About Orbits of Satellites, Moons, Planets and Spacecraft

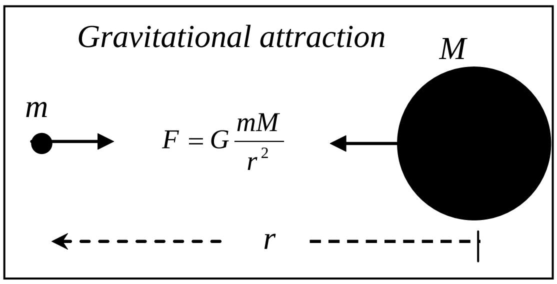
Personal introduction

The mathematics of the ellipse, parabola and hyperbola (the 'conic sections') was a favourite subject on which to spend many lessons for the more advanced schoolboys and schoolgirls in my younger days. Indeed, I had to buy an entire schoolbook on the subject (part III of Brown & Manson's *Elements of Analytical Geometry*). The topic was more complicated than the mathematics of circles but simple enough that exact results could often be deduced. As it happens, the possible orbits of a body under the gravitational influence of another are conic sections, so textbooks on Dynamics for students always had sections on orbital motion, largely as mathematical exercises decades before any satellites were launched. Indeed, generations of would-be physicists were brought up with the 19th century works of Routh, Lamb or Loney knowing much more about orbits than any modern Honours physics student.

The first artificial satellite (Sputnik) was launched in 1957 and over the next two decades mankind went to the Moon and probes were launched to all the nearby planets. At last, you may think, 'orbits' became a subject of real current interest. Maybe it did, but over the same period extensive conic section teaching fell out of the advanced school syllabus and many details about orbits fell out of most physicist's degree programme. Even basic orbital physics has now become a specialist topic, something one tends to learn only if there is a special need, not a topic of general interest. Yet 'orbits' have never been more relevant to topics of everyday life, from GPS satellites, environmental sensing and imaging satellites, space probes to interests in new moons discovered around solar system planets and new planets discovered around other stars. The orbits of asteroids and comets that might strike the Earth have never seen as much publicity as they now get. The intention of this piece is to say something about the principles involved, highlight some results about orbits and their generality, introduce a little of the vocabulary used without indulging in mathematical proofs. As I write this introduction I have no idea how this piece will turn out, for there is the electronic equivalent of a blank piece of paper in front of me.

Some time later: my 'paper' is no longer blank and I'll present 15 useful facts about orbits (the '**Results**') and give illustrations of their relevance. I'll end by introducing a number of other topics without going into fine detail about them.

The ingredients



The first ingredients are two orbiting bodies: a mass m, orbiting another mass M under the influence of their mutual gravity described by Newton's law, namely that the force each exerts on the other is $F = G\dfrac{mM}{r^2}$, the force being directed along the line between them. The symbol r represents the distance between the bodies and G is the 'universal gravitational constant' whose value in SI units is 6.67×10^{-11} $m^3 kg^{-1} s^{-2}$. Newton's law is strictly true for 'particles' of no significant size. Clearly even a mathematician would acknowledge that a planet or the Sun isn't a 'particle'. However, Newton himself deduced that the gravitational effect of a uniform spherical body was the same as that of a particle of the same mass located at its centre, so long as one was beyond the surface of the body. The sphere doesn't need to be homogeneous, just isotropic when viewed from its centre. This result greatly simplifies talking about satellites orbiting even close to the Earth.

Gravity is usually described as a 'central force' because the direction of the force is towards the centre of the attracting 'particle'. The spherical result also hints that small irregularities in the shape of the Earth will make real orbits slightly different from their ideal shape, which is true and an analysis of these differences has told us some facts about the internal structure of the Earth. Satellites around other planets (and they've orbited all the planets out to and including Saturn) provide equivalent information away from the Earth. The spherical result also hints that working out the orbit of a probe around a very irregular body such as a comet or any of the smaller asteroids will be a complicated problem, which it is.

The second ingredient in the presentation here is the simplification that the mass of the orbiting body is small compared with the mass of the parent body. This is obviously true for satellites and is even true for planets orbiting the Sun. The Sun's mass is a third of a million times greater than the Earth's mass. This simplification just makes the maths simpler but doesn't alter basic results. For instance the mutual orbits of binary stars are in most respects similar to planetary orbits but one has to be more careful about defining quantities such as the frame of reference for the orbit.

The third set of ingredients needed to describe orbits are Newton's laws of motion and concepts that derive from them. Basically '$\mathbf{F} = m\boldsymbol{a}$' where $\boldsymbol{a}$ is the acceleration of the body. From the acceleration can be derived velocity v, and such detail as distance travelled, period of the orbit, etc.

If you look back at the form of the gravitational force F you'll see that the mass m of the body occurs on both sides of Newton's law of motion '$\mathbf{F} = m\boldsymbol{a}$' and hence we can deduce our first result.

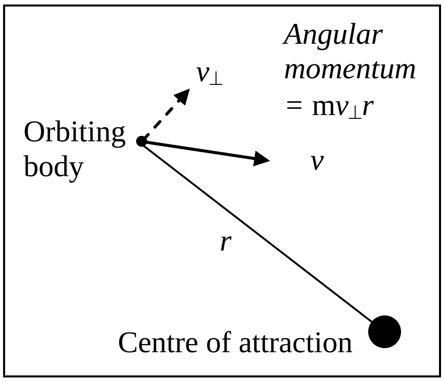
Result 1 *The orbit of a body doesn't depend on its mass*. Hence the mathematics relevant to orbital motion is usually written for unit mass (1 kg) and I'll do that in later sections. Of course the energy required to put something into orbit does depend on its mass but once initial conditions of velocity and location are specified then the mass of the orbiting body doesn't come into determining what the orbit will be.

The initial velocity of the orbiting body defines a line in space and this along with the centre of the attracting body defines a plane. Hence:

Result 2 *The orbit always lies in a plane containing the centre of attraction*. For example, Earth satellites always orbit in a plane containing the centre of the Earth. Geostationary satellites therefore must lie above the equator. It's not possible to have a satellite stationary above Aberdeen (latitude 57° N) or, indeed, any place not on the equator since the orbit required would be in a plane that doesn't contain the centre of the Earth.

A 'stationary' satellite can be imagined in a circular orbit sitting above a constant place on the equator of any planet or moon. In the absence of the influence of neighbouring bodies, its height (h) from the surface can be calculated if the body's radius R and rotational period T (seconds) is known. The body's gravitational attraction GmM/r^2 provides the force $m\omega^2 r$ necessary to hold the body in a circle, where r is its distance from the centre of attraction and $\omega = 2\pi/T$ is the angular rotational speed of the body (in radians s^{-1}). In a couple of lines you will see that the height $h = r - R = (GM/\omega^2)^{1/3} - R$. For the Earth this works out as ~35770 km;

for Mars 17030 km and for the slow rotating Moon a distance of over 86000 km, a distance too large to maintain an orbit because of the influence of the Earth.



The *angular momentum* of a body about a point is the product of three quantities: the mass of the body (*m*), the distance of the body from the point (*r*) and the component of the body's velocity perpendicular to the line from the point ($v_\perp$). To change the angular momentum of a body there must be a twist on it, as measured from the point. Taking the obvious 'point' as the centre of attraction, then another result immediately follows from the central nature of gravitational attraction, namely result 3.

Result 3 *The angular momentum of an orbiting body about the centre of attraction is constant*. This implies that orbital motion is in fact simpler than many other kinds of motion. We shan't make the deduction but Kepler's second law about a planet (or comet or asteroid) sweeping out equal areas in equal times follows from this result.

The constancy of angular momentum played an important role in the formation of the solar system from a rotating cloud of gas and dust. The outer parts of the cloud were orbiting around the inner material but the whole cloud was slowly pulled together by its own gravity. Collapse parallel to the axis of spin would have converted the cloud to a disk and collapse perpendicular to the axis of spin was less easy for it required an increase in rotational speed of the cloud as it shrunk because of conservation of angular momentum. This is the reason the solar system is pretty flat.

