ME 7953: Simulations in Materials

Fall 2002

Problem Set 2 (Friday, 9/6/2002)

Problems are due at the beginning of the class, Friday, 9/13/2002.

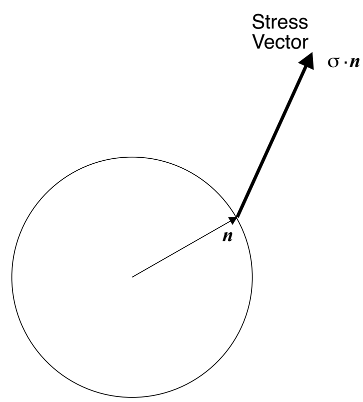
Visualization is an important tool to assist the interpretation and presentation of concept, and data resulting from numerical simulations and measurements. This is the focus of this week's problems.

1) 2D stress matrix visualization

(i) Design a method to view the image of a 2D stress matrix.
(ii) What information can we get by just look at the image?
(iii) Write an m-file to view the image of a 2D stress matrix. Apply the code to the following four 2D stress matrices:

(A) $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -2 & 3 \\ 3 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & -3 \\ -3 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & -3 \\ -3 & 2 \end{bmatrix}$.

In the class, I designed a *hedgehog method* to view the image of a 2D stress matrix.



In the method, the directional unit-vector $\boldsymbol{n}$ sweeps over all the directions along the circle. For each sweeping, put the stress vector $\boldsymbol{\sigma} \cdot \boldsymbol{n}$ on the circle point. The stress vector can point inside (under compression) or outside (under tension) of the circle. The radius of the circle is set as $\left(\sum_{i,j=1}^{2} \sigma_{ij}^2 \right)^{1/2}$.

By just a glance at the image we can get the following information:
(1) principal stresses and directions;
(2) which directions are under compression or under tension;
(3) shear stress information.

You can write code based on my image design. You are encouraged to have a different image design. If so, you can earn double points.

2) 3D stress matrix visualization

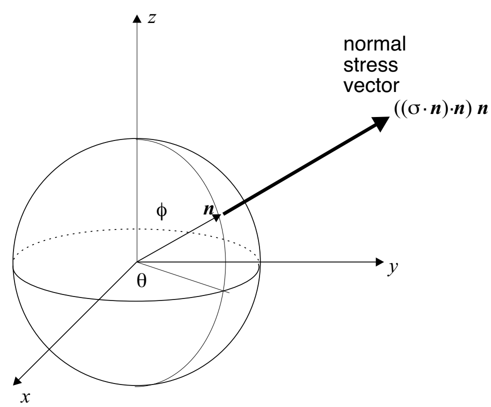
(i) Design a method to view the image of 3D stress matrix.

(ii) What information can we get by just look at the image?

(iii) Write an m-file to view the image of a 3D stress matrix. Apply the code to the following four 3D stress matrices:

$$(A)\ \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & -1 \\ 3 & -1 & 1 \end{bmatrix} \qquad (B)\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$

In the class, I designed a *bean-bag-method* to view the image of a 3D stress matrix.



In the method, the directional unit-vector $\boldsymbol{n}$ sweeps over all the directions along the sphere. For each sweeping, put the normal stress vector $((\boldsymbol{\sigma}\cdot\boldsymbol{n})\cdot\boldsymbol{n})\boldsymbol{n}$ on the sphere point. The normal stress vector can point inside (under compression) or outside (under tension) of the sphere. The radius of the sphere is set as $\left(\sum_{i,j=1}^{2}\sigma_{ij}^{2}\right)^{1/2}$.

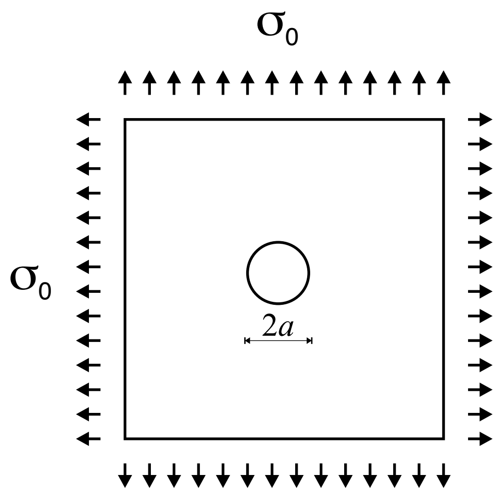
By just a glance at the image we can get the following information:

(1) principal stresses and directions;

(2) which directions are under compression or under tension.

You can write code based on my image design. You are encouraged to have a different image design. If so, you can earn double points.

3) Cymbal structure of the stress field around a hole



Lamé's solution for an elastic 2D plate, with a hole of radius a and uniform tensile stress σ_0 applied at the far field, gives the stresses distribution expressed in cylindrical polar coordinates as

$$\sigma_r = \sigma_0\left(1 - a^2/r^2\right)$$
$$\sigma_\theta = \sigma_0\left(1 + a^2/r^2\right),$$

for $a \le r < \infty$, $0 \le \theta < 2\pi$. The companion shear stress component is zero by virtue of the axisymmetry of the configuration.

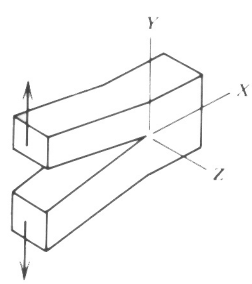
We can non-dimensionalize the equation by using

$$\sigma'_r = \frac{\sigma_r}{\sigma_0},\ \sigma'_\theta = \frac{\sigma_\theta}{\sigma_0}$$
$$r' = \frac{r}{a}$$

Write a MATLAB m-file to reveal the cymbal structure of the stress field around a hole. The spatial distribution of radial and tangential stress fields looks like a pair of cymbals.

4) Mountain structure of the crack tip field

The stress field in vicinity of the crack tip under the following opening loading is



$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right),$$
$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\tau_{zx} = \tau_{zy} = 0$$

where K is the stress intensity factor. Assume the case for a crack in a thin plate under opening load, we have

$$\sigma_{zx} \approx 0$$

From the equations, it is apparently that the local stresses of σ_y, σ_x, and τ_{xy}, at the crack tip could rise to extremely high levels as r approaches zero. This circumstance is precluded by the onset of plastic deformation at the crack tip.

In the maximum distortion energy or Von Mises' yield criterion, the equivalent stress can be described as

$$\sigma_e = \frac{\sqrt{2}}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)^2\right]^{1/2}$$

We can non-dimensionalize the equation by using

$$\sigma'_e = \frac{\sigma_e}{\sigma_{ys}}$$
$$r' = \frac{r}{(K/\sigma_{ys})^2}$$

If σ_e is greater than the yield strength σ_{ys}, the material become plastic deformation. We assume an perfect plasticity of the material, that is

$$\sigma_e = \sigma_{ys}, \text{ if } \sigma_e > \sigma_{ys}$$

Use MATLAB to describe the field distribution (in *x-y* plane) of the equivalent stress $\sigma'_e(r',\theta)$. Show the mountain structure of Von Mises's equivalent stress around the crack tip. The region of flat plasticity peak at the top of the mountain is called the crack tip plastic zone.