

# 1 Interface

The Mathematica interface in `correlations.nb` has different types of elements. An element of the interface is **interactive** if it captures the user's inputs, and **responsive** if it changes based on the user's inputs.

## Current state *responsive*

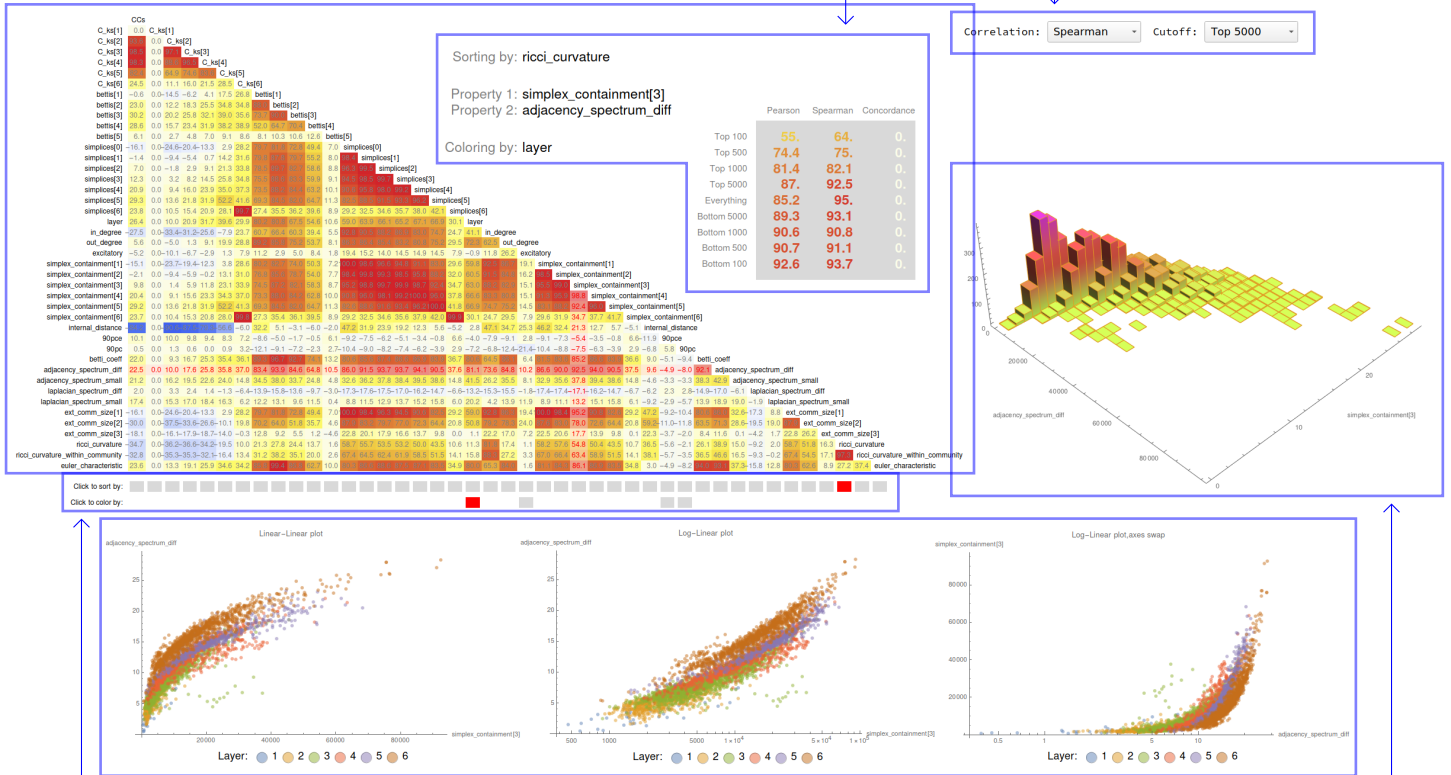
Indicates all the inputs chosen by the user. Bottom right shows correlation of chosen parameters if correlation or cutoff are changed.

## Correlation and cutoff *interactive*

Click dropdown lists to choose options.

## Main table *interactive, responsive*

Click on cells to change plots. Cell color reflects value of number (correlation).



## Scatter plots *responsive*

Same data in all three, presented in different scales: linear-linear, log-linear, linear-log. Legend changes based on chosen coloring and types of points seen in chosen cutoff.

## Density plot *responsive*

Shows the density of points in the linear-linear scatter plot. Click+drag to change view.

## Sort by and color by *interactive*

Click to choose which parameter to sort neurons by in the main table  
Click to choose which (available) parameter to color neurons by in the scatter plots

## 2 Parameters

There are 43 visualized parameters.