The energy of an orbiting body

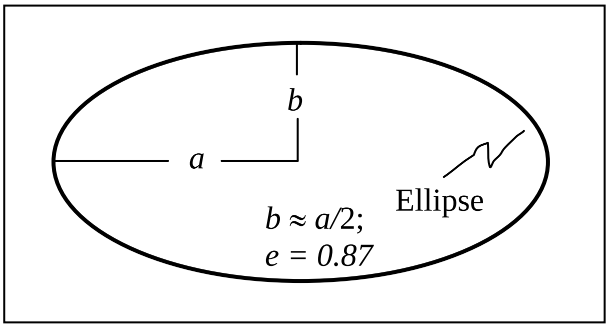
Orbiting bodies have two kinds of energy: kinetic energy (abbreviated *KE*) and potential energy (*PE*). The kinetic energy is just ½mv^2 as you would expect and hence depends on how fast the body is travelling. The potential energy is $PE = -\frac{GM}{r}$ and hence depends on how far from the centre of attraction (*r*) the body is. The total energy *E* is just the sum of the two, i.e. $E = KE + PE$.

Result 4 *If no force other than the central force acts on the orbiting body, then its total energy is constant*. This is just a statement of the law of conservation of energy. As the body changes its distance from the centre of attraction then it must change its speed and there is a conversion between potential energy and kinetic energy.

Result 5 a. *If the total energy (E) is positive, the orbit is a hyperbola*. The hyperbola is an open curve and the body will fly off 'to infinity'.

b. *If the total energy is zero, the orbit is a parabola*. The parabola is also an open curve and the body will eventually fly off.

c. *If the total energy is negative, then the orbit is an ellipse*. The ellipse is a closed curve so the body will circulate around the centre of attraction. This is the most interesting case. The magnitude of the gravitational *PE* is greater than the magnitude of the kinetic energy for all elliptically orbiting bodies.



An ellipse is a body whose shape is described by two parameters, slightly more complicated than a circle, which just needs one parameter, its radius. There is a choice of which parameters to use for an ellipse but a very common pair is its major and minor axes, denoted 2*a* and 2*b*. See the accompanying figure. Ellipses can vary from circular (the two axes are the same length) to very long and thin ($a >> b$). A useful quantity to designate how fat or thin an ellipse is is its *eccentricity*, always represented by the symbol *e*. $e = 0$ represents a circle and *e* near 1 a very thin ellipse. Venus' orbit is almost circular ($e = 0.0067$); Halley's comet has a highly eccentric orbit ($e = 0.967$). *b*, *a* and *e* are related: $b^2 = a^2(1 - e^2)$. It's not obvious but a parabola has $e = 1$ and a hyperbola has $e > 1$.

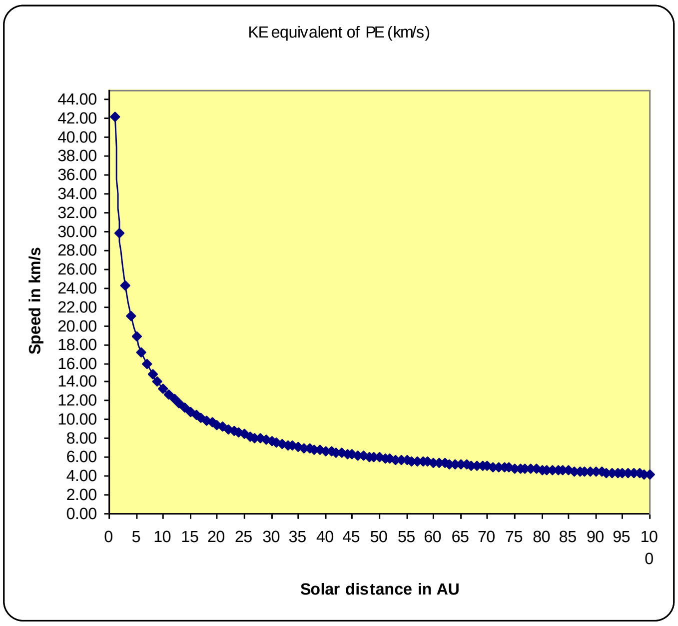
I said above that as a body varies its distance from the centre of attraction, the balance between *PE* and *KE* changes. Going towards the centre of attraction, the potential energy decreases and the kinetic energy increases. Since *PE* doesn't depend on direction, we have another result:

Result 6 *The speed of a body in orbit depends on how far it is from the centre of attraction not on which direction it is going at that distance.*

The speed that must be given to a body to put it into a parabolic orbit (in which it will never return) is known as *the escape velocity*. In this case $E = 0$ and the $KE = -PE$. i.e. $\frac{1}{2} v_{exc}^2 = GM/r$.

Result 7 *The escape velocity v_{esc} is given by $v_{exc}^2 = 2GM/r$, where r is the initial distance of the body from the centre of attraction.* This is the minimum speed necessary to ensure the body will not enter a returning orbit. Giving it more than this speed will put it in a hyperbolic orbit that will also ensure it won't return.

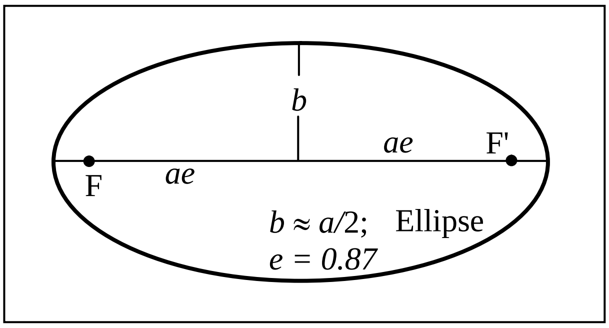
The potential energy has a large negative value for objects no further from the Sun than the planets. The Excel graph here shows the speed an object needs to have in km s^{-1} for its kinetic energy to match its potential energy in size. If that were to happen, the total energy would be zero and the body would have enough energy to escape from the Sun's gravitational pull. The points are plotted from 1 AU (the distance of the Earth from the Sun) to 100 AU, about twice the greatest distance of Pluto.



The focus of the orbit

It almost seems 'obvious' that the centre of attraction should be the centre of the orbit but like a good many 'obvious' implications, this one is wrong.

Result 8 *The centre of attraction lies at the focus of the orbit*, a special point in relation to a conic section. I'm going to concentrate on ellipses. An ellipse has two foci F and F' each a distance *ae* from the centre along the major axis (see the nearby figure). Since *e* is zero for a circle, then a circular orbit does indeed have the centre of attraction at the centre of the circle. For Halley's comet, the focus of the ellipse not occupied by the Sun is very near the end of the major axis and this does make sense in that the comet is known to go out a long way from the Sun, come in close to the Sun, swing round and then go out again. If the ellipse in the diagram represents an orbit, then the centre of attraction is at one of the foci, F or F'. One consequence of this is that the Earth is not at the same distance from the Sun in winter and summer. In fact it is closer to the Sun in northern hemisphere winters.



What orbit will a body follow?

Suppose a body (it could be a satellite, a spacecraft, an asteroid chunk or even a planet) is launched with velocity *v* in some direction at a distance *d* from the centre of attraction, what orbit will the body follow? Earlier it was said (Result 1) that the orbit didn't depend on the body's mass so **from here on I'll consider a body of unit mass**. Finding the answer is not quite straightforward. Even a good physics student given this question 'blind' would struggle to work out exactly how to manipulate the mathematics most effectively to come up with the answer. The initial conditions define the angular momentum (called *h* in many texts) of the body. Anyway, I'm going to quote the answer.

Result 9 $$a = -\frac{GM}{2E}, \quad e^2 = 1 + \frac{2Eh^2}{(GM)^2}.$$

That's it. At least these 2 parameters give the shape of the orbit. Notice that the extent of the ellipse, the semi-major axis *a*, is determined only by the body's energy. To find the orientation of the ellipse in space needs a bit more work, which we shan't do here. The relationships above give the major axis and the eccentricity of the orbit in terms of the initial angular momentum *h* per unit mass and the energy *E* per unit mass. I'm most interested in elliptical orbits for which, as we've seen, the total energy *E* is negative. Using the relationship above, the minor axis parameter *b* is therefore given by $b^2 = -\frac{h^2}{(2E)}$. These relationships don't tell you which focus of the ellipse is the location of the central body. The velocity of the orbiting body will be different in the two cases so you also need the velocity to determine the correct focus.

