Tutorial solutions

1. A shock wave travels along a copper bar with a velocity of 5km/s.
   (a) Determine the sound speed in the copper bar, before and after the shock.
   (b) Determine the pressure in the bar after the shock.
   (c) Determine the particle velocity in the bar after the shock.

You may need the following data. For copper, the shock velocity D is linearly related to the particle velocity u as

$$D = c_0 + su ,$$

where $c_0 = 3.940\text{km/s}$, $s = 1.489$. The Young's modulus and the density of the copper at rest are 120GPa and 8.930g/cm^3, respectively.

Solution:

(a) Shock velocity

$$D = 5\text{km/s}$$

Particle velocity

$$u = \frac{D - c_0}{s}$$
$$= \frac{5 - 3.94}{1.489}\text{km/s}$$
$$= 0.712\text{km/s}$$

Before shock:

Density: $\rho_0 = 8.93\text{g/cm}^3$

Sound speed: $\sqrt{\dfrac{E}{\rho_0}} = \sqrt{\dfrac{120\times 10^9}{8.93\times 10^3}} = 3.7\text{km/s}$

After shock:

Density: $\rho_1 = \rho_0 \dfrac{D}{D-u} = 8.93\times\dfrac{5}{5-0.712}\text{g/cm}^3 = 10.4\text{g/cm}^3$

Sound speed: $\sqrt{\dfrac{E}{\rho_1}} = \sqrt{\dfrac{120\times 10^9}{10.4\times 10^3}} = 3.4\text{km/s}$

(assuming that Young's modulus is not changed after shock)

(b) Pressure in the bar after the shock:

$$P = \rho_0 Du = 8.93\times 5\times 0.712\text{GPa} = 32\text{GPa}$$

(c) Particle velocity in the bar after the shock:

$$u = 0.712\text{km/s}$$

2. A slab made of aluminum (flying in the air at 2km/s) strikes the slab made of copper (at rest), and keeps pressing to sustain the pressure. Calculate

(a) the particle velocity and pressure at the interface between aluminum and copper slabs;

(b) the velocity of the shock wave generated in the aluminum slab;

(c) the velocity of the shock wave generated in the copper slab.

Solution:

(a) impact velocity: $u_{\text{imp}} = 2\text{km/s}$

Particle velocity at interface: u

Pressure at interface: P

<u>In copper:</u> (reference system is with copper at rest)

Shock velocity in copper: D_{Cu}

$$P = \rho_{0Cu} D_{Cu} u \ . \qquad (1)$$

$$D_{Cu} = c_{0Cu} + s_{Cu} u \ , \qquad (2)$$

where from problem 1, $c_{0Cu} = 3.940\times10^3 \text{m/s}$, $s_{Cu} = 1.489$, $\rho_{0Cu} = 8.93\times10^3 \text{kg/m}^3$.

<u>In aluminum:</u> (reference system is with aluminum flying at the velocity of u_{imp})

In this reference system, shock velocity and particle velocity in aluminum are denoted as D_{Al} and u_{Al}, respectively. One has the following relations:

$$P = \rho_{0Al} D_{Al} u_{Al} \ . \qquad (3)$$

$$u_{imp} - u_{Al} = u \ . \qquad (4)$$

$$D_{Al} = c_{0Al} + s_{Al} u_{Al} \ , \qquad (5)$$

where $c_{0Al} = 5.328\times10^3 \text{m/s}$, $s_{Al} = 1.338$, $\rho_{0Al} = 2.785\times10^3 \text{kg/m}^3$.

From Eqs. (1) - (5), one has the equation for u

$$\begin{aligned}&\left(\rho_{0Al} s_{Al} - \rho_{0Cu} s_{Cu}\right) u^2 - \left(\rho_{0Al} c_{0Al} + 2\rho_{0Al} s_{Al} u_{imp} + c_{0Cu}\rho_{0Cu}\right) u \\ &+ \rho_{0Al} u_{imp}\left(c_{0Al} + s_{Al} u_{imp}\right) = 0\end{aligned} \ . \qquad (3)$$

Solving the equation, $u = 0.63\text{km/s}$.

Substitute to Eq. (1), pressure $P = 27\text{GPa}$.

(b) From Eq. (5) $D_{Al} = 7.2\text{km/s}$.

(c) From Eq. (2) $D_{Cu} = 4.9\text{km/s}$.

