1 Application of Shock Wave Fragmentation

Shock wave can generate fragmentation. This is used in nuclear decommissioning engineering.

The best way for the demolition phase of nuclear decommissioning is to use explosives. 192 kilograms of explosives were used in the recently explosive demolition of the Calder Hall cooling towers (Figure 1).

(a) Explosive charges are placed under the towers.

(b) Controlled collapse of each tower.

Figure 1. The cooling towers of the shut down Calder Hall plant at Sellafield, UK – the world's first industrial-scale nuclear power plant – was demolished using explosives on 29 Sept 2007 (From World Nuclear News).

2 Shock Wave Simulation in a 1D String Model

Material properties: particle mass m, particle distance a. The inter-particle potential is described as a 6-12 Lennard-Jones potential.

3 Impact of Slab

We study the impact generated shock waves. In Figure 2, slab A is flying towards slab B (still) at a velocity u_{impact} . What are the shock waves generated from the collision?

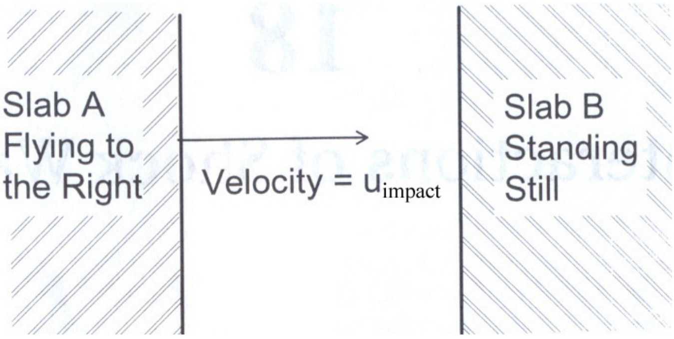

Figure 2. Impact of slab A on B.

Section 2.1 will review and extend previous learned Rankine-Hugoniot relation. The problem will be solved in Section 2.2.

3.1 Pressure - Particle Velocity Hugoniot

The term Hugoniot means the sequence of thermodynamic equilibrium states reached behind each shock for a sequence of different-strength shocks from a given initial state.

Based on the mass and momentum conservations, the Rankine-Hugoniot equations give the pressure-particle velocity relationship for shock wave passing a still media (Figure 3) as

$$P = \rho_0 D u_1 , \qquad (1)$$

where ρ_0 is the initial mass density, *D* is the shock velocity, and u_1 is the particle velocity after the shock.

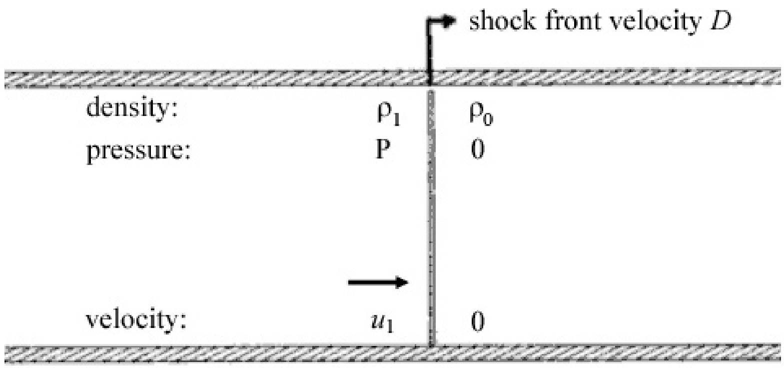

Figure 3. Shock transition from state 0 (initial velocity is 0) to state 1 (velocity is u_1)

It is found that the shock velocity *D* is linearly related to the particle velocity *u* for lots of materials, and can be expressed as

$$D = c_0 + s u_1 . \qquad (2)$$

where c_0 and *s* are material parameters that can be determined from experimental measurements.

Therefore, we have the pressure (*P*) – particle velocity (u_1) relationship as

$$P = \rho_0 c_0 u_1 + \rho_0 s u_1^2 . \quad (3)$$

Until now we looked at the pressure – particle velocity relationship with assumption that

(1) the coordinate direction and the shock direction are the same, and

(2) the initial particle velocity is zero.

For assumption (1), we can always adjust our coordinate system in the shock direction. The assumption (2), in effect, meant looking at the shock relative to the oncoming material. Now, allowing the initial state of the material to be in motion (velocity u_0) before shock arrival, we can change u_1 to $u_1 - u_0$, and rewrite Eq. (3) as

$$P = \rho_0 c_0 \left(u_1 - u_0\right) + \rho_0 s \left(u_1 - u_0\right)^2 . \quad (4)$$

Both u_0 and u_1 are in the adjusted coordinated system that shares the same direction of the shock wave.

3.2 Shock waves generated from slab impact

We now study the slab impact problem in Figure 2. When impact occurs, a pressure or shock pulse is formed. Slab A continues to press upon slab B, sustaining the pressure. Two shock waves are generated, one moves into B toward the right, the other into A toward the left. As long as the slabs are in contact, the pressure (*P*), as well as the particle velocity (*u*), on both sides of the interface, must remain the same and equal.

For slab B, the shock goes to the right, and particle velocity jumps from 0 to *u*,

$$P = \rho_{0B} c_{0B} u + \rho_0 s_B u^2 . \quad (5)$$

where ρ_{0B}, c_{0B} and s_B are material properties for slab B.

For slab A, the shock goes to the left, and the particle velocity after shock is *u* to the right. We set a coordinate system facing left. In this coordinate system, the particle velocity jumps from $-u_{\text{impact}}$ to $-u$. Therefore, we have

$$P = \rho_{0A} c_{0A} \left(u_{\text{impact}} - u\right) + \rho_{0A} s_A \left(u_{\text{impact}} - u\right)^2 . \quad (6)$$

where ρ_{0A}, c_{0A} and s_A are material properties for slab A.

Now we have two unknowns, *P* and *u*, and two equations, Eq. (5) and (6). The pressure *P* and particle velocity after impact *u* can be solved from the two equations.

Problem: A slab of 2024 aluminum alloy flying through air at $u_{\text{impact}} = 1.8\text{km/s}$ strikes a slab of 304 stainless steel. What particle velocity would be generated in the two materials at the impact surface? What shock pressure would be generated? How fast would be traveling into each material?

For 2024 aluminum, $\rho_{0A} = 2.785\times10^3\,\text{kg/m}^3$, $c_{0A} = 5.328\times10^3\,\text{m/s}$, $s_A = 1.338$.

For 304 stainless steel, $\rho_{0B} = 7.896\times10^3\,\text{kg/m}^3$, $c_{0B} = 4.596\times10^3\,\text{m/s}$, $s_B = 1.490$.

Solution:

Solving the equation system

$$\begin{cases} P = \rho_{0B}c_{0B}u + \rho_{0B}s_B u^2 \\ P = \rho_{0A}c_{0A}\left(u_{\text{impact}} - u\right) + \rho_{0A}s_A\left(u_{\text{impact}} - u\right)^2 \end{cases},$$

We have

$$\begin{cases} u = 0.56\text{km/s} \\ P = 24\text{GPa} \end{cases},$$

The shock velocity in the steel target D_B can be found by using the shock-particle velocity Hugoniot

$$D_B = c_{0B} + s_B u$$

for the 304 stainless steel, which yields

$$D_B = 5.4\text{km/s}.$$

The shock velocity running back into the aluminum is

$$D_A = c_{0A} + s_A\left(u_{\text{impact}} - u\right) \text{ (relative to the oncoming material),}$$

which gives $D_A = 7.0\text{km/s}$.