For a body of unit mass travelling in a circular orbit with velocity *v* the $PE = -v^2$ and the $KE = \frac{1}{2}v^2$. Hence the total energy $E = -\frac{1}{2}v^2 = PE/2$.

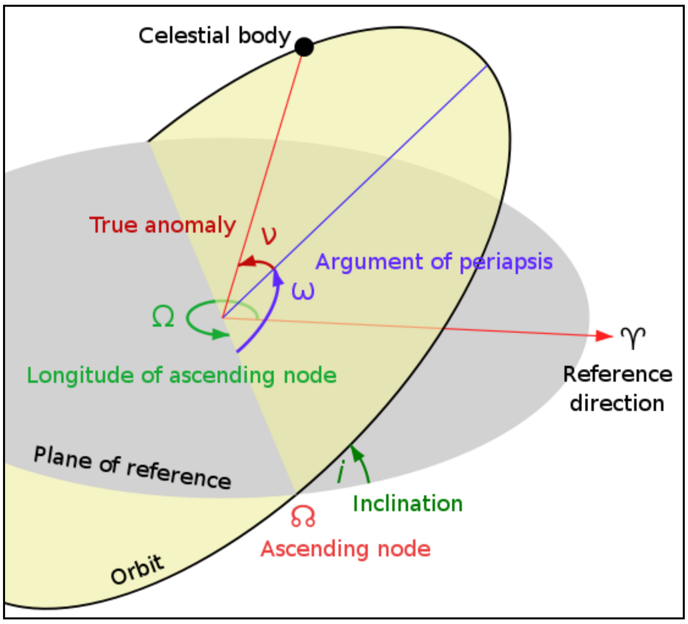
Result 10 *For a circular orbit the kinetic energy is equal to half the magnitude of the potential energy.*

The kinetic energy is therefore quite high in relation to the potential energy. Most of the planets have pretty circular orbits. (In addition to the example of Venus given above, Mercury is the least circular with e = 0.2056; Earth e = 0.0167, Mars e = 0.0935, Jupiter e = 0.0489, Saturn e = 0.0565, Uranus e = 0.0457, Neptune e = 0.013. On a timescale of millennia, as planets exchange small fractions of their energy, the eccentricities of their orbits change.) Increasing the speed of a body in circular orbit, if that were possible, by a factor of ✓2 will double its kinetic energy and hence make its total energy zero. This will be enough to turn the orbit into a parabola, taking the orbiting body off to infinity.

Another implication of this result is that since the potential energy is less in magnitude further from the centre of attraction, the further away a body is the slower it will be going. This is why the further out planets from the Sun travel more slowly in their orbit. More distant Earth satellites, which need more energy and hence fuel to put them into orbit, have less speed than lower satellites, which seems a bit counter-intuitive.

The result applies to non-circular orbits too: the larger the parameter a (equivalent to the radius of a circular orbit) the higher the energy of the orbit. (Result 9 can be written $E = -GM/2a$). E.g. the Earth travels faster than Mars so once a craft just leaves Earth orbit is its larger speed (inherited from the Earth) enough to get it to Mars? The answer is 'no'. Mars is in a higher energy orbit than the Earth and the extra speed of the Earth is not sufficient to get a craft to Mars, or anywhere further from the Sun than Earth.

Result 11 *6 parameters are needed to describe the position of a body in orbit, known as the 'orbital elements'.* This is mainly a geometrical statement. However, one of the implications is that if a new body is seen in the sky such as a possible asteroid or trans-Neptunian object, then its orbit can be deduced from 3 observations of its declination and right ascension (essentially its celestial latitude and longitude) made with a modest time in between them, say a couple of weeks. Of course the observations will have been made from the moving, rotating platform of the Earth so some mathematics must first be done to find the equivalent observations relative to a plane containing the Sun before the orbital parameters can be found. Gauss showed how to do this in the 19th century. Taking more observations or a longer time between them will give the orbital elements to higher accuracy. A related question is how do NASA and ESA find the orbits of their space-probes to pin-point accuracy when the probes are too far away to be seen? I'm leaving this question for your exploration!



The adjacent diagram from Wikimedia Commons will make it clearer why 6 elements are needed and what they are. We start with a reference plane in which there is a reference direction. The orientation of the plane of the orbit has to be defined in space. The orbit intersects the reference plane in two points, called the *ascending node* and *descending node*. These nodes turn out to be a central concept when trying to understand the occurrence of eclipses of the Sun and Moon. They are also important in understanding if an asteroid in an orbit that crosses the Earth's is likely to collide with the Earth. Anyway, defining the orbital plane in space takes 2 parameters, one to define its *inclination* (i) relative to the reference plane and the next to define the angle between the reference direction and the line to the ascending node (Ω in the diagram).

Next, the shape of the orbit is defined by the elliptical constants a and e as discussed earlier. The direction of the major axis (glorified with the words the *augment of the periapsis,* ω in the diagram) must be defined to determine the orientation of the orbit within its plane. The final element is the so-called *epoch* of the body in the orbit, basically how far around the orbit the particle is (designated the *true anomaly*, ν (Greek 'nu') in the diagram) the angle measured from the major axis. Phew.

Result 12 *The period of revolution T (in planetary terms, the body's year) can be shown to be given by* $T^2 = 4\pi \dfrac{a^3}{GM}$.

This result puts into mathematical symbols Kepler's third law. Kepler deduced his 'law' from the half dozen special cases for which he had planetary data but Newton showed that it was firmly based on the mathematics of elliptical orbits. Only one orbital parameter is involved, namely the semi-major axis of the elliptical orbit. Hence all bodies with the same value of a have the same period whatever the eccentricity or their orbits.

In terms of dynamical parameters:

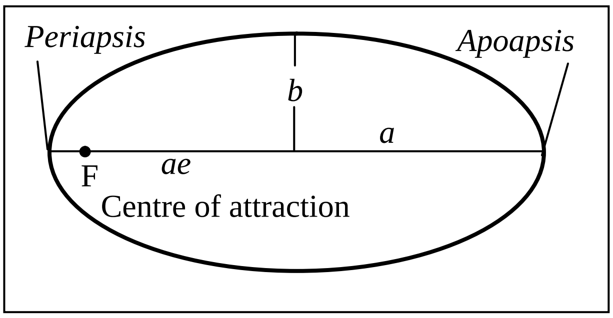
Result 13 $T^2 = 4\pi \dfrac{(GM)^2}{(-2E)^3}$, showing that the period depends only on the total energy of the orbiting body. Only one dynamical parameter is involved.

The distances of closest approach and furthest out

When the orbiting body is closest or furthest from the centre of attraction, it is travelling at right angles to the radius from the centre. These points are called the *apses* of the orbit. The *periapsis* is when the body is closest, the *apoapsis* when the body is furthest. [When the centre of attraction is the Sun, the words *perihelion* and *aphelion* are used for the nearest and furthest points; when the centre of attraction is the Earth, the words *perigee* and *apogee* are used; in relation to a star, *periastron* and *apastron*. This is rather a lot of Greek to get across a simple idea].

Result 14 From the picture of the ellipse, *the periapsis distance from the centre of attraction is a(1 – e) and the apoapsis distance a(1 + e).*

For a planet, these distances strongly determine how much solar heating it will get and the change in solar radiation and heating that will take place during a planetary year. For a satellite, the periapsis may be important in determining the drag it will experience in the upper atmosphere and hence its lifetime before it comes to Earth.



Will it crash and burn?

