ideal place to look away from the Earth at the cosmos at large. The second cosmic microwave anisotropy probe, WMAP, measured the cosmic microwave background to an accuracy of a few tens of a microKelvin while orbiting around L2. The results are stunning and were first reported in the press in February 2003. One outcome was a definitive measure of the age of the Universe, which is now known within about 1% to be 13.7 billion years. ESA sent its Planck cosmic microwave background probe there too in 2009, with even more sensitive receivers on board and its first results were published in March 2013. ESA's Herschel Space Observatory operated around L2 from 2009 - 2013, where it looked at the cooler universe in the far infra-red. The observatory was permanently shielded from the Earth's IR radiation (and the Sun's) and gradually swept its view around the sky once a year. The James Webb Space Telescope, successor to the Hubble Space Telescope, is also scheduled to go to L2, not to orbit the Earth, with a launch date in 2018. You'll hear more about the Lagrangian points in the future.

[Those who want to think in a bit more detail about L1 and L2 will realise that if you want a body to orbit at the same angular speed as the Earth but be at a different distance from the Sun, then that body must have less force pulling it round if it is closer to the Sun and a greater force if it is further from the Sun. (This should be clear by thinking again of the situation of twirling a stone on a string around you. The longer the string, the more force you need to twirl the stone around in the same time.) As you move from the Earth towards the Sun, the Sun's pull increases but the Earth's pull cancels out some of the Sun's pull. The point L1 is the point where the residual force is just what is needed to make a body orbit with the same period as the Earth. Likewise, moving away from the Earth in the opposite direction to the Sun, the pull towards the Sun from the Sun weakens but the pull from the Earth now helps in the same direction. Again, at the point L2 the combined strength of the gravity from the Sun and that from the Earth adds up to the extra force needed to make a mass that is further from the Earth orbit at the same angular speed as the Earth. Lagrange did well to work out the fine details of all this and the other 'Lagrange points' about two centuries ago. Some messy algebra is involved.]

Einstein (1879 – 1955)

Is Newton's word the last one on gravity? No. The last word, so far as modern science knows, is based on what was started by Einstein. It is called "The General Theory of Relativity". Einstein's general theory of relativity is our ultimate working theory of gravity, at least the ultimate theory that doesn't include quantum gravity. It predicts that light can be bent by masses (which effect has been seen); that 'gravitational lenses' can give multiple images of some very distant galaxies (which have been seen). The evolution of the Universe as a whole is intimately bound up with general relativity. There is a small section about it in Chapter 3 of our textbook, which you can read. Because its special predictions are particularly relevant to large-scale cosmology, which is not part of this course, I shall leave this topic to those interested in reading about it. Our level 2 general interest course on *Cosmology, Astronomy and Modern Physics* (PX2512) expands on the relevance of General Relativity to cosmology.

Gravity isn't just about objects falling to the ground, moons orbiting planets, planets orbiting stars and galaxies dancing around each other in space. Gravity is responsible for the very existence of moons, planets, stars and galaxies. The average density of matter in the universe is barely an atom per cubic metre. The density of ourselves and all the matter we see around us is in the region of 10^3 kg m^{-3}, which translates to a density of greater than 10^{28} atoms per cubic metre. What has given rise to this fantastically huge condensation of stuff that surrounds us and makes up the constituents of moons, planets and stars? It is gravity. If gravity had been significantly less strong, then matter wouldn't have clustered into objects like the stars and galaxies we know, and we wouldn't be here; if gravity had been significantly stronger, then the Universe would long ago have collapse in on itself, and we wouldn't be here. Understand gravity and you understand a lot about the universe. This has been a driving influence in Physics ever since Newton formulated the concept of gravity and gave us a good idea of how gravity works in our solar system and beyond. Einstein gave us a much better understanding of gravity, in the process showing that it was not nearly as simple as Newton had us believe.

Reflection

Astronomy isn't just about properties of planets and stars. It's about humanity and our interpretation of the Universe. The story of mankind's changing viewpoint, from ancient times through Copernicus, through Tycho Brahe, Kepler, Galileo, Newton, Einstein to our very modern picture of the Universe is one of the great stories in science. The modern view of the Universe is astonishingly different from the ancient view. I personally think that the modern view is all the more believable because it has evolved from the ideas of our predecessors. What makes it even more fascinating is that almost every apparently obvious deduction made in the past from the direct evidence of our senses has proved wrong. Deductions such as the Earth is stationary and the stars rotate around us; the stars are on a real celestial sphere; they are all made of the same stuff, quintessence, a fifth element not found on Earth (I didn't go into that); the Moon and the Sun are planets; that there are 7 planets out there and not much else beside the stars; the Moon is peppered with volcanic craters, and so on. All these ideas are completely wrong. The Universe is much more interesting than that.

The scale of the Universe is becoming clearer by the year. The person who first seemed to appreciate that the Universe really had a scale and the visible stars a distribution in space with a disc-like structure was William Herschel in the late 18th century. Our everyday experience is that you will see distant objects more clearly if you have a telescope or pair of binoculars with a large magnification rather than a small magnification, provided of course you can hold your optics steady. Herschel had the intuition to realise that this didn't apply in astronomy. Making telescopes of bigger diameter (i.e. wider) was more important than making them of larger magnification (i.e. longer). In astronomy, it is the wider telescope that lets you see more distant objects, not the one with greater magnification. Herschel pictured the observable stars, the observable Universe as far as he was concerned, as a disk with its diameter extending across the Milky Way. The larger telescopes of the age revealed hazy nebulae in space as well as stars, but were they evidence of very distant aggregations of stars or simply aggregations within the disk of our galaxy, our Universe? The quite famous English philosopher Herbert Spencer wrote in 1858 about the existence of nebulae beyond our galaxy *such a belief is next to impossible*. 41 years later, he came to a more positive conclusion and wrote *such a belief is impossible*. So much for the wisdom of old age.

How wrong could our predecessors in the 19th century have been? Far from the Milky Way being the only galaxy in the Universe, we now know there are in excess of a hundred, thousand, million galaxies spread over the observable universe. It was the advent in the early

20th century of telescopes with extra large diameters and accompanying extra large light gathering power, and astrophotography, that enabled astronomers to do the detailed work that would change the old picture. In the early 20th century, a good number of the nebulae were identified as 'Island Universes' or, as we now say, galaxies, far outside our own Milky Way. Telescopes of today see galaxies by the million, in all directions.

Observable objects can be seen stretching across many GLY (giga-light-years) in all directions; galaxies are found to be grouped in gravitationally interacting superclusters, with cosmic voids in between. Our local supercluster is centred in the direction of Virgo and is about 150 million LY across. Within each supercluster of galaxies are smaller clusters, ours being called the 'local group'. It is about 4 million light years across, includes 2 giant spirals (our galaxy called the "Milky Way", and Andromeda, M31) several elliptical galaxies and many dwarf galaxies, including the large and small Magellanic Clouds. The closer we look to 'home' the more detail we can see and our own Milky Way galaxy is a complex structure 150 thousand LY across of old stars, young stars, stars dying and stars being born, all spinning about a huge black-hole of several million solar masses, located about 27,000 LY from us. The majority view these days is that our quite large Milky Way galaxy is an aggregate over the lifetime of the Universe of many smaller galaxies. Our modern view of the Universe on a large scale is so new in its details of dimension and structure that it has come within your own lifetime. You can read a bit more on this topic by reading my supplementary piece '*Where am I?*' that you'll find in the blue panel of our astronomy web page.

As I said earlier, it is gravity that is the major force governing the large-scale structure of the Universe.

JSR