On the Interaction between Logic and Preference in Structured Argumentation

Anthony P. Young, Sanjay Modgil, and Odinaldo Rodrigues

Department of Informatics, King's College London

{peter.young, sanjay.modgil, odinaldo.rodrigues}@kcl.ac.uk

Abstract. The *structure-preference* (SP) order is a way of defining argument preference relations in structured argumentation theory that takes into account how arguments are constructed. The SP order was first introduced in the context of endowing Brewka's prioritised default logic (PDL) with sound and complete argumentation semantics. In this paper, we further articulate the underlying intuitions of the SP order in terms of how an agent should construct arguments. We also compare the SP order to other argument preference relations and illustrate the different results one would obtain. Finally, we prove that the SP order allows for the original version of PDL to satisfy Brewka's and Eiter's postulates.

1 Introduction

Argumentation theory [1,2,10,13] is a general framework for non-monotonic reasoning [3], where inference from an inconsistent knowledge base in a given non-monotonic logic (NML) can be expressed as the exchange of conflicting arguments with premises from that knowledge base, such that the inferred statements of the logic are the conclusions of justified arguments. As the study of how preferences are used to resolve conflicts has become a major topic in NML [5,14,17], argumentation theory has used preference relations to decide which arguments are justified. Such preferences over arguments could be taken as exogenously given, or be derived from more primitive concepts. Structured argumentation theories like ASPIC$^+$ [13], which treat arguments as structured objects made up of premises and inference rules, consider more primitive preferences that are given over argument components such as defeasible rules, such that these preferences over components are aggregated into an argument preference relation.

This paper makes the following contributions. We first motivate and define the *structure-preference* (SP) order. This is a rearrangement of the preference relations on the fallible components (i.e. the non-axiom premises and the non-deductive rules of inference) of a structured argumentation theory that takes into account the structure of arguments, understood as the actual order of applicability of the fallible components during argument construction. The SP order is an alternative preference relation that can also make use of the commonly-used aggregation techniques such as the elitist and democratic set-comparison relations, in accordance with the weakest-link and last-link principles [13, Section 5]. We also define the corresponding SP argument preference, which makes more certain arguments more preferred. The SP order was first devised to endow Brewka's prioritised default logic (PDL) [4] with argumentation semantics [20]. After recapping this result, our second contribution applies the insight of the SP order to show that Brewka and Eiter's principles for PDL [5,6] hold for the original version of PDL. We then discuss some related work, in particular the roots of the SP order in logic programming [8,17].

This paper is structured as follows. In Section 2 we review the relevant aspects of the ASPIC$^+$ framework for structured argumentation [10,13]. In Section 3 we define the SP order on the defeasible rules in the abstract context of ASPIC$^+$, establish its underlying intuitions, and compare it with different argument preference relations. In Section 4 we recap PDL [4] and its argumentation semantics in the case where the underlying priority is a total order [20]. We also recall Brewka and Eiter's claim that PDL "does not take seriously what they believe" [5,6], and their remedy by modifying PDL to satisfy two guiding principles. We prove that if PDL reasons with the SP order, then it will also satisfy these principles. Section 5 discusses related work [8,11,12,17] and Section 6 concludes.

2 Preferences in Structured Argumentation

To illustrate the idea of the SP order, we will use the ASPIC$^+$ framework for structured argumentation. However, it will become clear that any structured argumentation theory that considers preferences over the fallible components of arguments and has a well-defined notion of argument construction can accommodate the SP order.

2.1 The ASPIC$^+$ Framework for Structured Argumentation

We recap the relevant definitions of ASPIC$^+$ [10,13]. An *argumentation system* is a tuple $\langle \mathcal{L}, -, \mathcal{R}_s, \mathcal{R}_d, n \rangle$, where $\mathcal{L}$ is a set of well-formed formulae (wffs), $- : \mathcal{L} \to \mathcal{P}(\mathcal{L})$ is the contrary function[1] $\theta \mapsto \overline{\theta}$, where $\overline{\theta}$ is the set of wffs that disagree with θ. Let $m \in \mathbb{N}$ and $\theta_1, \ldots, \theta_m, \phi \in \mathcal{L}$.[2] $\mathcal{R}_s$ is *the set of strict inference rules*, where rules are denoted by $(\theta_1, \ldots, \theta_m \to \phi)$, meaning that if $\theta_1, \ldots, \theta_m$ are all true then ϕ is also true. $\mathcal{R}_d$ is *the set of defeasible inference rules*, where rules are denoted by $(\theta_1, \ldots, \theta_m \Rightarrow \phi)$, meaning that if $\theta_1, \ldots, \theta_m$ are all true then ϕ is tentatively true. We have that $\mathcal{R}_s \cap \mathcal{R}_d = \varnothing$. For $r = (\theta_1, \ldots, \theta_m \to / \Rightarrow \phi) \in \mathcal{R}_s \cup \mathcal{R}_d$ we define $Ante(r) := \{\theta_i\}_{i=1}^m \subseteq \mathcal{L}$ and $Cons(r) := \phi \in \mathcal{L}$.[3] Finally, $n : \mathcal{R}_d \to \mathcal{L}$ is a partial function that assigns a *name* to defeasible rules.

A *knowledge base* is a set $\mathcal{K} := \mathcal{K}_n \cup \mathcal{K}_p \subseteq \mathcal{L}$ where $\mathcal{K}_n \cap \mathcal{K}_p = \varnothing$. $\mathcal{K}_n$ is *the set of axioms*, and $\mathcal{K}_p$ is *the set of ordinary premises*. Given an argumentation system and knowledge base, ASPIC$^+$ arguments are constructed inductively:

[1] If X is a set then $\mathcal{P}(X)$ is its power set.
[2] In this paper, $\mathbb{N} := \{0, 1, 2, 3, \ldots\}$ and $\mathbb{N}^+ := \{1, 2, 3, 4, \ldots\}$.
[3] If $m = 0$ then rules like $(\to \phi)$ and $(\Rightarrow \psi)$ are well-defined, with $Ante(r) = \varnothing$.

1. (Base) $[\theta]$ is a *singleton argument* with $\theta \in \mathcal{K}$, with *conclusion* $Conc\left([\theta]\right) := \theta \in \mathcal{L}$, *premise set* $Prem\left([\theta]\right) := \{\theta\} \subseteq \mathcal{K}$, *top rule* $TopRule\left([\theta]\right) := *$[4] and *set of subarguments* $Sub\left([\theta]\right) := \{[\theta]\}$.
2. (Inductive) Let $n \in \mathbb{N}$ and $\{A_i\}_{i=1}^{n}$ be a set of arguments where for all $1 \leq i \leq n$, A_i has $Conc\left(A_i\right) \in \mathcal{L}$, $Prem\left(A_i\right) \subseteq \mathcal{L}$ and $Sub\left(A_i\right)$ well-defined. If we have a strict rule $r = \left(Conc\left(A_1\right), \ldots, Conc\left(A_n\right) \to \phi\right) \in \mathcal{R}_s$ and defeasible rule $s = \left(Conc\left(A_1\right), \ldots, Conc\left(A_n\right) \Rightarrow \psi\right) \in \mathcal{R}_d$, then $B := [A_1, \ldots, A_n \to \phi]$ and $C := [A_1, \ldots, A_n \Rightarrow \psi]$ are also arguments, with respective conclusions $Conc(B) := \phi$ and $Conc(C) := \psi$, premise sets $Prem(B)$, $Prem(C) := \bigcup_{i=1}^{n} Prem\left(A_i\right)$, top rules $TopRule(B) = r$ and $TopRule(C) = s$, and sets of subarguments $Sub(B) = \{B\} \cup \bigcup_{i=1}^{n} Sub\left(A_i\right)$ and $Sub(C) = \{C\} \cup \bigcup_{i=1}^{n} Sub\left(A_i\right)$.[5]

Let $\mathcal{A}$ denote the (unique) set of arguments constructed in this way.