Satellites in low Earth orbit, say 1000 km high, have to be in a nearly circular orbit even if they change their height by several hundred km. What counts is their distance from the Earth's centre and since that is over 7000 km then a few hundred km change in distance is still a nearly circular orbit. In short, they must have been given a kinetic energy that is about half the magnitude of their negative potential energy, which at 7000 km is $-GM/7\times10^6 = -3.4\times10^6$ J kg^{-1}. Half this magnitude implies a speed of 1.85 km s^{-1} or about 6600 km h^{-1}. You can see why most satellites are launched from near the equator because the rotation of the Earth gives points near the equator a speed of about 1600 km h^{-1}, which is at least a start. Many Earth monitoring satellites are in low-Earth orbit and if they aren't given the necessary speed they will indeed crash and burn. Another aspect is that satellites in low-Earth orbit don't have to lose much of their speed due to slight drag of the very thin extended Earth's atmosphere to have their orbital parameters changed enough to become slightly non-circular. They will then enter the lower atmosphere and burn up.

Objects in orbit around the Sun are not only launched by mankind but are launched as a result of interactions within the belts of asteroids, the Kuiper belt and the Oort cloud. If an object's orbits is conspicuously elliptical the object will head in towards the Sun. Is it likely any will crash and burn in the Sun, or at least disintegrate near the Sun? The answer is 'yes', as can be seen using the information in earlier sections. Actually, the answer is also 'yes' from images sent back by the Soho probe that has been looking almost unblinkingly at the Sun for over a decade. These images show that over a year quite a number of comets that hadn't been spotted before plunge into the outer reaches of the Sun.

Result 15 *If an object is 'launched' in the solar system so that most of its energy is its potential energy, then its perihelion distance will be approximately $h^2/2GM$.* (h is its angular momentum per unit mass about the Sun).

The result can be found by some manipulation of the relationships already given. If this distance is less than about 20 million km then a comet or man-made object isn't likely to survive such a close approach to the Sun. Icy bodies in the Kuiper belt at a distance of 50 AU from the Sun have orbital velocities about 4.2 km s^{-1}. Most of this speed must be removed by some interaction or collision if a fragment is going to end up by going close to the Sun. This doesn't happen often but it does happen several times a year.

A miscellany of more advanced issues

Here are 12 further topics, five particularly relevant to planets and moons, six relevant to satellites and space-probes and one to most of them.

The influence of other planets	*Changing satellite orbits*
Osculating orbital elements	*Aerobraking*
Resonance of orbits	*Manoeuvrability*
Gravitational assist	*The Hill radius*
Hyperbolic orbits	*The 5 Lagrangian points*
Sun synchronous orbit	*The Roche limit*

The influence of other planets

If the orbits of all bodies in a solar system were independent, then the trajectories of the planets and moons would all be fixed by the initial conditions that launched them. However, gravitational attraction is present between every two bodies and so every planet, for example, feels a slight influence from all the other planets. Newton's law of gravity lets one work out what the relative size of the interplanetary forces are and they are all very small compared with the force of the Sun. Even Jupiter, whose mass is greater than all the other planets put together, is less than 1/1000 times the mass of the Sun and at its nearest to the Earth it is more than 4 times further than the Sun. The net result of the pulling of the other planets is that the energy of a planet in its orbit varies only a little over time, the changes averaging out to zero. Since the major orbital axis (a) is determined only by the planet's energy then this doesn't change much. However, the change in eccentricity produced by the changing angular momentum of a planet as the other planets speed it up or slow it down do alter the shape of the orbit. The Earth's orbit, for example, can vary from completely circular ($e = 0$) to an eccentricity of about four times the present value. This has a very important effect on climate changes on a timescale of many tens of thousands of years, for it alters the focus of the orbit in relation to its centre and hence the variation in how far the Earth is from the Sun. In addition, the orbital inclination can vary by about 3 degrees and the whole orbit precesses around in space. This last effect is another influence on climate because the Earth's rotation axis is more or less fixed in space and hence the precession of the orbit changes the times during the year when the Earth is nearest and furthest from the Sun. Our meteorology course says more about these climatological effects. They are not little effects but are thought to be the main reasons for the approximately periodic appearances of ice-ages. Go to miscellany table

Osculating orbital elements

Surely if the Earth's orbit slowly changes in dimension, orientation and tilt over the centuries then it's not really the ellipse that astronomy books talk about. Ellipses are closed curves. Yes, that's quite true. The orbits of asteroids and comets show similar effects on a shorter timescale, partly because they travel closer to Jupiter and Jupiter is the biggest changer of orbits in the solar system. If in addition to the Sun you include the gravitational influence of Jupiter and other 'stuff' in the solar system on orbiting bodies then all orbits are immensely complex over a long period of time. However, for any short period of time (short on an astronomical timescale) the orbits look like ellipses. The elliptical parameters of any orbit are called its 'osculating orbital elements'. For some reason 'osculating' is quite a memorable word. The comparatively slow changes in the orbit are described by the changes in the osculating elements. For example Mercury's orbit is described as elliptical but its perihelion precesses at about 0.15° per century from this effect. Given that Mercury makes over 400 orbital revolutions per century, the description by the osculating elements is pretty good. If you need great precision, for example to guide a space probe to a rendezvous, then you need to factor in the changes in the osculating elements.

Asteroid osculating elements generally change faster than planetary ones. Take one example I have to hand, the asteroid Seneca, which sometimes comes closer to the Sun than the Earth does. Over a period of a century and a half, Seneca changes its semi-major axis by 4%, its eccentricity by 8%, its perihelion distance by 15% and the inclination of its orbit by about 2°. These changes are mainly caused by Jupiter and since Jupiter's orbit is predictable over many centuries so the orbit of Seneca is still predictable quite far into the future. An even more complicated situation is the orbit of our Moon. Here the main centre of attraction is the Earth while the Sun and the out-of-round shape of the Earth are the main causes of perturbations. For example the line of nodes of the Moon's orbit rotates once in 18.6 years, the line joining the perigee and apogee rotates once in 8.85 years. It's all rather complicated but over a period of a few months the Moon's orbit is still pretty elliptical and the parameters of the ellipse are given by the current osculating elements. To predict eclipses, the changes in these elements must be known. For a final example, showing ESA's INTEGRAL a satellite in a constantly changing elliptical orbit, see the video. Go to miscellany table

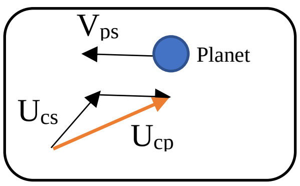
Resonance of orbits

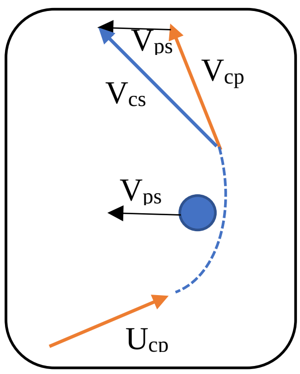
Orbital resonance is quite common in the solar system. It can happen when more than two bodies interact over a long period of time. The periods of Neptune and Pluto, for example, are in a resonance very close to 2:3. The periods of Jupiter's moons Io, Europa and Ganymede are in resonance close to 1:2:4. The conspicuous gap in Saturn's rings is because of a 1:2 resonance in the period of a body within the gap and the moon Mimas. There are Kirkwood gaps in the asteroid belt for asteroids whose orbital periods resonate with that of Jupiter in several ways, the 1:2 resonance being clearest. There is even a very close 8:13 resonance between the orbits of Earth and Venus. Is this last one co-incidence? It's actually hard to tell because working out exactly the result of many bodies interacting over a few billion years is not possible. The special feature of resonance is that it is more stable than non-resonance so that once a resonance is set up, it tends to persist. Go to miscellany table

Gravitational assist

Often referred to as a gravitational slingshot, the assist can be used to speed up or slow down a spacecraft by making its trajectory pass close to a planet. Prior to the space age, nobody was really interested in this problem because planets, comets and asteroids were considered by most to travel in their own independent orbits. The really fussy would take into account 'perturbations' of the orbits due to the influence of other objects but it was the orbit change that was of most interest, not the speed change. The gravitational assist of the space age is most importantly a way of changing a craft's speed. To work out exactly what will happen is a tricky 3-body problem (Sun, planet and craft) in 3 dimensions. One can get an idea of why it works by looking at a 2-dimensional example with some simplifications.