3. A slab made of copper (flying in the air at 2km/s) strikes the slab made of aluminum copper (at rest), and keeps pressing to sustain the pressure. Calculate

(a) the particle velocity and pressure at the interface between aluminum and copper slabs;

(c) the velocity of the shock wave generated in the aluminum slab;

(d) the velocity of the shock wave generated in the copper slab.

Solution:

(a) impact velocity: $u_{\text{imp}} = 2\text{km/s}$

Particle velocity at interface: u

Pressure at interface: P

In aluminum (reference system is with aluminum at rest)

Shock velocity in aluminum D_{Al}

$$P = \rho_{0Al} D_{Al} u \,. \tag{1}$$

$$D_{Al} = c_{0Al} + s_{Al} u \,, \tag{2}$$

In copper: (reference system is with copper flying at the velocity of u_{imp})

In this reference system, shock velocity and particle velocity in copper are denoted as D_{Cu} and u_{Cu}, respectively. One has the following relations:

$$P = \rho_{0Cu} D_{Cu} u_{Cu} \,. \tag{3}$$

$$u_{imp} - u_{Cu} = u \,. \tag{4}$$

$$D_{Cu} = c_{0Cu} + s_{Cu} u_{Cu} \,. \tag{5}$$

From Eqs. (1) - (5), one has the equation for u

$$\begin{aligned}&\left(\rho_{0Cu} s_{Cu} - \rho_{0Al} s_{Al}\right) u^2 - \left(\rho_{0Cu} c_{0Cu} + 2\rho_{0Cu} s_{Cu} u_{imp} + c_{0Al}\rho_{0Al}\right) u \\ &+ \rho_{0Cu} u_{imp}\left(c_{0Cu} + s_{Cu} u_{imp}\right) = 0\end{aligned} \,. \tag{3}$$

Solving the above equation, take the solution $u = 1.4\text{km/s}$ ($< u_{\text{imp}}$).

Substitute to Eq. (1), pressure $P = 27\text{GPa}$.

(b) From Eq. (2) $D_{Al} = 7.2\text{km/s}$.

(c) From Eq. (5) $D_{Cu} = 4.9\text{km/s}$.

4. An explosive of 500kg TNT charge detonates at 60m underwater. Can this be treated as a deep underwater explosion? Estimate the maximum bubble diameter and oscillation period for the first bubble pulsation cycle, assuming 50% of the explosive energy contributes to the bubble expansion.

Solution:
The yield of the explosive is

$$500 \times 2.2 = 1100 \text{ lb of TNT.}$$

Therefore, $W = 1100$.

The explosive is placed about

$$60 \text{ meter} = 196.8 \text{ ft}$$

under the water level. Therefore, $d = 196.8$.

Since

$$\frac{d}{W^{1/3}} = \frac{196.8}{1100^{1/3}} = 19 > 16,$$

so this is a deep underwater explosion.

The hydrostatic pressure at the burst point is

$$\begin{aligned} P &\approx \rho g d + 1atm \\ &= 1\times10^3 \times 9.8 \times 60\text{Pa} + 1.01\times10^5\,\text{Pa} \\ &= 6.89\times10^5\,\text{Pa} \end{aligned}$$

where $g = 9.8\text{m/s}^2$ is the gravity.

The energy available in the TNT is $4.84\times10^6\,\text{J/kg}$. So the total explosive energy is

$$\begin{aligned} E_{\text{total}} &= 500\times4.84\times10^6\,\text{J} \\ &= 2.4\times10^9\text{J} \end{aligned}$$

The energy contribute to the bubble expansion is

$$\begin{aligned} E &= E_{\text{total}} \times 50\% \\ &= 1.2\times10^9\text{J} \end{aligned}$$

The maximum bubble radius for the first cycle can be estimated as

$$\begin{aligned} R_{\max} &= J\left(\frac{E}{P}\right)^{1/3} \\ &= 0.58\times\left(\frac{1.2\times10^9}{6.89\times10^5}\right)^{1/3} \\ &= 7.0m \end{aligned}$$

Oscillation period for the first bubble pulsation cycle

$$
\begin{aligned}
T &= K\frac{E^{1/3}}{P^{5/6}}\sqrt{\rho} \\
&= 1.12\times\frac{\left(1.2\times10^{9}\right)^{1/3}}{\left(6.89\times10^{5}\right)^{5/6}}\sqrt{10^{3}}\,s\,. \\
&= 0.51s
\end{aligned}
$$