Example 1. Working in propositional logic where $\mathcal{L}$ denotes the set of all propositional wffs and $\mathcal{R}_s$ contains all of the usual rules of proof. Let $a, b, c \in \mathcal{L}$ be propositional variables. Suppose we have $\mathcal{R}_d = \{(a \Rightarrow b), (b \Rightarrow c)\}$ and $\mathcal{K}_n = \{a\}$. Then we have arguments $A_0 := [a]$, $A_1 := [A_0 \Rightarrow b]$ and $A := [A_1 \Rightarrow c]$.

Two arguments are *equal* iff they are constructed identically in the above manner with syntactically identical formulae. For $A, B \in \mathcal{A}$ we say A is a *subargument of* B iff $A \in Sub(B)$ and write $A \subseteq_{\text{arg}} B$; it is clear that $\subseteq_{\text{arg}}$ is a preorder on $\mathcal{A}$. An argument A is *firm* iff $Prem(A) \subseteq \mathcal{K}_n$. For $A \in \mathcal{A}$ let $DR(A) \subseteq \mathcal{R}_d$ be the defeasible rules applied in constructing A. We say an argument is *strict* iff $DR(A) = \varnothing$; non-strict arguments are *defeasible*. We say that A *attacks* B *on* $B' \subseteq_{arg} B$ iff at least one of the following hold.

1. *Undermine* iff $(\exists \theta \in Prem(B) \cap \mathcal{K}_p)\left[B' = [\theta] \text{ and } Conc(A) \in \overline{\theta}\right]$.
2. *Rebut* iff $r := TopRule\left(B'\right) \in \mathcal{R}_d$ and $Conc(A) \in \overline{Cons\left(r\right)}$.
3. *Undercut* iff $Conc(A) \in \overline{n\left(TopRule\left(B'\right)\right)}$.

Example 2. (Example 1 continued) Let $- : \mathcal{L} \to \mathcal{P}\left(\mathcal{L}\right)$ be the contrary function representing *classical syntactic negation*, i.e. $\overline{\theta} := \{\psi\}$ where if θ is syntactically of the form $\neg\phi$, then $\psi = \phi$, else $\psi = \neg\theta$. Suppose a further defeasible rule $(a \Rightarrow \neg c) \in \mathcal{R}_d$, then $B := [A_0 \Rightarrow \neg c]$ is an argument such that $B \rightharpoonup A$, and $A \rightharpoonup B$, both rebutting each other at their conclusions.

ASPIC^{+} also includes preferences on each arguments' fallible components, namely ordinary premises in $\mathcal{K}_p$ and defeasible rules in $\mathcal{R}_d$. These sets are equipped with the respective strict partial orders $<_K$ and $<_D$, assumed to be exogenously given, such that $\langle \mathcal{K}_p, <_K \rangle$ and $\langle \mathcal{R}_d, <_D \rangle$ are respectively lifted to relations $\vartriangleleft_K$ and $\vartriangleleft_D$ that compare finite subsets of $\mathcal{K}_p$ and $\mathcal{R}_d$ respectively.[6]

[4] In this paper, undefined quantities are denoted with $*$.

[5] From Footnote 3: when $n = 0$ then arguments like $[\to \phi]$ and $[\Rightarrow \psi]$ are well-defined, each with empty premises and only itself as a subargument.

[6] The subscript K stands for "knowledge" and D stands for "defeasible".

This information is aggregated to an argument preference relation $\precsim$ on $\mathcal{A}$ (see Section 2.2). We can then use this preference relation to determine which attacks succeed as defeats. The *defeat* relation $\hookrightarrow$ on $\mathcal{A}$ is defined as: $A \hookrightarrow B$ on $B' \Leftrightarrow [A \rightharpoonup B$ on $B' \subseteq_{\text{arg}} B,\ A \not\prec B']$, in the cases where $A \rightharpoonup B$ is an undermine or rebut.[7] This gives us a directed graph called an *argumentation framework* $\langle \mathcal{A}, \hookrightarrow \rangle$ where the usual methods of calculating the justified arguments from abstract argumentation apply [10]. For our purposes we say that a set of arguments $S \subseteq \mathcal{A}$ is *justified* (i.e. a *stable extension*) iff it is conflict free, $\hookrightarrow \cap S^2 = \varnothing$, and $(\forall B \notin S)\,(\exists A \in S)\, A \hookrightarrow B$.

2.2 Principles for Argument Preferences

We now elaborate on how ASPIC$^+$ derives argument preference relations from the strict partially ordered set (poset) $\langle \mathcal{K}_p, <_K \rangle$ and $\langle \mathcal{R}_d, <_D \rangle$ [13, Section 5]. Let X be $\mathcal{K}_p$ or $\mathcal{R}_d$ and $<$ be either $<_K$ or $<_D$. To lift $\langle X, < \rangle$ to $\langle \mathcal{P}_{\text{fin}}(X), \triangleleft \rangle$,[8] we have the following formulae [13,15]: for $\Gamma,\ \Gamma' \in \mathcal{P}_{\text{fin}}(X)$ and $\triangleleft \in \{\triangleleft_{Eli},\ \triangleleft_{Dem}\}$:

$$\Gamma \trianglelefteq_{Eli} \Gamma' \Leftrightarrow [\Gamma = \Gamma' \text{ or } \Gamma \triangleleft_{Eli} \Gamma'], \tag{1}$$

$$\Gamma \triangleleft_{Eli} \Gamma' \Leftrightarrow (\exists x \in \Gamma)\,(\forall y \in \Gamma')\ x <_D y, \tag{2}$$

$$\Gamma \trianglelefteq_{Dem} \Gamma' \Leftrightarrow [\Gamma = \Gamma' \text{ or } \Gamma \triangleleft_{Dem} \Gamma'],\ \text{and} \tag{3}$$

$$\Gamma \triangleleft_{Dem} \Gamma' \Leftrightarrow \begin{cases} \text{true if } \Gamma \neq \varnothing,\ \Gamma' = \varnothing, \\ \text{false if } \Gamma = \Gamma' = \varnothing \text{ or } (\Gamma = \varnothing, \Gamma' \neq \varnothing), \\ \text{else, } (\forall x \in \Gamma)\,(\exists y \in \Gamma')\ x <_D y. \end{cases} \tag{4}$$

The relation $\triangleleft_{Eli}$ (Eq. 2) is the *strict elitist set-comparison relation*. The relation $\triangleleft_{Dem}$ (Eq. 4) is the *strict democratic set-comparison relation*. Notice in all cases $\varnothing$ is a maximal element to reflect that what should be most preferred should be arguments with no such fallible component.