Far from the planet, the orbit of the craft is controlled by the Sun and will have been plotted in the frame of reference of the Sun. Look instead a how the craft appears in the frame of reference of the planet. Quite close to the planet, the gravity of the planet will exceed that of the Sun. The solar orbit provides an initial velocity for a (hyperbolic) orbit that the craft will take around the planet. Assuming the craft does not enter the planet's atmosphere, there is no process that takes any energy from the craft. Therefore, retreating from the planet the craft will then continue in a different direction but with the same speed as it had initially. At first sight there seems to be no slingshot but look at how the velocities appear in the frame of reference of the Sun.





The following two diagrams show velocities relative to the Sun's frame of reference. Let U_{cs} be the velocity of the craft relative to the Sun as it approaches the planet; V_{ps} the velocity of the planet relative to the Sun (taken as constant over the time of the slingshot) and U_{cp} the initial velocity of the craft relative to the planet. Then $U_{cp} = U_{cs} - V_{ps}$. This combination of velocities is shown in the first diagram on the right. U_{cp} is shown in orange. The velocity relative to the planet is larger than the velocity relative to the Sun, for the planet is heading towards the craft.

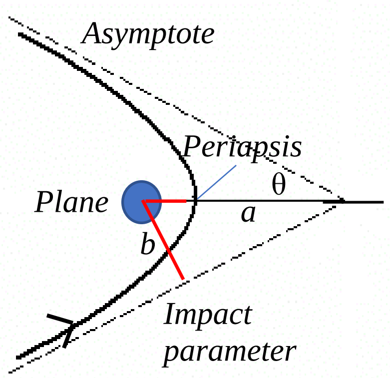
After partly orbiting the planet, the craft's velocity is V_{cp}, the same magnitude as U_{cp} but in a different direction. Combining velocities as before, the velocity of the craft relative to the Sun is $V_{cs} = V_{cp} + V_{ps}$. The diagram shows the result in blue. Its magnitude (the speed of the craft) is greater than U_{cp} and even greater than the original velocity of the craft in its orbit. The closer the craft goes to the planet, the more its trajectory is turned in the direction of the planet and the larger the slingshot. You can draw similar diagrams for the craft passing in front of the planet and see that in this case the slingshot can reduce the energy of the craft. This looks to be the opposite of what you might expect but remember that gravity is an attractive force, not a repulsive one.

Taking a specific example, the BebiColombo mission to Mercury (now launched as I update this piece) will have one flyby of Earth and two of Venus to reduce its perihelion to about that of Mercury, followed by 6 flybys of Mercury before Mercury orbit is achieved. Fine control of the trajectory will be provided by electric propulsion thrusters generating xenon ions, though they might better be called 'nudgers' than thrusters since the force provided is only a few hundred mN on a craft of mass about 4 tonnes. You can ponder whether the slingshots are needed to give the craft energy or take away energy. The kinetic energy of an orbiting body at Mercury's distance from the Sun is greater than its kinetic energy at Earth distance but its potential energy is a lot less. It must also lose energy from orbiting the Sun to orbiting Mercury. The dynamics are complicated. The journey from Earth to Mercury orbit will take about 7 years. Go to miscellany table

Hyperbolic orbits

Most of the previous sections concern elliptical orbits but the section on gravity assist above, the interest in near misses and the first tracked appearance of an object from outside the solar system in 2017 (Oumuamua) all involve hyperbolic orbits. What are they?

As mentioned in Result 5, the hyperbolic orbit is the orbit passed a planet or moon of mass M_p when the total energy E of the body is positive. The initial speed of the body must therefore exceed the escape velocity of the planet. The hyperbola is a conic section looking a bit like a bent rod that goes off to infinity at either end effectively in straight lines. These are called the *asymptotes* of the hyperbola. The hyperbola has an eccentricity e greater than 1 and, like the ellipse, can be described with parameters a and b, though these no longer correspond to axes as they do in an ellipse. Strictly speaking the hyperbola equation describes a pair of similar curves but a spacecraft or orbital rock can of course be on only one curve. 'a' is given by the total energy of the orbiting body *in the frame of reference of the planet*: $E = GM_p/2a$, chosing a positive (see Result 9). In the planet's frame of reference, the energy of a body that just escapes is zero so E represents the kinetic energy above the escape energy. If v is the incoming velocity, this can be written $\frac{1}{2}v_\infty^2$ per unit mass = $\frac{1}{2}v^2 - \frac{1}{2}v_{esc}^2$.



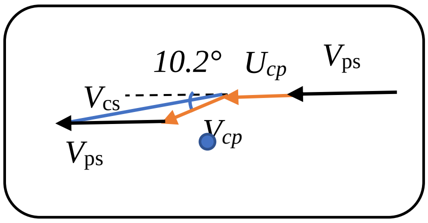
As with an ellipse, a, b and e are related, in this case $b = a(e^2 - 1)^{½}$. 'b' has the convenient interpretation as the *impact parameter*, namely the perpendicular distance from the centre of the attracting body to the incoming particle asymptote – see the nearby figure. The energy and the impact parameter determine a and b and hence $e = (1 + b^2/a^2)^{½}$. The asymptotes make an angle to the axis of $\theta = \tan^{-1}(b/a)$. Their intersection point is conveniently taken as the origin, if coordinates are needed. The distance from there to the periapsis point is 'a'.

The closest approach of the orbiting body to the planet is the *periapsis*, whose distance from the planet is $a(e - 1)$. The amount of bending that the planet exerts on the trajectory is given by $180° - 2\theta$.

To interpret the symbols above a bit, hyperbolic orbits with only a slight bend have large eccentricity e and a large periapsis distance. Tightly bent orbits have an eccentricity close to 1.0 and a small periapsis distance. If b is less that the radius of the central body, then the orbiting body will collide.

For a freely travelling visitor to the solar system like Oumuamua, it is likely to be the Sun and not a planet that creates the hyperbolic path. For a craft or asteroid approaching a planet, both the planet and the object experience similar forces from the Sun, though they are not likely to be travelling in the same direction. The force of the planet, mass M_p, on the object at distance r_p from the planet will equal the force on the object of the Sun, mass M_s, at distance r_s from the Sun when $M_p/r_p^2 = M_s/r_s^2$. This gives $r_p = r_s \times (M_p/M_s)^{½}$. For the Earth, this is about 260,000 km, the distance when the Earth's force is as big as the Sun's force on an approaching object. A quoted estimate of when the planet's gravity starts to have an effect is $r_p = r_s \times (M_p/M_s)^{0.4}$. For the Earth, this works out as approaching a million km.

As an example, suppose an Earth-crossing asteroid is approaching us at 15 km s^{-1} (which is 3.8 km s^{-1} faster than the Earth's escape velocity of 11.2 km s^{-1}) with an impact parameter of 7 Earth radii or 44646 km. This is just larger than the height of geostationary satellites and if this happens the press will be flooded with red-letter headlines. Ignore any complication given by the Moon. The relevant energy per unit mass is $½v_\infty^2 = 50$ MJ. From above, $a = GM_p/2E = 3981$ km and $b = 44646$ km. Hence the eccentricity of the orbit $e = (1 + b^2/a^2)^{½} = 11.26$. The periapsis distance $= a(e - 1) = 40842$ km. The bending of the orbit by the Earth is only 10.2°. Certainly the asteroid would continue to miss the Earth, though it would skim below the height of geostationary satellites. The press will call it a 'lucky escape', though on that trajectory it was not going to collide.



The change in energy of the asteroid depends on its initial direction relative to the direction of the Earth. Suppose it approaches on a nearly parallel trajectory to the Earth which is travelling at speed V_{ps} of 30 km s^{-1} in its orbit, giving the asteroid a speed U_{cs} of 45 km s^{-1} relative to the Sun. (I've used the same symbols as in the previous section). After being deflected it has a speed V_{cs} of 44.77 km s^{-1} relative to the Sun (see the 'gravity assist' section) and has therefore been slowed a little by swinging in front of the Earth. A space probe wanting to make a significant change in its speed needs to go a lot closer to a planet than 7 planetary radii. Probes using the inner planets for gravity assist may well come within 1000 km of the surface. The BepiColombo mission mentioned earlier will execute flybys within a few hundred km of Earth, Venus and Mercury. Go to miscellany table

Sun synchronous orbit

A range of Earth observation satellites, including weather observation satellites such as EUMETSAT's polar orbiters, employ sun synchronous orbits. The essence of a sun synchronous orbit is that the satellite will pass over the same place at about the same solar time throughout a year. If you imagine yourself on the Earth's orbit going around the Sun once a year, then you have to turn around through 360° to keep pointing at the Sun. Turning its plane around in space once a year is what must be done for the sun synchronous satellite.

