UNIVERSITY OF ABERDEEN **SESSION 1BC–1AD**

DEGREE EXAMINATION

MX4549 Geometry – MOCK

1–13–32 (23:00–25:00)

Only calculators approved by the Department of Mathematical Sciences may be used in this examination. Calculator memories must be clear at the start of the examination.

Marks may be deducted for answers that do not show clearly how the solution is reached.

Answer THREE questions. All questions carry equal marks.

1. Let X be the surface of the unit cube with two adjacent faces removed (a surface made of 4 squares) equipped with the usual polygonal metric. Let A and B be the distinct vertices belonging to exactly one face each.

(a) Draw the surface X and mark the vertices A and B.

(b) What is the distance between A and B.

(c) Define the concept of a geodesic in a metric space.

(d) How many distinct geodesics are there from A to B in X.

2. **(a)** Define the concept of a surface patch $f\colon U \to \mathbf{R}^3$.

(b) Give an example a surface patch $f\colon U \to \mathbf{S}^2 \subset \mathbf{R}^3$, where $\mathbf{S}^2$ is the unit sphere and $U = (-\pi/2, \pi/2) \times \mathbf{R}$.

(c) Compute the second fundamental form for the surface patch f defined above.

(d) Define the Gauss curvature and prove that it is positive for the above sphere.

3. **(a)** Define the concept of the curvature of a vertex in a polygonal surface.

(b) State the Gauss-Bonnet theorem for closed polygonal surfaces.

(c) Prove the Gauss-Bonnet theorem for closed polygonal surfaces.

(d) Draw examples of closed polygonal surfaces with **(i)** all vertices positively curved, **(ii)** vertices positively, negatively curved and flat.