To relate these set-comparison relations to arguments, we recall the *last link principle* (LLP) [13, Definition 20]. For $A, B \in \mathcal{A}$ and $\trianglelefteq \in \{\trianglelefteq_{Eli},\ \trianglelefteq_{Dem}\}$, define

$$A \precsim B \Leftrightarrow \begin{cases} Prem_p(A) \trianglelefteq Prem_p(B) & LDR(A) = LDR(B), \\ LDR(A) \trianglelefteq LDR(B) & \text{else}, \end{cases} \tag{5}$$

where for $A \in \mathcal{A}$: if A is singleton then $LDR(A) = \varnothing$, else if $A = [A_1, \ldots, A_n \Rightarrow Conc(A)]$ then $LDR(A) = \{(Conc(A_1), \ldots, Conc(A_n) \Rightarrow Conc(A))\}$, else we have $LDR(A) = \bigcup_{i=1}^{n} LDR(A_i)$. Alternatively, one can use the *weakest link principle* (WLP) [13, Definition 21]. For $A, B, \trianglelefteq$ as above,

$$A \precsim B \Leftrightarrow \begin{cases} Prem_p(A) \triangleleft Prem_p(B) \text{ if } A, B \text{ are strict}, \\ DR(A) \triangleleft DR(B) \text{ if } A, B \text{ are firm}, \\ Prem_p(A) \triangleleft Prem_p(B) \text{ and } DR(A) \triangleleft DR(B) \text{ else}. \end{cases} \tag{6}$$

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[7] This is adequate for our purposes. For a discussion of the subtleties of how this depends on the contrary function and for the case of undercutting attacks, see [13].

[8] If X is a set then $\mathcal{P}_{\text{fin}}(X)$ is the set of all finite subsets of X.

Both the LLP and WLP are commonly used ways of defining preferences $\precsim$ on arguments from more primitive preferences on the fallible components of arguments.[9] The strict preference is defined as $A \prec B \Leftrightarrow [A \precsim B,\ B \not\precsim A]$.

3 The Structure-Preference Order

3.1 Guiding Intuition – How to Construct Arguments

We now articulate the guiding intuition of the SP order, which is related to how agents should construct and compare arguments. Preferences have long been used to guide reasoning in non-monotonic logics (NMLs) and logic programming. In [9], Delgrande et al. review the ways preferences are treated in NMLs. They distinguish between two types of preferences. *Prescriptive preferences* provide information on which of the applicable rules *should* be selected, i.e. "applicable" in the sense of having all of the antecedents of a rule known. *Descriptive preferences* specify the exact order of how the rules *are actually* applied. How do these ideas translate to structured argumentation theory?

Assume we have an inferentially ideal agent who, when constructing arguments, is able to apply all applicable strict rules in $\mathcal{R}_s$ when it is possible to do so. Such an agent would begin with all premises $\mathcal{K}$ (as singleton arguments) and deductively close under all possible strict rules to form a *core*. Of the applicable defeasible rules, the agent would choose the $<_D$-most preferred ones to be applied. The agent then continues deductively closing with respect to the strict rules, and then adding the $<_D$-most preferred defeasible rules... and so on. This view of argument construction gives a canonical enumeration of *how far* a given argument is from the agent's core, in terms of the number of times the agent has added a defeasible rule and closed under all possible strict rules. This canonical enumeration also creates a preference over the defeasible rules that is descriptive in the sense of Delgrande et al.

How can we define such a descriptive preference on $\mathcal{R}_d$ in structured argumentation theory? For simplicity, we will assume arguments are *firm* and that the agent has a prescriptive preference relation $<_D$ over $\mathcal{R}_d$ that is a strict *total* order [18, Chapter 3], and that $\mathcal{R}_d$ is a finite set. We define a preference $<_{SP}$ on $\mathcal{R}_d$, where SP stands for *structure-preference*, as follows. The most $<_{SP}$-preferred defeasible rule, a_1, is the most $<_D$-preferred *applicable* rule after all strict arguments are constructed, i.e. the core. The next most $<_{SP}$-preferred defeasible rule, a_2, is the next most $<_D$-preferred applicable rule after a_1... etc. and so on until all defeasible rules are added. If $<_D$ is a total order then $<_{SP}$ is also a total order. We will formalise this idea in Definition 2 below.

3.2 The SP Order - Definitions, Comparisons, Properties

We give the following definitions. We work with an arbitrary argumentation system and knowledge base with $\mathcal{K}_p = \varnothing$ (Section 2).

[9] The infallible components of arguments, i.e. the axiom premises and deductive rules of inference, are by convention incomparable because they are all true.

Definition 1. *Let $R \subseteq \mathcal{R}_d$. The set $Args(R) \subseteq \mathcal{A}$ is defined as: $A \in Args(R) \Leftrightarrow DR(A) \subseteq R$. This is* ***the set of arguments freely constructed with defeasible rules restricted to those in*** *R.*

The set $Args(R)$ has all arguments with premises in $\mathcal{K}$, strict rules in $\mathcal{R}_s$ and defeasible rules in R. Given R, $Args(R)$ exists and is unique. Further, we will assume that there are no *irrelevant rules*, i.e. there is no $r \in \mathcal{R}_d$ such that $r \notin DR(\mathcal{A})$. Therefore, all rules in $\mathcal{R}_d$ feature in some argument. Also, we generalise the conclusion function (Section 2) to sets of arguments. For $S \subseteq \mathcal{A}$, $Conc(S) := \bigcup_{A \in S} \{Conc(A)\}$.

We will define the SP order for argumentation systems where

1. $\mathcal{K}_p = \varnothing$ (i.e. all arguments are firm),
2. $\mathcal{R}_d$ is finite and
3. $<_D$ is a total order on $\mathcal{R}_d$.

We will briefly consider how assumptions 1 and 3 above might be lifted in Section 6. Definition 2 below formalises our discussion of Section 3.1. Each $<_D$ over $\mathcal{R}_d$ can be transformed into $<_{SP}$ that incorporates the logical relationship of the defeasible rules, which is determined by the order they are applied when constructing arguments.