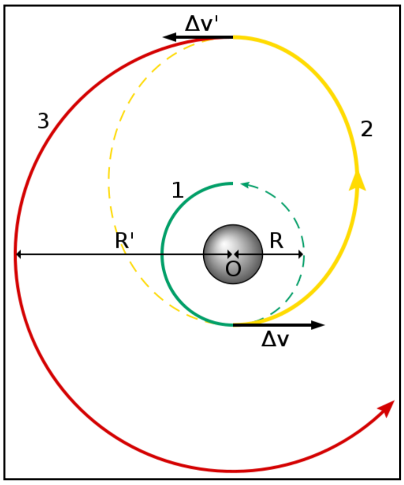
If you set a top spinning on a polished table so that it is exactly upright, it keeps spinning upright until the slight friction at the table slows it and induces wobble. If the top is spun at a slight angle to the vertical, it precesses because there is a torque on it created by the combination of its weight and the reaction of the table. The precession causes the spin axis of the top to circle around the vertical. Similarly, a satellite launched in an exact polar orbit will tend to stay in the same orbit. A satellite launched in an orbit inclined to a plane containing the poles will precesses due to a torque provided by the ellipsoidal shape of the Earth. The trick with the sun synchronous satellite is to get the plane of the orbit to precess at the rate of one turn per year, which is not much.

With the precession rate fixed, there is only one inclination of orbit for a given orbital height (and hence a given orbital period, for that too is controlled by the height). At least it doesn't matter what the mass of the satellite is, as the earlier results show that orbits are mass independent. For example, EUMETSAT's polar orbiters are at an altitude of just over 800 km; they orbit in about 100 minutes on a trajectory inclined to the equator at 98.7°. They don't go directly over the poles. This gives about 14.25 orbits per day. Because there are not an exact number of orbits in a day, the satellite won't be seen in the sky at exactly the same time each day and hence looking straight down won't see the same track every day. In addition, because the precession is constant, it doesn't even follow the true direction of the Sun in the sky, which varies slightly during the year because the Earth's orbit is elliptical with the Sun off-centre. Still, it is a reasonable approximation. The higher the sun synchronous satellite orbit, the more inclined to a polar plane it has to be, resulting in their being a limit over which sun synchronicity cannot be achieved. About 6500 km is the limit for an Earth satellite.

Orbital precession is something all satellite operators have to contend with, either compensating by firing thrusters while fuel is available or accepting that the orbits will change. In terms of orbital parameters (Result 11) the nodes of the orbit will precess and this rate can be found quoted for many satellites. Go to miscellany table

Changing satellite orbits

If you want to put a satellite into a medium or high orbit (e.g. a geostationary satellite at over 40,000 km from the Earth's centre), then blasting it all the way up with a rocket and then releasing it would be hugely wasteful of energy because it takes a lot of energy to put a rocket that high and the once the rocket has released the satellite the rocket is useless. It doesn't fall back to Earth because it has acquired a similar orbital speed to the satellite. In fact it would pose a hazard as 'space junk'. The technique is to put the satellite into low-Earth orbit and give the satellite some propulsive ability of its own, or for larger transfers an attached transfer orbit stage, that is capable of raising it to a higher orbit. The simplest but quite frequently required case is to change from a circular low-Earth orbit to a circular medium- or high-Earth orbit. This is done by a Hohmann transfer orbit, a technique devised by Hohmann as early as 1925, long before any artificial satellites existed.



The Hohmann transfer orbit is an elliptical orbit whose periapsis is the lower orbital radius and whose apoapsis is the higher radius. Of course you now know what this means (!) but a diagram always helps. In the lower orbit (1 in the nearby diagram), a burst of rocket power gives the satellite an extra speed Δv necessary to put it into the elliptical transfer orbit (2 in the diagram). When the satellite reaches the apoapsis another burst of speed $\Delta v'$ is given to take it out of its elliptical orbit and put it into the (higher energy) circular orbit required (3 in the diagram). If all this makes sense, then you can congratulate yourself on reading this document well.

The results given earlier even allow us to work out quite easily the parameters a and e of the transfer orbit. Using the symbols in the diagram, R is the radius of the lower orbit and R' the radius of the higher orbit. Result 14 gives $a(1 - e) = \mathrm{R}$ and $a(1 + e) = \mathrm{R'}$, from which the eccentricity $e = (\rho - 1)/(\rho + 1)$, where $\rho = \mathrm{R'/R}$, the ratio of the two orbital radii. It also follows that $a = (\mathrm{R'} + \mathrm{R})/2$, which is clear from looking at the diagram. Given a and e you could work out the two bursts in speed Δv and $\Delta v'$ from the energy relationships given earlier.

Orbital transfer can take place accidentally if you are not careful. Suppose you want to dock with a space station orbiting 1 km in front of you in the same circular orbit. Giving yourself some more speed 'to catch up' quickly will alter your energy and hence move you into an orbit with a larger value of a. You will no longer be in the same orbit as the station and will miss it. Manoeuvres like docking are trickier in orbit than similar ones on Earth. It is even worse if what you are trying to dock with is slowly rotating. The Russian cosmonaut Vladimir Dzhanibekov who was very experienced in docking in the era of the Salyut space stations, commented "*To give you an idea of the manoeuvrability of a spaceship, we could compare it to a 7 tonne dumper truck on a lake covered with bare ice, without being equipped with studded winter tyres. Suppose the task is getting the dumper truck to a designated spot for unloading. Several men have volunteered to steer the truck by pushing it from various sides. Imagine how much effort they have to expend to direct this load to where it has to go at a set speed and then to stop it bang on target.* It's probably even harder than this. Dzhanibekov's job was to dock in the minimum time using the least fuel. Now it's all computerised, though astronauts have to be able to do it manually in the event of computer failure. Go to miscellany table

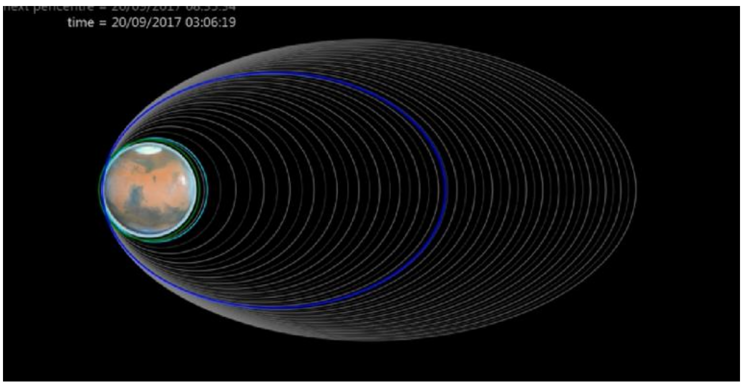
Aerobraking

The problem is speed. A spacecraft needs to be going quickly to get anywhere in a reasonable time, since distances in space are so great. Making matters worse, anywhere large enough to have an atmosphere also has gravity that will provide additional speed for a craft hurtling towards it. Without some slowing mechanism, a craft will either fly past or crash into the surface. Early space exploration settled mainly for fly-bys, though some impactor missions gave close-up pictures prior to crashing and created plumes of debris for analysis. Landing missions employ variants of the seed-pod strategy. Enclose the cargo in an expendable shell. A heat-shield followed by a parachute, for example as used by the Huygens' mission to Titan. This can be supplemented by a bouncy shell, such as used by the early Mars rovers, a crush-pad or retrorockets, as used for the Apollo Moon landings or the 'sky crane' of the Mars Curiosity rover. Timing is of the essence, as the ExoMars lander *Schiaparelli* demonstrated when it crashed into Mars at about 500 km h^{-1} following mistiming of the parachute release and retro rockets. Getting rid of speed is the problem.

