# **British Topology Meeting 2024**

# University of Aberdeen, August 28-30

# Participants:

Macarena Arenas	University of Cambridge
Miguel Barrero	Radboud University Nijmegen
Valerio Bianchi	Cardiff University
Rudradip Biswas	University of Warwick
Rachael Boyd	University of Glasgow
Lukas Brantner	University of Oxford
Robert Burklund	University of Copenhagen
Sebastian Chenery	University of Southampton
Gabriel Corrigan	University of Glasgow
Jack Davidson	University of Sheffield
Briony Eldridge	University of Southampton
Salvatore Elia	University of Southampton
Andrew Fisher	University of Sheffield
Mark Grant	University of Aberdeen
Daniel Graves	University of Leeds
Richard Hepworth	University of Aberdeen
Daniel Kasprowski	University of Southampton
Jarek Kędra	University of Aberdeen
Michael Kohn	Durham University
Ran Levi	University of Aberdeen
Kerr Maxwell	University of Birmingham
Ehud Meir	University of Aberdeen
Juliano Morimoto	University of Aberdeen
Csaba Nagy	University of Glasgow
John Nicholson	University of Glasgow
Patrick Orson	California Polytechnic State University
Irakli Patchkoria	University of Aberdeen
Ulrich Pennig	Cardiff University
Mark Powell	University of Glasgow
Patrick Ramsey	Lancaster University
Sam Richardson	Cardiff University
Ananya Satoskar	LSGNT
Jacob Saunders	University of Sheffield
Dirk Schuetz	Durham University
Baylee Schutte	University of Aberdeen
Malthe Sporring	University of Edinburgh
Jan Steinebrunner	University of Cambridge
Ulrike Tillmann	University of Oxford
Markus Upmeier	University of Aberdeen
Ruzhen Yang	University of Warwick

# Schedule

## Wednesday

13:00-13:30 :	Arrival and Registration
13:30-14:30 :	Daniel Kasprowski, Stable equivalence relations on 4-manifolds
14:30-15:00	Coffee break
15:00-15:30	Daniel Graves, Cohomology of diagram algebras
15:40–16:40	Robert Burklund, Beyond the telescope conjecture at height 2
16:50–17:20	Sebastian Chenery, Gyration Stability for Projective Planes
Thursday	
09:00-10:00	Macarena Arenas, Cubically presented groups, strong asphericity, and applications
10:10–10:40	Jan Steinebrunner, Moduli spaces of 3-manifolds with boundary are finite
10:40-11:10	Coffee break
11:10–11:40	Mark Powell, Corks for exotic diffeomorphims
11:50-12:20	Patrick Orson, Unknotting nonorientable surfaces topologically
12:20-14:00	Lunch break
14:00-15:00	Lukas Brantner, Integrating partition Lie algebroids
15:00-15:30	Coffee break
15:30-16:00	Baylee Schutte, Projective span of Wall manifolds
16:10–16:40	John Nicholson, Comparing equivalence relations on 4-manifolds
19:00-	Conference dinner @ Palm Court Hotel

# ScheduleFriday09:00–09:30Valerio Bianchi, Units of equivariant K-theory and bundles of<br/>C\*-algebras09:40-10:10Sam Richardson, Exponential Functors and Twists of K-Theory10:10–10:40Coffee break10:40–11:40Ulrike Tillmann, Homology stability for generalised Hurwitz spaces<br/>and asymptotic monopoles11:40Close

### Abstracts

Daniel Kasprowski, Stable equivalence relations on 4-manifolds
 Kreck's modified surgery gives an approach to classify 2n-manifolds up to stable diffeomorphism, i.e., up to a connected sum with copies of S<sup>n</sup> × S<sup>n</sup>. In dimension 4, we use a combination of modified and classical surgery to compare the stable diffeomorphism classification with other stable equivalence relations. Most importantly, we consider homeomorphism and homotopy equivalence up to connected sum with copies of S<sup>2</sup> × S<sup>2</sup>. This is joint work with Mark Powell and with John Nicholson and Simona Veselá.

• Daniel Graves, Cohomology of diagram algebras

In this talk I will recap known results about homology of the partition algebras and state some new results, joint with Andrew Fisher, about the (co)homology of related families of algebras.

### • Robert Burklund, Beyond the telescope conjecture at height 2

I will survey what we currently know about the relationship between T(2)-local spectra and K(2)-local spectra and what this tells us about the stable homotopy groups of spheres. This talk is based on projects joint with Carmeli, Clausen, Hahn, Levy, Schlank and Yanovski.

### • Sebastian Chenery, Gyration Stability for Projective Planes

Gyrations are operations on manifolds that arise in geometric topology, where a manifold M may exhibit multiple gyrations depending on the chosen "twisting". A natural question arises: for a given M, do all gyrations share the same diffeomorphism, homeomorphism, or homotopy type regardless of the twisting we took? This property is known as gyration stability. Inspired by recent work by Duan, which demonstrated that the quaternionic projective plane is not gyration stable (with respect to diffeomorphism) by invoking spin structures, we will explore gyration stability of projective planes from the homotopy theoretic perspective. Our generalization provides new results and leads to a complete description of gyration stability for the complex, quaternionic, and octonionic projective planes. This is joint work with Stephen Theriault.

### • Macarena Arenas, *Cubically presented groups, strong asphericity, and applications* We'll explore the problem of finding effective models for the classifying spaces of certain quotients of cubulated groups, we'll discuss the framework – cubical small-cancellation theory – that provides the necessary tools to do so, and we'll explain how this viewpoint allows us to compute the homology and cohomology of various examples, including many Artin groups and other related families.