Definition 2. *Let $N := |\mathcal{R}_d|$ and $1 \leq i \leq N$. We define a rearrangement of the defeasible rules $r \in \mathcal{R}_d$ to $a_i \in \mathcal{R}_d$ as follows:*

$$a_i := \max_{<_D} \left[\left\{ r \in \mathcal{R}_d \,\middle|\, Ante(r) \subseteq Conc \left[Args \left(\bigcup_{k=1}^{i-1} \{a_k\} \right) \right] \right\} - \bigcup_{j=1}^{i-1} \{a_j\} \right]. \quad (7)$$

The ***(strict) structure-preference (SP) order on*** *$\mathcal{R}_d$, denoted by $<_{SP}$, is:*

$$(\forall 1 \leq i, j \leq N)\, a_i <_{SP} a_j \Leftrightarrow j < i. \quad (8)$$

The ***non-strict SP order*** *is $a_i \leq_{SP} a_j \Leftrightarrow [a_i = a_j \text{ or } a_i <_{SP} a_j]$.*[10]

As $<_D$ is a total order and $\mathcal{R}_d$ is finite, a_i exists and is unique. The agent first constructs all strict (and firm) arguments $Args(\varnothing)$, then adds the $<_D$-most preferred applicable rule $a_1 = \max_{<_D} \{r \in \mathcal{R}_d \,|\, Ante(r) \subseteq Conc(Args(\varnothing))\}$. Then the agent adds the next $<_D$-most preferred applicable rule a_2... etc. until all rules are exhausted. Note that the second union after the set difference in Eq. 7 ensures that each rule is only applied once. The result is such that $<_{SP}$-larger defeasible rules belong to smaller arguments or are more preferred. Clearly, $<_{SP}$ is also a strict total order on $\mathcal{R}_d$, and the transformation $<_D \mapsto <_{SP}$ is functional.

[10] This is well-defined because $i \mapsto a_i$ is bijective between $\mathcal{R}_d$ and $\{1, 2, 3, \ldots, |\mathcal{R}_d|\}$.

3.3 The SP Argument Preference

Inspired by preferences in NML, $<_{SP}$ provides a new way of defining argument preference relations, because it takes into account how arguments are constructed. We now lift $<_{SP}$ to its corresponding argument preference, $\prec_{SP}$. The guiding intuition is that arguments further away from the core should be less preferred, because (as $\mathcal{K}_p = \varnothing$) arguments in the core are certain (strict and firm); one might expect an agent to prefer arguments that are more certain (closer to the core) by virtue of it having less fallible elements and thus being less susceptible to attack. We formalise this as $A \subseteq_{\text{arg}} B \Rightarrow A \precsim B$, and investigate through examples whether the other ASPIC$^+$ preferences satisfy this property.

Example 3. (Example 1 continued) Consider the arguments A_1 and A from Example 1. Clearly $LDR(A_1) = \{(a \Rightarrow b)\}$ and $LDR(A) = \{(b \Rightarrow c)\}$. Suppose $(a \Rightarrow b) <_D (b \Rightarrow c)$. By $\trianglelefteq_{Eli}$-LLP (Eqs. 1, 2 and 5), we have $A_1 \prec A$. By Eq. 7 $[a] \in Args(\varnothing)$ and $Ante(a \Rightarrow b) = \{a\} \subseteq Conc(Args(\varnothing))$ so $a_1 = (a \Rightarrow b)$. Similarly, $a_2 = (b \Rightarrow c)$ hence $(b \Rightarrow c) <_{SP} (a \Rightarrow b)$. Therefore, by $\trianglelefteq_{Eli}$-LLP under $<_{SP}$, we have that $A \prec A_1$.

Example 3 shows that under $<_D$, it is possible for $\trianglelefteq_{Eli}$-LLP to rank an argument A that is further from the core (because it has two defeasible rules composed in series) to be more preferred than an argument A_1 that is closer to the core.

The next example shows that $<_{SP}$ does not completely capture that arguments should be less preferred than their (smaller) subarguments under $\trianglelefteq_{Dem}$.

Example 4. (Examples 1 to 3 continued) Let $r_1 := (a \Rightarrow b)$, $r_2 := (b \Rightarrow c)$ and $r_3 := (a \Rightarrow \neg c)$. Suppose $r_1 <_D r_3 <_D r_2$. Applying $\trianglelefteq_{Dem}$-WLP, we have $A_1 \prec B \prec A \prec A_0$, which by Example 2 means that $A \hookrightarrow B$ on B, so c is a justified conclusion. From Eq. 7, we have $r_2 <_{SP} r_1 <_{SP} r_3$, hence the new preference is A, $A_1 \prec B \prec A_0$, with A and A_1 incomparable.

As $A_1 \subseteq_{\text{arg}} A$, we would like $A \precsim A_1$. Does $\trianglelefteq_{Eli}$ fare any better?

Example 5. (Example 4 continued) Consider the same situation but with $\trianglelefteq_{Eli}$-WLP. From $r_1 <_D r_3 <_D r_2$ we have A, $A_1 \prec B \prec A_0$ with A and A_1 incomparable. However, from $r_2 <_{SP} r_1 <_{SP} r_3$ we have $A \prec A_1 \prec B \prec A_0$.

Example 5 makes the larger argument A less preferred than its subargument A_1. However, even when using $\trianglelefteq_{Eli}$, this does not generally hold true.

Example 6. Consider a different example where $a, b, c \in \mathcal{L}$, $\mathcal{K}_n := \{a\}$, $\mathcal{R}_d := \{r_1 := (a \Rightarrow b),\ r_2 := (b \Rightarrow c),\ r_3 := (b \Rightarrow \neg c)\}$. We can construct the arguments $A := [[[a] \Rightarrow b] \Rightarrow c]$ and $B := [[[a] \Rightarrow b] \Rightarrow \neg c]$, with $DR(A) = \{r_1, r_2\}$ and $DR(B) = \{r_1, r_3\}$. Suppose we have that $r_3 <_D r_2 <_D r_1$, which gives $r_3 <_{SP} r_2 <_{SP} r_1$ by Eq. 7. Under $\trianglelefteq_{Eli}$-WLP, both A and B are incomparable.

The $\prec_{SP}$-smaller argument should be that which has the $<_{SP}$-smallest rule, i.e. be further from the core. Example 6 shows that $\trianglelefteq_{Eli}$ does not behave well when comparing arguments with shared rules. We now define a set-comparison relation that compares arguments at their non-shared rules.

Definition 3. *Given* $<_D$ *and* $<_{SP}$ *from Eq. 8, the* ***structure preference set-comparison relation****,* $\triangleleft_{SP}$ *is the following binary relation on* $\mathcal{P}_{fin}(\mathcal{R}_d)$*:*

$$\Gamma \triangleleft_{SP} \Gamma' \Leftrightarrow (\exists x \in \Gamma - \Gamma')\,(\forall y \in \Gamma' - \Gamma)\, x <_{SP} y\,. \tag{9}$$

It can be shown that as $<_{SP}$ is a strict total order on $\mathcal{R}_d$, then $\triangleleft_{SP}$ is also a strict total order on $\mathcal{P}_{\text{fin}}(\mathcal{R}_d)$ [20, Lemma 4.2].[11] We specialise this relation to obtain the corresponding argument preference relation:

Definition 4. *Given* $<_D$*,* $<_{SP}$ *from Eq. 8 and* $\triangleleft_{SP}$ *from Eq. 9, the* ***(strict) structure-preference (SP) argument preference relation*** *is the relation* $\prec_{SP}$*, whch is Eq. 9 specalised to WLP:*

$$A \prec_{SP} B \Leftrightarrow DR(A) \triangleleft_{SP} DR(B), \tag{10}$$

with the ***non-strict SP argument preference relation*** *defined as* $A \precsim_{SP} B \Leftrightarrow [A \prec_{SP} B$ *or* $DR(A) = DR(B)]$*.*

We use WLP to avoid situations like Example 3. It follows that $\precsim_{SP}$ is a total preorder on $\mathcal{A}$. In particular, $\precsim_{SP}$ satisfies the following two properties that reflect how arguments further from the core are $\prec_{SP}$-less preferred.