The aerobraking in this piece is about the use of an atmosphere to change the orbit of a satellite and reduce its speed. Approaching close to a planet or moon with some atmosphere, retro-rockets can be used to take away enough speed to put the craft in orbit. The orbit, though will be very elongated if not much more than the minimum thrust is used. Relative to the central body, the craft will have a small negative energy. Result 9 shows that it will be in an orbit with large semi-major axis *a*. The orbit will have an eccentricity close to unity; the craft will be positioned to skim 'the top' of the atmosphere, not that an atmosphere has a well-defined top for it gradually fades out.

Now we have our craft in orbit around the chosen body but it spends a lot of time far away from it, like Haley's comet does from the Sun. This is a poor option for exploration. A near circular orbit close to the planet is usually what is wanted. We now have the reverse problem that many satellite operators face. They launch their satellite into low-earth orbit with large rockets and then want to move them to higher orbits. The section on *changing satellite orbits* shows how this can be done by two thruster burns and a Homann transfer orbit. The thruster burns each give a positive Δv (a change in speed) to the satellite. Aerobraking does the reverse, giving a negative Δv to the craft during the short part of the orbit it is close to the central body. Each braking reduces the total energy and hence decreases the semi-major axis of the orbit.

Aerobraking is a very delicate operation, definitely a case of 'slowly, slowly, catchy monkey'. As I'm writing this, the ExoMars satellite (image here, courtesy ESA) has been aerobraking for a year, knocking off about 1 m s^{-1} of speed on every orbit as it dips into the top of the very thin atmosphere, about 110 km from the surface. That may seem very little speed loss but travelling at 3700 m s^{-1}, a loss of 1 m s^{-1} corresponds to a loss of kinetic energy of a few megajoules for every 1000 kg of mass of the craft. Every joule saved by aerobraking is a joule that doesn't have to be supplied by thrusters. The aerobraking force is exerted mainly through the solar panels that have to be oriented correctly. If the force isn't oriented correctly then the orbit will be changed wrongly; if the force isn't applied correctly to the craft, it will start to tumble and likely loose communication with the Earth. A craft can't do much scientific operation during aerobraking since orientation control for aerobraking is the over-riding priority. Every aerobraking manoeuvre in every orbit is controlled, overseen by the orbital dynamics team, so there is a cost in manpower and operations time. For the ExoMars satellite, its highly elliptical four-day orbit extending to some 98 000 km was changed to a near-circular path at about 400 km. The final phase of creating a near circular orbit was done by thrusters.



Aerobraking is what the Space Shuttle did and what all Earth re-entry systems use. In this case the change of orbital parameters is not the big issue, slowing without using fuel is. Mars and Venus have so far been the targets for aerobraking orbits. The accompanying diagram, courtesy ESA, illustrates the approach towards circularity of the ExoMars satellite created by aerobraking. Some 800 orbits of aerobraking are involved. At the time of writing a video showing the orbit changes can be seen at https://www.esa.int/spaceinvideos/Videos/2018/02/Good_progress. A second video by ESA at http://www.esa.int/spaceinvideos/Videos/2016/12/ExoMars_first_year_in_orbit gives a more detailed look of the orbital changes in 3D. Go to miscellany table

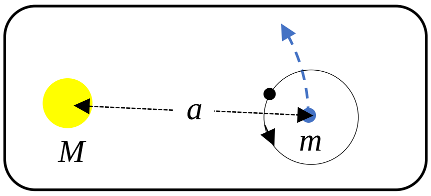
Manoeuvrability

In interplanetary space there is little to slow you down. Equally, there is little to let you change direction. On land, in the sea or in the air we turn corners by pressing against the land, the sea or the air. How do we turn corners in space? What about side thrusters? They aren't as effective as you might think. If the thrust is not exactly aligned with the centre of mass then it will provide a torque that will put the craft in a spin. Turn off the thrusters and the craft will continue to rotate '*ad infinitum*'. This is a particularly serious problem if parts of the craft are moveable, for example with solar panels that deploy or crew that can move around. In this case the centre of mass will change and several thrusters must operate together to cater for this. It's no surprise that every asteroid and comet we have found is spinning around. Any impact is very unlikely to align with its centre of mass so impacts will induce rotation, or alter existing rotation. Re-orienting a craft, though, just requires inducing a rotation with a short thrust and then halting it by a reverse thrust. That does not make any significant change to the orbit. In fact the same re-orientation can be achieved by spinning up an internal flywheel so the craft will rotate in the opposite direction for the desired time, and then braking the fly wheel. No rocket fuel needed. For making substantial course changes, side thrusters exert too little force and are ineffective, as we'll see.

The real problem with turning corners is that it is fuel greedy. No interplanetary craft is cruising along in a straight line – all are in some kind of orbit around the Sun. A spacecraft in orbit will reach other points in its orbit in due course if nothing is done. It will go nowhere else and the time taken is also fixed. If the crew or operators want it to go anywhere else then its velocity components must be changed. That implies acceleration and acceleration requires a force, as Newton's laws of motion tell us. Force controls the rate of change of momentum or to put it in almost the same way, a force applied for a given time changes the momentum over that time. Everyone knows, by repute if not experience, that turning a fully laden oil tanker at sea takes a lot of time inspite of continuous pressure from the rudder. The problem is its momentum. A tanker of mass 200,000 tonnes travelling at a mere 5 $m\,s^{-1}$ (9.7 knots) has a momentum of a billion (10^9) $kg\,m\,s^{-1}$. A spacecraft of mass 100 tonnes, surely about the minimum to support a manned mission, travelling at 30 $km\,s^{-1}$ (the speed of the Earth in its orbit – craft have at least the speed of where they came from) has a momentum of 3×10^9 $kg\,m\,s^{-1}$, with no sea to push against to turn it around. To turn the craft around 'in space' a comparable force has to be provided for a comparable time. If this is done by burning rocket fuel, a good rocket engine might be able to generate an exhaust speed of 3 $km\,s^{-1}$. You'll soon see that discharging almost the entire 100 tonnes of mass at this speed will only change the momentum by 3×10^8 $kg\,m\,s^{-1}$. Spacecraft take 'lumbering' to extremes not seen on Earth. The faster the craft, the more lumbering it is.

What can you do? The answer is 'not a lot'. You can't even stop, given the example figures above. Stopping in interplanetary space, though, will only get you falling into the Sun, for the Sun is still exerting its gravitational force on you. It's worth remembering the basics of Newtonian mechanics that to get an object to go in a circular arc a force needs to applied at right angles to the direction of travel. The traditional strategy for a course change is to give the craft a short burst of rocket thrust, making a minor change to a new orbit that will, over a long period of time, make a sizeable change in destination. The more recent variant is to take a leaf out of the Sun's method of changing the direction of orbiting objects, applying a small force for a very long time. In the Sun's case, that force is mainly gravity. The spacecraft can use an ion engine that ejects more massive atoms than rocket exhaust (xenon is the material of choice) at higher speeds. The force available with the current version of the technology is only a Newton or so but applied for days on end will make a controlled orbital change. The best of the rest is the gravitational slingshot, using a planet to deflect the craft as well as alter its speed (see above). The Parker Solar Probe speeding from Earth as I write is a good example of these techniques in action. As I see it, interplanetary craft are hugely unmanoeuvrable. Thrusters can make minor course corrections but the technique is basically to point the craft in the right direction, use occasional speed changes from the main thruster and let the orbital mechanics of the solar system do the rest. Science fiction films and TV series depicting spacecraft as manoeuvrable as cars are, well, complete fiction. There will be no handbrake turns in space. Go to miscellany table