### • Jan Steinebrunner, Moduli spaces of 3-manifolds with boundary are finite

In joint work with Rachael Boyd and Corey Bregman we study the classifying space BDiff(M) of the diffeomorphism group of a connected, compact, orientable 3-manifold M. By a theorem of Milnor every such M has a unique prime decomposition as a connected sum of prime 3-manifolds. The purpose of this talk is to explain how one can compute the moduli space BDiff(M) in terms of the moduli spaces of prime factors. We show that certain space of systems of reducing spheres is contractible. (This can be thought of as saying that the modular infinity-operad of 3-manifolds is freely generated by irreducible manifolds.) We use this to prove that if M has non-empty boundary, then  $BDiff(M, \partial M)$  has the homotopy type of a finite CW complex, as was conjectured by Kontsevich.

### • Mark Powell, Corks for exotic diffeomorphims

In this project, joint with Krushkal, Mukherjee, and Warren, we show that often exotic diffeomorphisms of simply connected 4-manifolds can be localised to a contractible submanifold, and we give some applications.

### • Patrick Orson, Unknotting nonorientable surfaces topologically

Knot invariants are typically used to give a negative answer to the question of when two embeddings are ambiently isotopic, and rarely to give a positive answer. An exception is the celebrated result of Freedman and Quinn that if the complement of a 2-sphere embedded in the 4-sphere has cyclic fundamental group then that 2-sphere is topologically unknotted. I will discuss recent work, where we proved the analogous result for closed nonorientable surfaces in the 4-sphere (in most cases). This is joint work with Anthony Conway and Mark Powell.

### • Lukas Brantner, Integrating partition Lie algebroids

Given a finite purely inseparable field extension F/K, intermediate fields  $K \subset E \subset F$  lie in Galois correspondence with certain partition Lie algebroids on the point Spec(F). In this talk, I will focus on a more geometric setting, and formally integrate partition Lie algebroids on a (reasonable) R-scheme X to (suitably finite) stacks under X. This talk is based on joint works with Waldron and Magidson-Nuiten, and ties into the forthcoming work of Jiaqi Fu.

### • Baylee Schutte, Projective span of Wall manifolds

The projective span of a smooth manifold is defined to be the maximal number of linearly independent tangent line fields. In this talk, we introduce projective span and highlight its relationship with the span (a more classical invariant). Chiefly, we outline our computation of the projective span of all Wall manifolds, which are certain mapping tori of Dold manifolds. To carry out this computation, we utilize the representation theory of complex Clifford algebras to construct quasi-invariant vector fields on coverings of Wall manifolds, for such vector fields descend to line fields on the quotient.

### • John Nicholson, Comparing equivalence relations on 4-manifolds

Two CW-complexes are said to be simple homotopy equivalent if they are related by a sequence of collapses and expansions of cells. This notion interpolates between homeomorphism and homotopy in the sense that simple homotopy equivalent implies homotopy equivalent, and homeomorphic implies simple homotopy equivalent. In this talk, I will present the first examples of two 4-manifolds which are homotopy equivalent but not simple homotopy equivalent, as well as in all higher even dimensions. This is joint work with Csaba Nagy and Mark Powell. I will also discuss progress on the question of whether smooth 4-manifolds exist with these properties, through joint work with Daniel Kasprowski and Simona Veselá. Finally, I will mention joint work in progress with Ian Hambleton in which we construct arbitrarily large families of smooth 4-manifolds which are stably diffeomorphic but not homotopy equivalent. A key feature of these results is the development of methods from algebra and number theory to both construct and distinguish non-simply connected 4-manifolds.

### • Valerio Bianchi, Units of equivariant K-theory and bundles of C\*-algebras

Every  $E_{\infty}$ -ring spectrum has an associated spectrum of units. Locally trivial  $C^*$ -algebra bundles with compact operators as fibres provide elements in the first group of the cohomology theory associated to the units of complex topological K-theory by a classical result of Dixmier and Douady. Dadarlat and Pennig have shown that this generalises to the much larger family of bundles of stabilised strongly self-absorbing  $C^*$ -algebras. Building on work of Evans and Pennig I consider  $\mathbb{Z}/p\mathbb{Z}$ -equivariant  $C^*$ -algebra bundles over  $\mathbb{Z}/p\mathbb{Z}$ -spaces. The fibres of these bundles are infinite tensor products of the endomorphism algebra of a  $\mathbb{Z}/p\mathbb{Z}$ -representation. In joint work with Pennig we show that the theory refines completely to this equivariant setting. In particular, we prove a full classification of the  $C^*$ -algebra bundles via equivariant stable homotopy theory.

### • Sam Richardson, Exponential Functors and Twists of K-Theory

An overview of the work I have done together with my PhD supervisor Dr. Ulrich Pennig. This work achieves its  $C^*$ -algebra flavour from its connection to Dixmier-Douady classes. I will introduce the idea of exponential functors and discuss how they induce maps between certain spaces which in turn induce (not quite) natural transformations of cohomology theories. In our quest to understand these induced maps, we require an understanding of the Weyl map  $W: SU(n)/TxT \rightarrow SU(n)$ , the class of this map in K-theory, and the tautological line bundles over SU(n). Homotopies and extraordinary cohomology theories abound!

• Ulrike Tillmann, *Homology stability for generalised Hurwitz spaces and asymptotic monopoles* Configuration spaces have played an important role in mathematics and its applications. In particular, the question of how their topology changes as the cardinality of the underlying configuration changes has been studied for some fifty years and has attracted renewed attention in the last decade. While classically additional information is associated "locally" to the points of the configuration, there are interesting examples when this additional information is "non-local". With Martin Palmer we have studied homology stability in some of these cases, including Hurwitz space and moduli spaces of asymptotic monopoles.