1. Larger arguments are less preferred than smaller arguments, i.e. $A \subseteq_{\text{arg}} B \Rightarrow B \precsim_{SP} A$ [20, Lemma 4.1].
2. Infallible arguments, in this case strict arguments, are $\precsim_{SP}$-maximal. It follows from the definition of the defeat relation that (e.g.) strict arguments concluding θ will defeat any defeasible argument concluding $\neg\theta$.

In summary, $\prec_{SP}$ is a new argument preference relation, based on $<_{SP}$, which captures the intuition that arguments further from the core are less certain and therefore less preferred. As shown in the preceding examples, these properties do not hold for LLP or $\trianglelefteq_{Dem}$, and also fails for $\trianglelefteq_{Eli}$ when there are shared defeasible rules.

4 Applications to Prioritised Default Logic

In this section, we remind the reader that $\prec_{SP}$ has been used to endow Brewka's prioritised default logic (PDL) [4] with sound and complete argumentation semantics [20]. Further, we show that whereas when reasoning according to $<_D$ in PDL does not saitsfy Brewka and Eiter's two principles (articulated in Section 4.2 below, also see [5,6]), the principles are satisfied if $<_{SP}$ is used instead. We work in first order logic where the set of formulae is $\mathcal{FL}$, and the set of formulae without free variables is $\mathcal{SL} \subset \mathcal{FL}$, with the usual quantifiers and connectives. Classical entailment is denoted by $\models$. Given $S \subseteq \mathcal{FL}$, the *deductive closure of* S is $Th(S) \subseteq \mathcal{FL}$, and given $\theta \in \mathcal{FL}$, the *addition operator* is $S + \theta := Th(S \cup \{\theta\})$.

[11] Eq. 9 has previously been considered in a different context [7].

4.1 Brewka's Prioritised Default Logic as Argumentation

In this paper, we assume *closed normal defaults* of the form $\frac{\theta:\phi}{\phi}$ read as: if the *antecedent* $\theta \in \mathcal{SL}$ is the case and the *consequent* ϕ is consistent with what we know, then ϕ is also the case. Given $S \subseteq \mathcal{SL}$, a default is *active (in S)* iff $[\theta \in S,\ \phi \notin S,\ \neg\phi \notin S]$. Active defaults are precisely those that can be applied, such that the information gained is new and consistent with what we know. A *finite prioritised default theory* (PDT) is a structure $T := \langle D,\ W,\ <\rangle$, where $W \subseteq \mathcal{SL}$ is a possibly infinite set of known facts and $\langle D,\ <\rangle$ is a *finite* strict poset of defaults, where $d < d' \Leftrightarrow$ means d' is *more prioritised than* d. Intuitively, D consists of the defaults that nonmonotonically extend W. The inferences of a PDT are defined by its extensions. Let $<^+ \supseteq <$ be a linearisation of $<$. A *prioritised default extension (with respect to $<^+$)* (PDE) is a set $E := \bigcup_{i\in\mathbb{N}} E_i \subseteq \mathcal{SL}$ built inductively as

$$E_0 := Th\,(W) \text{ and } E_{i+1} := \begin{cases} E_i + \phi\,, & \text{if property 1} \\ E_i\,, & \text{else} \end{cases} \tag{11}$$

where "property 1" iff "ϕ is the consequent of the $<^+$-greatest default d active in E_i". Intuitively, one first generates all classical consequences from the facts W, and then iteratively adds the nonmonotonic consequences from the most prioritised default to the least. The set of defaults thus added are called the *generating defaults of* E, denoted by $GD(<^+) \subseteq D$. Notice if W is inconsistent then $E_0 = E = \mathcal{FL}$. It can be shown that the ascending chain $E_i \subseteq E_{i+1}$ stabilises at some finite $i \in \mathbb{N}$ and that E is consistent provided that W is consistent. E does not have to be unique because there are many distinct linearisations $<^+$ of $<$. We say the PDT T *sceptically infers* $\theta \in \mathcal{SL}$ iff $\theta \in E$ for *all* PDEs E.

Henceforth, we will assume a linearised PDT (LPDT) $T = \langle D, W, <\rangle$ where $<$ is a strict total order unless otherwise stated. By Eq. 11, since $<$ is total, there is only one way to apply the defaults in D, hence the PDE is unique and all inferences are sceptical. We say that θ *follows from* T iff $\theta \in E$ where E is the PDE of T. Further, we will assume W is consistent.

Given an LPDT $T := \langle D, W, <\rangle$ we translate directly into an argumentation system and knowledge base. For the argumentation system, we have that $\mathcal{L} = \mathcal{FL}$, $-$ is classical syntactic negation (as in Example 2),

$$\mathcal{R}_s := \{(\theta_1,\ \ldots,\ \theta_n \rightarrow \phi) \mid \theta_1,\ \ldots,\ \theta_n \models \phi\}\,, \tag{12}$$

$$\mathcal{R}_d := \left\{(\theta \Rightarrow \phi) \,\middle|\, \frac{\theta : \phi}{\phi} \in D\right\},\ (\theta \Rightarrow \phi) <_D (\theta' \Rightarrow \phi') \Leftrightarrow \frac{\theta : \phi}{\phi} < \frac{\theta' : \phi'}{\phi'}. \tag{13}$$

Also, $n \equiv *$ (we do not need undercuts), $\mathcal{K}_n = W$ and $\mathcal{K}_p = \varnothing$. Arguments and attacks are defined as in Section 2. It has been shown that a sound and complete argumentation semantics for PDL is obtained if $<_{SP}$ is used rather than $<_D$ [20]. Further, it has been shown why the ASPIC$^+$ argument preferences (Section 2.2) cannot give a sound and complete argumentation semantics based on $<_D$ [19, Section 4.2.1]. Intuitively, the inference mechanism of PDL (Eq. 11) picks out those defaults that are most preferred *and active*. This requirement of being active is not a property of $<$, but rather a property of the way PDEs are defined (Eq. 11). When translating into argumentation, $<_D$ only contains the information from $<$. To achieve soundness and completeness, we must explicitly incorporate the idea for a default to be active, such that arguments containing rules corresponding to blocked defaults are defeated by being less preferred, which is what $<_{SP}$ captures. Further, common rules are ignored because either they are included in E or not, which is what $\prec_{SP}$ captures (Eq. 9).