The Hill radius

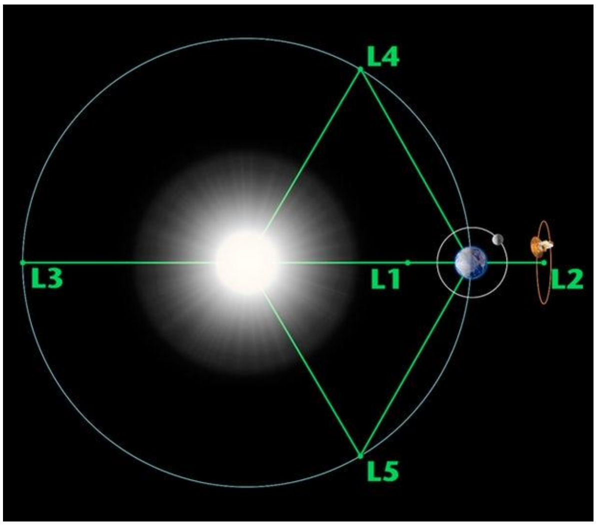


Mankind has sent up thousands of satellites that orbit the Earth. The Moon has been orbiting the Earth since the early years of the solar system, yet more distant asteroids, comets and planets all orbit the Sun. Clearly there's a distance beyond which the much more massive Sun takes control. The distance within which a body will orbit in the presence of a much larger but more distant body is known as the *Hill radius*, denoted r_H. For the Earth the Hill radius is significantly larger than the Earth-Moon distance (it's about 1.5 million km). The Hill radius is a 3-body issue involving a large body (mass M), a smaller body (mass m) about which an orbit might exist for a comparatively much smaller orbiting body. In general, 'm' could be in an elliptical orbit about M and the body orbiting m could also be in another elliptical orbit, making the calculation of r_H messy. In the simplest case where m orbits with radius a, the Hill radius $r_H = a \times (m/3M)^{1/3}$. There are issues of the stability of orbits near the limit, whether the orbit is in the same direction as the orbit of m or in the opposite direction, and other complications. The Hill radius above should be taken as an outer limit.

For example, when Moon bases are established (or even beforehand), it would very useful to have a small constellation of lunar positioning satellites around the Moon, doing what the GPS satellites do for us on Earth. The further from the Moon the constellation is, the bigger the range of each satellite but too far and they will drift away. If they were to be placed at half the Hill radius, how far from the Moon's surface would they be? In this case M is the mass of the Earth and m is mass of the Moon and we'll leave the Sun out of it. $m/M = 0.0123$ and $a =$ 384400 km and the Moon's equatorial radius is 1738 km. Using the formula above gives the Hill radius as 61524 km from the Moon's centre or about 60,000 km from the surface. Half this is 30,000 km. This is further than the GPS satellites are from Earth but if the lunar versions are put in place from Earth then distance from the lunar surface is not a big issue. Signal strength would be greater if they were lower and in fact there are many other considerations for a practical system. Go to miscellany table

The 5 Lagrangian points

The Lagrangian points are another 3-body concept, locating 5 points in space where a comparatively small third body will orbit a large mass M with the same period as an intermediate mass m. The most common example are the 5 points for the Sun/Earth system. The Earth orbits the Sun once a year. A free body a little closer to the Sun will orbit slightly quicker and one further away will take more than a year. However, a body (e.g. a space probe) on the Earth-Sun line that is nearer the Sun is influenced both by the gravity of the Sun and the Earth. There is a point, L1, where the outward pull of the Earth on the body reduces the stronger gravitational pull towards the Sun so that the body orbits in the same time as the Earth. L1 is about 1.5 million km from Earth. At this point the body stays on the Earth/Sun line as it orbits the Sun. L1 is almost a good place to locate a space-probe for monitoring the Sun, since it will be a constant distance from Earth. Unfortunately, the position at L1 is unstable in that a small drift away once begun will increase with time. This is not a project killer, for a space craft can be maintained in an orbit around L1 by the active use of thrusters. The ESA/NASA SOHO probe is there at the time of writing, as is NASA's ACE probe. Both these 1990s missions have already had long lives. [L1 is not to be confused with the point where the Sun's gravity and the Earth's gravity act equally in opposite directions. This point is about 260,000 km from the centre of the Earth (ignoring the gravity of the Moon)].



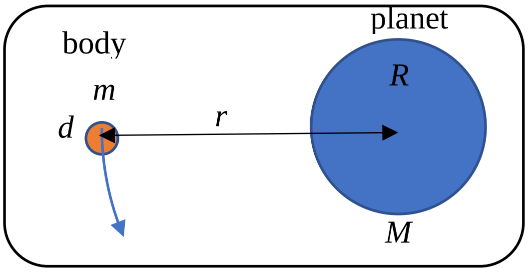
Commonly used diagram showing the location of the Lagrangian points for the Sun/Earth system. Not to scale.

L2 is a similar point on the far side of the Earth from the Sun where the net gravitational force on a craft from both Sun and Earth produces a period of rotation around the Sun of one year. This point is favoured by space craft mapping the universe at large that need to be shielded from the Sun's radiation. WMAP & PLANCK (cosmic microwave background mapping), HERSCHEL (infra-red) and GAIA (astrometry) all worked from L2, and the James Webb Space Telescope is scheduled to operate from there when it is finally launched. Craft at L2 also need active propulsion to maintain an orbit there. The instability at the locations of L1 and L2 is almost a guarantee that no space debris from comets and asteroids is trapped there, for there is no trap.

L3 is on the other side of the Sun from Earth and is not of interest for space exploration since any craft there is always out of direct communication with Earth. L4 and L5 are 60° around the Earth's orbit on either side of the Earth. They are stable points in that a body can orbit around either without any active power system. Naturally they have collected a myriad of solar system debris particles and small asteroids. ESA are seriously considering sending a solar observation probe to L5 to get a sideways look at coronal mass ejections heading earthwards. Projects like this take at least a decade to come to fruition. All comparable 2-body systems have analogous Lagrangian points, e.g. Sun/Jupiter, Earth/Moon, etc. Go to miscellany table

The Roche limit

The Roche limit is an example of the effect of tidal forces on an orbiting body. The Roche limit is responsible for the rings around the four gas giants in the solar system. It's easy to get the impression that bodies like the Earth and Moon are as solid as the granite used to build traditional houses here. They aren't. The Earth and Moon are held together mainly by their own gravity. So are most moons in the solar system. Asteroids and comets have even less internal cohesion than moons. If a body held together by gravity gets into an orbit close to a planet then it is in trouble. The gravitational force on its near surface will be appreciably stronger than the gravitational force on the centre. The result is a tidal force trying to pull the body apart. The Roche limit is the smallest distance from the planet that a body held together by gravity can survive. To put it another way, if such a body gets within the Roche limit, it will break apart into myriad pieces. Yet another consequence is that if a large number of bodies are orbiting within the Roche limit, they will not come together as a single gravitationally bound body.



It's not hard to estimate the Roche limit. If the planet has mass M, radius R, and the circulating body at the Roche limit r has mass m and diameter d, then the tidal force on unit mass facing the planet is $GM/(r - d/2)^2 - GM/r^2 \approx GMd/r^3$ taking $d \ll r$, a force directed towards the planet. The self-gravity on the surface of the body for unit mass is $Gm/(d/2)^2 = 4Gm/d^2$, directed towards the body's centre. Equating these two forces gives $r^3 = (M/4m)d^3$, with r an estimate of the Roche limit. Since the masses are proportional to the diameters cubed and the densities ρ_P and ρ_b of planet and body respectively, a variant of this relationship is $r = R(2\rho_P/\rho_b)^{1/3}$.

The expression above shows that the Roche limit will depend a bit on the composition of the orbiting body. Unsurprisingly, Saturn's rings are within their Roche limit. Several moons of Saturn are not far outside their Roche limit. A largish comet or asteroid approaching the Earth will break in pieces once within the Roche limit of about 18000 km. The Roche limit doesn't apply to objects like spacecraft that are held together by the strength of their construction materials and not by gravity. Go to miscellany table

Conclusion

The point of the above is not really to turn anyone into a space scientist but to show that the subject is based on understandable physics: it makes sense, it's relevant to astronomy, space science and indeed to what's happening in the world around us. In fact space science is worth over £15 billion per year to the UK economy, including the building of satellites, the contributions to space exploration programs and the sale of knowledge and services earned from remote sensing and satellite communications. Knowing some of the science of orbits should not be the preserve of specialists only. We all have a vested interest in the success of space science and many technically literate people are employed in this business.

JSR