Name	Explanation	Value
CCs	clustering coefficient of the community	$\mathbf{R}_{\geq 0}$
C_ks[n]	$k$ -clustering coefficient, for $k = n$	$\mathbf{R}_{\geq 0}$
bettis[n]	$k$ th Betti number of the flag complex of the community, for $k = n$	$\mathbf{Z}_{\geq 0}$
simplices[n]	number of $k$ -simplices in the flag complex of the community, for $k = n$	$\mathbf{Z}_{\geq 0}$
layer	layer in which the neuron is	$\{1, \dots, 6\}$
in_degree	in-degree of the neuron	$\mathbf{Z}_{\geq 0}$
out_degree	out-degree of the neuron	$\mathbf{Z}_{\geq 0}$
excitatory	1 if the neuron is excitatory, 0 if not	$\{0, 1\}$
simplex_containment[n]	number of $k$ -simplices in the flag complex of the community that the head neuron is a vertex of, for $k = n$	$\mathbf{Z}_{\geq 0}$
internal_distance	internal distance of the community	$\mathbf{R}_{\geq 1}$
90pce	1 if the community is among the top ones (excitatory only sorted by CC) whose union covers 90 percent, 0 otherwise	$\{0, 1\}$
90pc	1 if the community is among the top ones (all sorted by CC) whose union covers 90 percent, 0 otherwise	$\{0, 1\}$
betti_coeff	Betti coefficient of the community	$\mathbf{R}_{\geq 0}$
adjacency_spectrum_diff	difference between the two largest eigenvalues of the adjacency spectrum of the community	$\mathbf{R}_{\geq 0}$
adjacency_spectrum_small	smallest non-zero eigenvalue of the adjacency spectrum of the community	$\mathbf{R}_{> 0}$
laplacian_spectrum_diff	difference between the two largest eigenvalues of the Laplacian spectrum of the community	$\mathbf{R}_{\geq 0}$
laplacian_spectrum_small	smallest non-zero eigenvalue of the Laplacian spectrum of the community	$\mathbf{R}_{> 0}$
ext_comm_size[n]	extended community size, for $n = 1, 2, 3$	$\mathbf{Z}_{> 0}$
ricci_curvature	Ricci curvature of the whole brain graph at the neuron	$\mathbf{R}_{\geq 0}$
ricci_curvature_within_community	Ricci curvature of the community at the neuron	$\mathbf{R}_{\geq 0}$
euler_characteristic	Euler characteristic of the flag complex of the community	$\mathbf{Z}$

**Internal distance:** For each neuron in a community, the average distance to every other neuron is computed. The internal distance of the community is the average of this number over all the neurons in the community.

**Betti coefficient:** Given a community with betti coefficients  $b_0, \dots, b_k$ , the Betti coefficient of this community is  $b_0/w_0 + \dots + b_k/w_k$ , where  $w_i$  is the average  $i$ th Betti number among all communities.

**Extended community size:** Defined as number of neurons within a certain path length from a given neuron. The extended community size at step 0 is always 1. At step 1, the extended community is the neighborhood of the neuron. At step 2, the extended community is the neighborhoods of all the neighbors of the neuron, and so on.

**Ricci curvature:** For every directed edge, its curvature is  $2 - (\text{in degree of tail}) - (\text{out degree of head})$ . For every vertex, its Ricci curvature is the sum of the curvatures of its in-edges minus the sum of the curvatures of its out-edges.

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## 3 Variables

There are 5 different variables.

1. **Parameter 1 and 2:** By clicking on a cell in the main table, the two parameters indicated by the row and column at the chosen cell will be compared in the scatter plots on the bottom and the density plot on the right.
    - *Options:* Any two of the 43 parameters described in Section 2
    - *Affects:* Scatter plots, Density plot
  2. **Correlation:** A correlation coefficient of the two chosen parameters. This is a number that ranges from -100 to +100, with -100 indicating a perfect negative relationship between the two chosen parameters (one decreases at the same rate as the other increases), +100 indicating a perfect positive relationship, and 0 indicating no relationship. The color of the cell reflects the number, with -100 corresponding to blue, -100 corresponding to red, 0 corresponding to white.
    - *Options:* Pearson (for linear relationships), Spearman (for monotonic relationships)
    - *Affects:* Values in main table
  3. **Cutoff:** The 31346 neurons may have too much data, so a cutoff can be chosen, either from the top or the bottom. That is, when sorting by a parameter, either a collection from the highest-ordered or lowest-ordered neurons can be chosen. The choice of cutoff is the number of neurons whose correlation is compared and which are visualized in the plots.
    - *Options:* 100, 500, 1000, 5000 (from top or bottom), no cutoff
    - *Affects:* Values in main table, Scatter plots, Density plot
  4. **Sort by:** If a cutoff less than 31346 is chosen, the “Sort by” parameter allows the user to choose to what sorting the cutoff refers to. That is, choosing parameter  $X$  and cutoff  $n$ , means only the top  $n$  neurons, as sorted by  $X$  from largest to smallest, are shown.
    - *Options:* All 43 parameters
    - *Affects:* Values in main table, Scatter plots, Density plot
  5. **Color by:** The parameters which take on a small number of values can be viewed concurrently with the other parameters. Choosing one of these will color the scatter plots with common colors corresponding to neurons with the same value in this parameter.
    - *Options:* Layer, Excitatory, 90% excitatory membership, 90% membership
    - *Affects:* Scatter plots
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## 4 Plots

Every choice of two parameters and a cutoff gives 4 different plots.

### 4.1 Scatter plots

All three scatter plots compare the two parameters on the two axes. The plot on the left has both linear axes, and the middle and right plot have one linear and one logarithmic axis, to visualize any potential relationship between the variables.

### 4.2 Density plot

If there are too many neurons, it may be hard to understand the defining features of the scatter plots. The density plot counts the number of neurons in rectangular bins of the linear-linear scatter plot on the bottom left. The higher a bar, the more neurons are located within that bin.