Using $<_{SP}$ and the associated defeat relation $\hookrightarrow$, it can be shown that there is a unique stable extension of $\langle \mathcal{A}, \hookrightarrow \rangle$ [20, Theorem 5.2]. We then have the following soundness and completeness result.

Theorem 1. *Let T be an LPDT where W is consistent, and $DG(T) := \langle \mathcal{A}, \hookrightarrow \rangle$ be its defeat graph with $\hookrightarrow$ defined under $\prec_{SP}$.*

1. *Let E be the extension of T. Then there exists a unique stable extension $\mathcal{E} \subseteq \mathcal{A}$ of $DG(T)$ such that $Conc(\mathcal{E}) = E$.*
2. *If $\mathcal{E} \subseteq \mathcal{A}$ is the stable extension of $DG(T)$, then $Conc(\mathcal{E})$ is the extension of T.*

Proof. See [20, Theorem 5.3]. ■

PDL is thus endowed with argumentation semantics. The following definition (Definition 5) is important for proving Theorem 1 because given the PDE E of T we can show that $Args(F(NBD(E)))$ is the unique stable extension $\mathcal{E}$ (Definition 1). The set $NBD(E)$ will become important in the next section.[12]

Definition 5. *(From [20, Eq. 5.3 and Lemma 5.1]) Let $T = \langle D, W, < \rangle$ with extension E. Define* ***the set of non-blocked defaults w.r.t.*** *E*

$$NBD(E) := \left\{ \frac{\theta : \phi}{\phi} \in D \middle| \theta \in E \text{ and } \neg\phi \notin E \right\}. \tag{14}$$

The corresponding set of defeasible rules is denoted by the obvious order isomorphism $F : D \to \mathcal{R}_d$ implicit in Eq. 13.

$$F(NBD(E)) := \{(\theta \Rightarrow \phi) \in \mathcal{R}_d \,|\, \theta \in E \text{ and } \neg\phi \notin E\}. \tag{15}$$

4.2 On Brewka and Eiter's Principles for Priorities

When PDL was defined, Prakken offered an alternative intuition of the preference that differed from Eq. 11 [4, Section 5]. Brewka modified PDL in order to accommodate Prakken's intuition, at the cost of a less intuitive, non-constructive inference mechanism. Brewka and Eiter later formalised this version of PDL with two intuitive principles which they argue all PDLs should satisfy (see below),

[12] Given T and its PDE E is generated by the total order $<$, we have $NBD(E) \neq GD(<)$ in general. See [20, Section 5.1] for an explanation.

which are satisfied by the non-constructive inference mechanism but not by Eq. 11. We will show that by importing $<_{SP}$ from the argumentation semantics of PDL back into PDL (Definition 6), Eq. 11 satisfies both of these principles as well. This allows us to retain the constructive inference mechanism of PDL.

Brewka and Eiter articulated two general principles that should hold true for any prioritised default logic [5,6].

1. **Principle I - Preference:** Let T be a Reiter default theory[13] [16] with extensions E_1 and E_2 respectively generated by the defaults $R \cup \{d_1\}$ and $R \cup \{d_2\}$ where $d_1, d_2, \notin R \subseteq D$. Let $< \neq \varnothing$ be a strict partial order on D such that T is now a PDT. If $d_2 < d_1$ then E_2 cannot be a PDE of T.
2. **Principle II - Relevance:** Let T be a PDT with PDE E. Let $d = \frac{\theta:\phi}{\phi} \in D$ such that $\theta \notin E$. Define a new PDT $T' = \langle D \cup \{d\}, W, <' \rangle$. If $<' \cap D^2 =<$ then E is also a PDE of T'.

Principle I states that if E_1 and E_2 are Reiter extensions of the PDT that have almost the same generating defaults but one, such that d_1 generates E_1 and d_2 generates E_2, and if $d_2 < d_1$, then E_1 should be the PDE of the PDT. Principle II states that the addition of irrelevant defaults cannot change the PDEs unless the preference changes. Principle I is not satisfied by PDL (Eq. 11).

Corollary 1. *Eq. 11 does not satisfy Principle I.*

Proof. (Based on [4, Section 5] and [6, Example 4]) Consider the PDT $T = \langle D, W, < \rangle$ with $W = \{a\}$, $D = \left\{d_1 := \frac{b:c}{c}, d_2 := \frac{a:b}{b}, d_3 := \frac{a:\neg c}{\neg c}\right\}$ and $d_2 < d_3 < d_1$. Applying Eq. 11, we have $E_0 = Th(\{a\})$, $E_1 = E_0 + \neg c$, $E_2 = E_1 + b$ so $E := Th(\{a, b, \neg c\})$. The equivalent Reiter default theory gives E and also $E' := Th(\{a, b, c\})$ as extensions. E is generated by the defaults $\{d_2, d_3\}$ and E' is generated by the defaults $\{d_1, d_2\}$. However, the original preference states $d_3 < d_1$. Therefore, Principle I states that E cannot be an extension. However, by Eq. 11, E is an extension. ■

We show here that Principle II is satisfied by Eq. 11 for general PDTs.

Theorem 2. *Let E be a PDE of $T = \langle D, W, < \rangle$ where $<$ does not have to be total. Let $d := \frac{\theta:\phi}{\phi} \notin D$ be a default such that $\theta \notin E$. Define a new PDT $T' := \langle D \cup \{d\}, W, <' \rangle$ where $<' \cap D^2 =<$. E is also a PDE of T'.*

Proof. If E is a PDE of T then by Eq. 11 there exists some $LPDT$ $T^+ = \langle D, W, <^+ \rangle$ where $<^+$ is a linearisation of $<$ such that E is the unique PDE of T^+. As $<' \cap D^2 =<$, we can place d at any position along the chain $<^+$ to make a linearisation $<'^+$ of $<'$. As $\theta \notin E$, then d is not $<'^+$-least active at $E_0 = Th(W)$ (else $\theta \in E$). Similarly, the addition of any of the defaults in D to $E_i \subseteq E$ will not make d $<'^+$-least active either because $\theta \notin E$ implies $\theta \notin E_i$. Therefore, E is also a PDE of T'. ■

[13] Here, T is a PDT with no priority (partial order $<= \varnothing$), see [4, Proposition 6].

Brewka and Eiter showed that their non-constructive version of PDL, modified to accommodate Prakken's intuition, does satisfy both these principles, unlike Eq. 11 [5,6]. We now prove that Eq. 11 can satisfy Principle I when it reasons with the PDL version of $<_{SP}$ on its defaults.

Example 7. (Continued from the Example in Corollary 1) We can transform T into its argumentation framework with $d_2 < d_3 < d_1 \mapsto r_2 <_D r_3 <_D r_1$.[14] It can be shown that $r_1 <_{SP} r_2 <_{SP} r_3$. Now consider the equivalent preference to $<_{SP}$ on the side of PDL, denoted as $d_1 <_{PDLSP} d_2 <_{PDLSP} d_3$. The Reiter extensions are $E = Th(\{a, b, \neg c\})$ and $E' = Th(\{a, b, c\})$, respectively generated by $\{d_2, d_3\}$ and $\{d_1, d_2\}$. Notice now that E is indeed the extension *and* $d_1 <_{PDLSP} d_3$, which means Principle I is satisfied.

