

## MA1006 ALGEBRA – Week 2

In all the questions below when asked about the image or preimage give three types of answer: describe the set using the mathematical notation, draw it and describe it in words. In all questions justify your answer.

1. Read the wikipedia articles about the Cartesian coordinate system, a function and the image of a function.
2. Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be a function defined by  $f(x) = -x^2 + 3$ .
  - a. Draw a graph of the function  $f$ .
  - b. Is the function  $f$  injective?
  - c. Is the function  $f$  surjective?
  - d. What is the image of the function  $f$ .
  - e. What is the image of the interval  $[-1, 3]$  with respect to the function  $f$ ?
  - f. What is the preimage of the interval  $(0, 1)$ ?
  - g. What is the preimage of the interval  $[3, 10]$  and the interval  $(3, 4)$ ?
3. Let  $f: \mathbf{R} \rightarrow \mathbf{R}^2$  be a map defined by  $f(x) := (\cos(x), \sin(x))$ .
  - a. What is the image of the map  $f$ ?
  - b. What is the image of the interval  $[0, 1]$  with respect to the map  $f$ ?
  - c. What is the image of the interval  $[0, 10]$  with respect to the map  $f$ ?
  - d. Is the map  $f$  injective?
  - e. Is the map  $f$  surjective?
  - f. Try to draw the graph of  $f$ . Notice that the graph is a subset of three dimensional space which makes it quite difficult to draw.
  - g. What is the preimage of the subset  $\{(1, 0)\}$  with respect to  $f$ ?
4. Let  $f: \mathbf{R}^2 \rightarrow \mathbf{R}$  be a map defined by  $f(x, y) = x^2 + y^2$ .
  - a. Is  $f$  injective?
  - b. Is  $f$  surjective?
  - c. What is the image of  $f$ ?
  - d. What is the preimage of the point  $1 \in \mathbf{R}$ ?
  - e. What is the preimage of the interval  $[0, 1]$ ?
  - f. What is the preimage of the interval  $[-3, 1]$ ?
  - g. What is the preimage of the interval  $[1, 2]$ ?
5. Let  $f: \mathbf{R}^2 \rightarrow \mathbf{R}$  be a map defined by  $f(x, y) = |x - y|$ .
  - a. Is  $f$  injective?
  - b. Is  $f$  surjective?
  - c. What is the image of  $f$ ?

- d. What is the preimage of the point  $1 \in \mathbf{R}$ ?
  - e. What is the preimage of the interval  $[0, 1]$ ?
  - f. What is the preimage of the interval  $[-3, 1]$ ?
  - g. What is the preimage of the interval  $[1, 2]$ ?
6. Let  $f: \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be a map defined by  $f(x, y) = (x + y, y)$ .
- a. Is  $f$  injective?
  - b. Is  $f$  surjective?
  - c. What is the image of the square on the plane with vertices at  $(0, 0), (1, 0), (1, 1), (0, 1)$ ?
  - d. What is the image of the  $x$ -axis? What is the image of a horizontal line?
  - e. What is the image of the  $y$ -axis? What is the image of a vertical line?
  - f. Write the formula for the composition of  $f$  with itself.
7. Let  $f, g: \mathbf{R} \rightarrow \mathbf{R}$  be functions defined by  $f(x) = ax + b$  and  $g(x) = cx + d$ , where  $a, b, c, d \in \mathbf{R}$  and  $a \neq 0$  and  $c \neq 0$ . Write the formulae for the compositions  $f \circ g$  and  $g \circ f$  and for the inverse functions  $f^{-1}$  and  $g^{-1}$ .
8. Give an example of a map  $f: \mathbf{R}^2 \rightarrow \mathbf{R}$  such that the preimage of the set  $\{1\}$  is equal to:
- a. the  $x$ -axis;
  - b. the  $y$ -axis;
  - c. the circle of radius one centered at the origin;
  - d. the circle of radius  $r > 0$  centered at  $(a, b)$ ;
  - e. the boundary of the square of area one with vertices on the axes.

### Summary of Week 2:

- Coordinates in many dimensions.
- Maps and functions.
- Image and preimage.