LECTURE 3. POWER SERIES AND ITERATIVE SOLUTION OF EQUATIONS
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Teaching materials are at
http://personalpages.manchester.ac.uk/staff/henry.tan/teaching/computation2/computation2.htm

I. Power series

Any sufficiently smooth function $f(x)$ can locally be approximated by polynomials.

$$f = \sum_{i=0}^{n} a_n x^n. \tag{1}$$

A simple example is the approximation of the exponential function e^x near $x = 0$:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}, \tag{2}$$

The approximation is called the n-th order Taylor approximation to e^x because it approximates the value of the exponential function by a polynomial of degree n.

1. Using Excel spreadsheet

Normal view

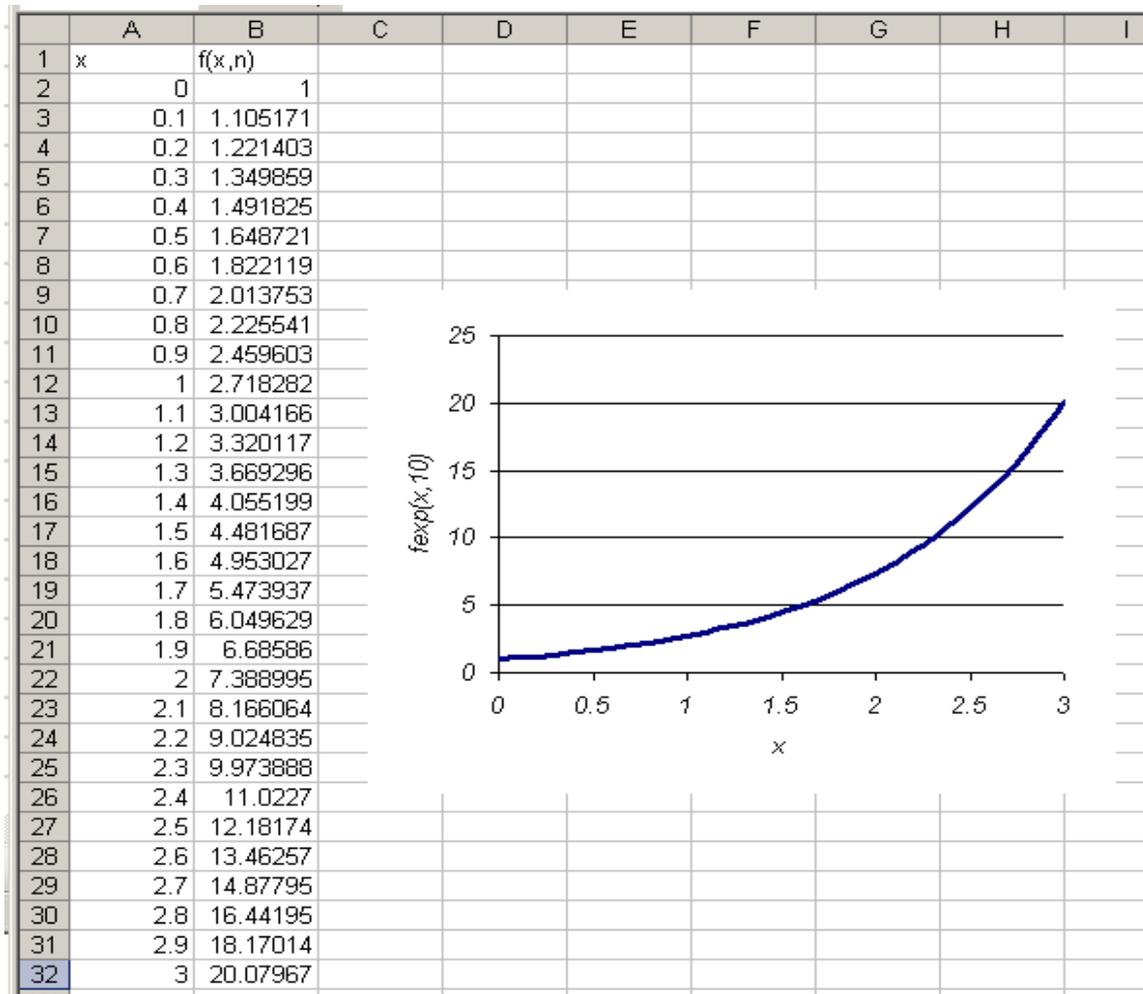Formula view

2. Using VBA

```
Function fexp(x as double, n as integer)
    term = 1
    fexp = term
    For i = 1 To n
        term = term * x / i
        fexp = fexp + term
    Next i
End Function
```

The function `fexp(x, n)` has two arguments, x and n, where x is the variable of the function, and n is the highest order of Taylor polynomials in Equation (2).

Plot the curve of function `fexp(x, 10)`

	A	B	C	D	E	F	G	H	I
1	x	f(x,n)							
2	0	1							
3	0.1	1.105171							
4	0.2	1.221403							
5	0.3	1.349859							
6	0.4	1.491825							
7	0.5	1.648721							
8	0.6	1.822119							
9	0.7	2.013753							
10	0.8	2.225541							
11	0.9	2.459603							
12	1	2.718282							
13	1.1	3.004166							
14	1.2	3.320117							
15	1.3	3.669296							
16	1.4	4.055199							
17	1.5	4.481687							
18	1.6	4.953027							
19	1.7	5.473937							
20	1.8	6.049629							
21	1.9	6.68586							
22	2	7.388995							
23	2.1	8.166064							
24	2.2	9.024835							
25	2.3	9.973888							
26	2.4	11.0227							
27	2.5	12.18174							
28	2.6	13.46257							
29	2.7	14.87795							
30	2.8	16.44195							
31	2.9	18.17014							
32	3	20.07967							

II. Iterative solution of equations

Iterative method attempts to solve an equation or system of equations by finding successive approximations to the solution starting from an initial guess.

Solve the equation

$$x^2 - x - 1 = 0.$$ (3)

Rewrite the equation as

$$x = f(x), \text{ where } f(x) = 1 + \frac{1}{x}.$$ (4)

```
Function f(x)
      f = 1 + 1 / x
End Function
```

normal view

	A
1	0.589
2	2.697793
3	1.370673
4	1.729568
5	1.578179
6	1.633642
7	1.612129
8	1.620298
9	1.617171
10	1.618364
11	1.617908
12	1.618082

formula view

	A
1	0.589
2	=f(A1)
3	=f(A2)
4	=f(A3)
5	=f(A4)
6	=f(A5)
7	=f(A6)
8	=f(A7)
9	=f(A8)
10	=f(A9)
11	=f(A10)
12	=f(A11)
13	

Try

$$x = x^2 - 1.$$

Try

$$x = \frac{1}{2}\left(x^2 - 1 + x\right).$$