Example 7 leads to the following definition, which formalises the idea of importing $<_{SP}$ from the argumentation semantics of PDL back into PDL.

Definition 6. *Let T be an LPDT. Let $< \cong <_D \mapsto <_{SP}$ be the corresponding SP order on the defeasible rules in its argumentation framework (Eqs. 7, 8 and 13). The* ***SP default priority****, $<_{PDLSP}$ on D is the total order that is order isomorphic to the SP order $<_{SP}$ on $\mathcal{R}_d$.*

Definition 6 transforms the prescriptive preference $<$ of a PDT to its corresponding descriptive preference $<_{PDLSP}$. This does not change the PDE, because Eq. 11 already selects the most active default at each stage, so explicitly incorporating "active" into $<$ to form $<_{PDLSP}$ means that selecting the $<_{PDLSP}$-greatest active rule is the same as selecting the $<$-greatest active rule.

We now prove that Principle I is always satisfied by Eq. 11 when using the SP default priority $<_{PDLSP}$. To do this, we first prove some properties of $Args(R)$ (Definition 1).

Lemma 1. *If $S = Args(R) \subseteq \mathcal{A}$ for some $R \subseteq \mathcal{R}_d$, then $DR(S) \subseteq R$, where $DR(S) := \bigcup_{A \in S} DR(A)$.*

Proof. For all $A \in S = Args(R)$, $DR(A) \subseteq R$ by definition. Let $r \in DR(S)$, then $(\exists A \in S)\, r \in DR(A) \subseteq R$, hence $r \in R$. ■

Lemma 2. *If $S = Args(R) \subseteq \mathcal{A}$ for some $R \subseteq \mathcal{R}_d$ and $(\forall r \in R)\, Ante(r) \subseteq Conc(S)$, then $DR(S) = R$.*

Proof. Let $r \in R$, then $Ante(r) \subseteq Conc(S)$ and $(\exists n \in \mathbb{N})\, r = (\theta_1, \ldots, \theta_n \Rightarrow \phi)$. If $n = 0$ then $Ante(r) = \varnothing$, so r is always applicable and the argument $[\Rightarrow \phi] \in S$, so $r \in DR(S)$. If $n > 0$, as $Ante(r) \subseteq Conc(S)$, then for each θ_i for $1 \leq i \leq n$ there is some argument $A_i \in S$ such that $Conc(A_i) := \theta_i$. By Lemma 1, $DR(A_i) \subseteq R$. As $Args(R)$ contains all arguments freely constructed from all premises, strict rules and defeasible rules in R, we can construct the argument $B := [A_1, \ldots, A_n \Rightarrow \phi]$ such that $TopRule(B) = r$. Clearly, $DR(B) = \bigcup_{i=1}^{n} DR(A_i) \cup \{r\} \subseteq R$, hence $B \in S$ and hence $r \in DR(S)$. As r is arbitrary, we conclude $R \subseteq DR(S)$. By Lemma 1, it follows that $R = DR(S)$. ■

[14] Where (e.g.) $d_1 := \frac{b:c}{c}$ means that $r_1 := (b \Rightarrow c)$, and similarly for d_i to r_i, $i \in \{2, 3\}$.

Theorem 3. *Let* $T := \langle D, W, < \rangle$ *be an LPDT with corresponding Reiter default theory* $T_\varnothing := \langle D, W \rangle$. *For* $i = 1, 2$ *let* E_i *be an extension of* $T_\varnothing$ *generated by* $R \cup \{d_i\} \subseteq D$, *for* $d_i \notin R$. *Let* $<_{PDLSP}$ *be the SP default priority (Definition 6). If* $d_2 <_{PDLSP} d_1$ *then* E_2 *is not an extension of* T.

Proof. (Sketch) Let $i = 1, 2$. By [6, Section 2] and Definition 5, $NBD(E_i) = R \cup \{d_i\}$. By Definition 5 and Footnote 14, $F(NBD(E_i)) = F(R) \cup \{r_i\} \subseteq \mathcal{R}_d$. Applying Definition 6 and Eq. 9, $d_2 <_{PDLSP} d_1 \Leftrightarrow r_2 <_{SP} r_1 \Rightarrow F(R) \cup \{r_2\} \triangleleft_{SP} F(R) \cup \{r_1\} \Leftrightarrow F(NBD(E_2)) \triangleleft_{SP} F(NBD(E_1))$.

Assume for contradiction that E_2 is the extension of T. By Theorem 1, $DG(T)$ has its unique stable extension $\mathcal{E}_2$ such that $Conc(\mathcal{E}_2) = E_2$. By Definition 5 and Theorem 1, $(\forall r \in F(NBD(E_2)))\, Ante(r) \subseteq E_2 = Conc(\mathcal{E}_2)$. By Lemma 2 and the proof of Theorem 1, $DR(\mathcal{E}_2) = F(NBD(E_2))$.

Now consider E_1. Define $\mathcal{E}_1 := Args(F(NBD(E_1))) \subseteq \mathcal{A}$, which is well-defined and *not* a stable extension of $DG(T)$. As E_1 is consistent by Eq. 11, it can be shown that $\mathcal{E}_1$ is conflict-free. Clearly, $r_1 \in DR(\mathcal{E}_1)$ so there is some argument $A \in \mathcal{E}_1$ such that $r_1 := TopRule(A)$. If $A \in \mathcal{E}_2$ then $DR(A) \subseteq DR(\mathcal{E}_2)$ and hence $r_1 \in DR(\mathcal{E}_2) = F(NBD(E_2)) = F(R) \cup \{r_2\}$, which is impossible. Therefore, $A \notin \mathcal{E}_2$. Hence $(\exists B \in \mathcal{E}_2)\, B \hookrightarrow A$ because $\mathcal{E}_2$ is a stable extension. As $B \in \mathcal{E}_2$, $DR(B) \subseteq DR(\mathcal{E}_2) = F(R) \cup \{r_2\}$. Either $r_2 \in DR(B)$ or $r_2 \notin DR(B)$. If $r_2 \notin DR(B)$ then $DR(B) \subseteq F(R)$, and hence $B \in \mathcal{E}_1$ – contradiction as $\mathcal{E}_1$ is conflict free. If $r_2 \in DR(B)$, then $r_2 \in DR(B) - DR(A)$ and $r_1 \in DR(A) - DR(B)$, but by our hypothesis, $r_2 <_{SP} r_1$ and hence $B \prec_{SP} A$ (Eqs. 9 and 10), so $B \not\hookrightarrow A$ – contradiction. Therefore, $\mathcal{E}_2$ cannot be a stable extension of $DG(T)$. By Theorem 1, $E_2 = Conc(\mathcal{E}_2)$ cannot be a PDE of T. ■

One way to see why Principle I failed for Eq. 11 is because if $<$ does not take the order of applicability of defeasible rules into account, then it is possible for a default of low priority to block a default of high priority because the former is applicable before the latter; this was what happened in the example of Corollary 1 (where $d_3 < d_1$ but $d_1 <_{PDLSP} d_3$). This is remedied by transforming the prescriptive preference $<$ into its corresponding descriptive preference $<_{PDLSP}$.

