Quantitative Problems in Astronomy

In the Astronomy course we meet a number of universal laws of Physics that relate different quantities through exact equations. Most of these laws can be applied to a very wide range of science, not simply to astronomy. Each law allows the exact calculation of one quantity in the formula if the other quantities in the formula are all known. You will need a simple calculator to work out the following examples. The formulae are given here in the notation of the course. Put the correct units after each calculated number.

1) Kepler's third law relates the period *P* of a planet, in years, to its average distance from the Sun, *a* in AU: $P^2 = a^3$

   Find the period in years for the average distances of
   a) 0.387 AU (*Mercury*)
   b) 9.539 AU (*Saturn*)
   c) 17.9 AU (*Haley's comet*)

   Find the average distance from the Sun of asteroids that are observed with periods of
   a) 2.0 years
   b) 3.721 years
   c) 11.8 years

2) Newton's second law of motion relates acceleration, *a* in ms^{-2}, produced by a force *F* in Newtons (N) acting on a mass *m* in kg: $F = ma$

   Find the
   a) force that produces an acceleration of 2 ms^{-2} on a mass of 10 kg
   b) mass that is accelerated by 9.8 ms^{-2} using a force of 20 N
   c) acceleration caused by a force of 20 N acting on a mass of 0.5 kg
   d) acceleration produced by a force of 3.54×10^{22} N acting on a mass of 5.975×10^{24} kg

3) Weight, *W* in Newtons, is related to mass, *m* in kg, and the local acceleration due to gravity, *g* in ms^{-2}: $W = mg$

   Find the
   a) weight of an 80 kg person on Earth where g = 9.82 ms^{-2}
   b) weight of an 80 kg person on the Moon where g = 1.62 ms^{-2}
   c) weight of a 100 kg rover on Mars where g = 3.73 ms^{-2}
   d) mass in kg of a lander whose weight on Mercury where g = 3.73 ms^{-2} will be 50 N

4) Newton's law of gravitation gives the size of the attractive force, *F* in Newtons, between two masses m_1 and m_2 in kg separated by a distance *d* in metres: $F = G\frac{m_1 m_2}{d^2}$, where $G = 6.67 \times 10^{-11}\ m^3kg^{-1}s^{-2}$

   Find the
   a) force exerted by two lead balls each of mass 3 kg separated by a distance *d* of 0.3 m

b) force between the Earth of mass 6x10^{24}kg and a satellite of mass 100 kg at a distance of 6.5x10^{6} m from the centre of the Earth

c) force exerted by the Sun of mass 1.989x10^{30} kg on the Earth of mass 5.975x10^{24} kg in orbit at a distance of 1.5x10^{11} m

5) The energy *E*, in Joules, associated with a mass *m*, in kg, is given by Einstein's famous formula involving the speed of light c, in ms^{-1}: $E = mc^2$, $c = 3.0\text{x}10^8\,\text{ms}^{-1}$

Find the

a) energy in 1 g of hydrogen

b) energy in 1 kg of uranium

c) mass equivalent of 1MW hour $(=10^6\,\text{x}3600\text{J})$, the electrical energy taken by 200 houses each consuming 5 kW for an hour.

6) The inverse square law of radiation says that the energy per m^2 per second at a distance *d* from a source decreases as $1/d^2$: $\text{Energy}\,(\text{m}^{-2}\text{s}^{-1}) \propto 1/d^2$.

If the energy per square metre of sunlight at the Earth (distance 1AU) is about 1380W, what is the Sun's energy flux on

a) Mercury at a distance of 0.387 AU

b) Mars at a distance of 1.524 AU

c) Pluto at a distance of 39.44 AU

7) The kinetic energy (*KE*), in Joules, of a body is given by its mass *m*, in kg, and its velocity *v*, in ms^{-1}: $KE = \frac{1}{2}mv^2$

Find the

a) kinetic energy of a person of 80kg cycling at a speed of 5 ms^{-1}

b) kinetic energy of a lorry of mass 40 tonnes (1 tonne = 1000kg) travelling at 30 ms^{-1}

c) kinetic energy of a meteorite of weight 0.1 kg arriving at the Earth with a speed of 20 km s^{-1}

d) kinetic energy of the Earth of mass 5.975x10^{24}kg travelling around the Sun at a speed of 29.87 km s^{-1}

8) The *density*, in kg m^{-3}, of a body is determined by its mass *m*, in kg, and its volume *v*, in m^3: $density = \frac{m}{v}$. The volume *v*, in m^3, of a sphere of radius *r*, in m, is: $v = \frac{4}{3}\pi r^3$

Find the

a) density of an apple of mass 0.12kg and volume 1.5x10^{-4} m^3

b) density of the Earth, taken as a sphere of mass 5.975x10^{24}kg and radius 6.378x10^{6}m

c) density of Titan, taken as a sphere of mass 1.35x10^{23}kg and radius 2575 km

Units and geometry

9) Angles in Astronomy are traditionally measured in degrees (°), minutes (′) and seconds (″) of arc: $1^{\circ} \equiv 60'$; $1' \equiv 60''$

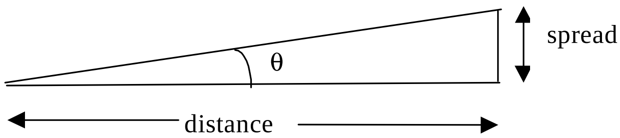
How many arc seconds are there in
a) 1.5°
b) 0.1°
c) 1.5′

9) Distances in astronomy can be measured in m, km, astronomical units (AU), light years (LY) or parsecs (pc):

$$1\,\text{pc} = 3.262\,\text{LY} = 3.086\times10^{16}\,\text{m}$$
$$1\,\text{LY} = 6.324\times10^{4}\,\text{AU} = 9.461\times10^{15}\,\text{m}$$
$$1\,\text{AU} = 1.496\times10^{8}\,\text{km} = 1.496\times10^{11}\,\text{m}$$
$$1\,\text{km} = 1.000\times10^{3}\,\text{m}$$

Find the
a) distance in m to the nearest star Proxima Centauri at 4.2 LY
b) distance in LY to the Large Magellanic Cloud measured as 48 kpc
c) distance to Saturn in km when it is 12 AU from the Earth

10) Small angles frequently arise in astronomy. They are a measure of *spread/distance*.



On a calculator, the (small) angle in degrees is given by: $\theta = \tan^{-1}\left(\dfrac{spread}{distance}\right)$.

a) The Moon has a diameter of 3476 km. What angle is measured across its equator when the Moon is at an average distance of 384,000 km?
b) As the Huygens probe begins its decent into Titan’s atmosphere it looks back at Saturn, diameter 120,660 km at a distance of 1.22 10^6 km. What angle does Saturn appear in the field of view of the probe?
c) The Earth’s orbit is about $300\text{x}10^6$ km in diameter. What angle does it subtend when seen from a nearby star at a distance of 6.5 LY $\left(1\text{ LY} = 9.461x10^{12}\,\text{km}\right)$?

10) The surface area of a sphere *A*, in m^2, is found from the radius *r* of the sphere in m as: $A = 4\pi r^2$

Find the
a) surface area of the Earth of radius $6.378\text{x}10^6$ m
b) surface area of the Sun of radius $6.962\text{x}10^5$ km
c) radius of a sphere whose surface area is 1000 m^2