To summarise: PDL is a way of using preferences to guide default reasoning. One inference mechanism is Eq. 11, which offers a constructive definition of extending facts with non-monotonic conclusions from the defaults. Brewka and Eiter articulated two principles (Principles I and II) that they argued any PDL should satisfy, and pointed out that Eq. 11 does not satisfy Principle I (Corollary 1). We apply the insights from the argumentation semantics of PDL to show that Eq. 11, when reasoning with the PDL analogue of $<_{SP}$ (Definition 6), does satisfy Principle I. Principle II is already satisfied (Theorem 2).

5 Related Work

Preferences have been used to upgrade NMLs and logic programs. A variety of approaches and attempts have been made to classify them [9]. For example, Schaub and Wang have uniformly characterised three different approaches to preferences in logic programming [17], where they clarified that an answer set (analogous to extensions in argumentation) is *preference-preserving* if the preference on the rules also reflects their order of applicability. Delgrande, Schaub and Tompits developed a transformation of arbitrary preferences on the rules of a logic program into a preference that is aligned with the applications of the rules such that the answer sets are preserved [8]. Our work here investigates analogues of such a transformation in PDL inspired by ideas from preferences in structured argumentation.

We are not the first to investigate descriptive preferences in argumentation. Dung has investigated the analogue of a preference-preserving answer set, called an *enumeration-based extension*, while articulating axioms suitable for the study of structured argumentation with preferences [11,12]. Dung defined an *ordinary attack relation* (a type of defeat relation) that satisfies all of his axioms as well as Brewka and Eiter's two principles. Dung then investigated soundness and completeness of enumeration-based extensions with respect to the ordinary attack relation. He discovered that enumeration-based extensions are stable with respect to this attack, but only exist when the underlying knowledge base is *well-ranked*. Intuitively, this means that the underlying preference is already descriptive. Our work in this paper has provided a way of transforming a prescriptive preference into a descriptive preference, such that a corresponding stable extension can always be shown to exist.

6 Conclusions and Future Work

We have defined the *structure-preference (SP) order* $<_{SP}$ on ASPIC$^+$ defeasible rules, which provides a descriptive account of the use of preferences in structured argumentation theory (Section 3). This argument preference is interesting because it can endow Brewka's PDL with sound and complete argumentation semantics (Section 4.1) and it makes the original PDL inference mechanism satisfy Brewka and Eiter's postulates (Section 4.2, Theorems 2 and 3).

Future work would incorporate $\mathcal{K}_p$ into $<_{SP}$. As there is no explicit structure,[15] intuitively $<_K$ should be unchanged but still somehow "prior" to $<_{SP}$ on $\mathcal{R}_d$, because rules cannot be applied without premises. By representing W as $\mathcal{K}_p$ instead of K_n, we can consider PDTs where W is inconsistent. Further, what would the SP order look like when $<_K$ and $<_D$ are *partial* orders instead of total orders? One approach could be to consider all possible linearisations $<_K^+$ and $<_D^+$, transform each into the appropriate total order $<_{SP}$ and then aggregate preferences using some appropriate social welfare function [18, Chapter 9]. Finally, it will be interesting to study the SP argument preference under other ASPIC$^+$ argument preference relations, or in other structured argumentation theories with preferences over the defeasible rules and a well-defined notion of argument construction.

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[15] This is unlike defeasible rules, which can be composed in series.

References

1. P. Besnard, A. Garcia, A. Hunter, S. Modgil, H. Prakken, G. Simari, and F. Toni. Intro. to Structured Argumentation. *Argument & Computation*, 5(1):1–4, 2014.
2. A. Bondarenko, P. M. Dung, R. A. Kowalski, and F. Toni. An abstract, argumentation-theoretic approach to default reasoning. *Artificial Intelligence*, 93(1):63–101, 1997.
3. G. Brewka. *Nonmonotonic Reasoning: Logical Foundations of Commonsense*, volume 12 of *Cambridge Tracts in Theoretical Computer Science*. Cambridge University Press, 1991.
4. G. Brewka. Adding Priorities and Specificity to Default Logic. In *Logics in Artificial Intelligence*, pages 247–260. Springer, 1994.
5. G. Brewka and T. Eiter. Preferred answer sets for extended logic programs. *Artificial Intelligence*, 109(1):297–356, 1999.
6. G. Brewka and T. Eiter. Prioritizing Default Logic. In *Intellectics and Computational Logic*, pages 27–45. Springer, 2000.
7. G. Brewka, M. Truszczynski, and S. Woltran. Representing Preferences Among Sets. In *AAAI*, 2010.
8. J. P. Delgrande, T. Schaub, and H. Tompits. A framework for compiling preferences in logic programs. *Theory and Practice of Logic Programming*, 3(2):129–187, 2003.
9. J. P. Delgrande, T. Schaub, H. Tompits, and K. Wang. A Classification and Survey of Preference Handling Approaches in Nonmonotonic Reasoning. *Computational Intelligence*, 20(2):308–334, 2004.
10. P. M. Dung. On the Acceptability of Arguments and its Fundamental Role in Nonmonotonic Reasoning, Logic Programming and n-Person Games. *Artificial Intelligence*, 77:321–357, 1995.
11. P. M. Dung. An Axiomatic Analysis of Structured Argumentation for Prioritised Default Reasoning. In *ECAI2014*, pages 267–272. IOS Press, 2014.
12. P. M. Dung. An axiomatic analysis of structured argumentation with priorities. *Artificial Intelligence*, 231:107–150, 2016.
13. S. Modgil and H. Prakken. A General Account of Argumentation with Preferences. *Artificial Intelligence*, 195:361–397, February 2013.
14. G. Pigozzi, A. Tsoukiàs, and P. Viappiani. Preferences in Artificial Intelligence. *Annals of Mathematics and Artificial Intelligence*, pages 1–41, 2014.
15. H. Prakken. An Abstract Framework for Argumentation with Structured Arguments. *Argument and Computation*, 1(2):93–124, 2010.
16. R. Reiter. A Logic for Default Reasoning. *Artificial Intelligence*, 13:81–132, 1980.
17. T. Schaub and K. Wang. A Comparative Study of Logic Programs with Preference: Preliminary Report. In *Answer Set Programming*, 2001.
18. Y. Shoham and K. Leyton-Brown. *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2008.
19. A. P. Young, S. Modgil, and O. Rodrigues. Argumentation Semantics for Prioritised Default Logic. *arXiv preprint arXiv:1506.08813v2*, 2015. Available from `http://arxiv.org/abs/1506.08813`, last accessed 22/5/2016.
20. A. P. Young, S. Modgil, and O. Rodrigues. Prioritised Default Logic as Rational Argumentation. In *Proceedings of the 2016 International Conference on Autonomous Agents & Multiagent Systems*, pages 626–634. International Foundation for Autonomous Agents and Multiagent Systems, 2016.