A SELECT CLOCK
by John S. Reid

JAMES Ferguson's "Select Mechanical Exercises" was in its day a treatise of some popularity, running through four editions between 1773 and 1823. Among thirty-six assorted articles mainly on horological topics there are several designs for astronomical clocks of ingenuity. One in particular stands out as being a splendid creation. Shown by its dial is the month, the day and solar time to within five minutes; lunar time in days and hours since full moon, with a visual illustration of the moon's state of illumination; the position in the sky of the major constellations of stars, enabling one to say where the stars are relative to horizon, zenith and the north-south line (the meridian); which stars are rising or setting at any given moment and also the times of rising, southing and setting of sun and moon. Ferguson's skill presents all this information in a compact, simple and readily understandable form.

Ferguson himself made a wooden model of his invention in 1747 which he subsequently used, presumably rather like an orrery, to illustrate his astronomical lectures. In spite of the wide dissemination of this design it seems that no one took up its commercial manufacture. It was therefore a great pleasure to find that Aberdeen University possessed the parts of such a clock which had been made there in the late eighteenth century. Ferguson was a self-educated farmer's son from Aberdeenshire who left the region as a young man and later successfully established his reputation in London as a lecturer on Mechanics.

The clock illustrated in Figs. 1 and 2 was constructed in the late 1780's for a similar educational purpose that motivated Ferguson in the first place. Although unsigned it was apparently made by Professor Patrick Copland of Marischal College and his assistant John King. Copland himself was by all accounts a mechanic of substantial abilities. John King, who was employed by Copland for much of the 1780's as a maker of instruments and demonstration apparatus, served a prior apprenticeship with an Aberdeen watchmaker. Copland comments² that his "workmanship I have
fit, 2. The dial seen through the inscribed lines on the cover glass, parallax from the camera position destroys the exact registration of the horizon curve over the star positions.

As well as making serious astronomical observations from this observatory (the equipment included a Master Clock by Marriott of London and a secondary by Gartly of Aberdeen) Copland used it for educational purposes. The majority of the students at Marischal College spent most of their third year studying Natural Philosophy with Copland, whose course included some astronomy. Copland was an enthusiast for practical demonstrations and no doubt the clock was shown on every appropriate occasion. Curiously enough one of Copland's first students was Ferguson's youngest son John, who

found equal, both in wood and metals, to that of the best London artists." There seems no doubt that they had between them the abilities to undertake such a time consuming enterprise as the construction of Ferguson's clock.

Professor Stuart writing in 1798² recalls that the clock was kept in the Castlehill observatory which was demolished in 1795 and its contents removed to the College buildings. The observatory was also equipped and run by Patrick Copland who raised money for its construction and instrumentation from "the Gentlemen in Aberdeen and the neighbouring country".

Fig. 2. The dial seen through the inscribed lines on the cover glass. Parallax from the camera position destroys the exact registration of the horizon curve over the star positions.
attended Marischal College between 1773 and 1777, before the observatory was built. John Ferguson may have put the idea into Copland's mind, directly or indirectly, of making the clock but on this point we may never be sure.

A poorly heated observatory, at times with its interior exposed to the elements when the cupulas are thrown open, is not a good environment for a clock. It is quite possible that by the demise of the replacement College observatory in 1837 upon the demolition of the whole of the old College buildings, the clock had already fallen into a poor state. Certainly we read nothing more of it as a working exhibit and in all likelihood it has not functioned for well over a century. It was therefore something of a surprise to find that almost all the parts had survived.

When cleaned the wheelwork was found to be in excellent condition with the pallets of the regulating dead-beat escapement not much worn. Ferguson describes the gearing in both "Select Mechanical Exercises" and in his "Tables and Tracts", the latter account changing the pinion pivoting the astronomical motions from one of 8 leaves to one of 16. This improvement is incorporated in the Aberdeen clock. The lunar and sidereal trains are the relatively simple ones used by Ferguson in his orrery. As a result the lunation is 57 seconds per month slow on the synodic month of 29 days 12 hours 44 minutes 3 seconds. Since the astronomical wheelwork drives motions relative to the solar day, it is entirely floating on a plate which rotates once every twenty-four hours. A heavy lenticular pendulum bob weighing approximately 8 kg regulates the driving clockwork which runs for over three weeks between windings.

The dial requires some explanation, partly because Fig. 2 does not completely reveal the depth involved. Examining Fig. 3, which is Ferguson's own drawing from "Select Mechanical Exercises", some more detail can be seen and a comparison made with the Aberdeen clock. Within the fixed outer 24 hour ring there are two rotating dials. The first is a narrow ring with a fleur-de-lis mark and the lunar days circumscribed, which rotates once every solar day. The second is the star plate sunk some 12 mm below and circumscribed with the days, months and constellation positions, which rotates once every sidereal day. The stars are mapped onto this plate with the celestial north pole at the centre, the stellar right ascension converted to an angle round the plate and the colatitude mapped into a proportional radial distance from the centre. With this mapping the celestial equator becomes a circle (152 mm in diameter on the Aberdeen clock) and both ecliptic and horizon are distorted circles. On Ferguson's copper-plate engraving the ecliptic is incorrectly drawn, the crossings of ecliptic and equator not occurring exactly half a year apart, as they should. Copland and King have rectified this error. Ferguson describes the horizon curve, which is actually inscribed on the door glass, as "an ellipsis", a word not now found in geometry textbooks. However it is not difficult to calculate the true form of this curve and one finds that it is slightly more circular than an ellipse for the latitude of Aberdeen, and the same form of curve (but one even more nearly circular) should be used for the ecliptic. On Ferguson's engraving stars are only shown up to declination 50° North. Copland and King engraved stars up to declination 70° North, thereby including the commonly recognised constellations of Ursa Major, Cassiopeia and Draco, as well as engraving other stars of lesser magnitude than those shown by Ferguson. These northerly constellations are the ones which in practice one might use at night to tell the time with the aid of a nocturnal or merely by eye.

The symbols for earth, moon and sun are an integral part of the display. The earth is a fixed disc at the centre, not mapped as the stars but engraved with lines of longitude and latitude shown in polar orthogonal projection. With the longitude of the clock's position (Aberdeen) set vertically, then on knowing the longitude of any place on earth, the position of the marked stars, and the sun and moon, relative to the meridian at that place can be observed on the dial. The sun wire is attached to the rotating fleur-de-lis ring and extends to the tropic of cancer. The intersection of this wire and the ecliptic shows the position of the sun in the sky. The sun symbol does not slide along the wire to cover the intersection point but merely sits at the equatorial distance. The moon wire rotates from a central mechanism hidden by the earth plate at a slower rate than the star plate, thereby showing lunar time anticlockwise. The simultaneous rotation of the wire
about its axis by a bevel gear shows how the moon is illuminated by the sun. When the moon is diametrically opposite the sun it shows completely white (full moon); when on top of the sun, it shows completely black, representing the moon between the sun and earth.

Finally, the "ellipsis" representing the horizon is drawn on the cover glass along with the meridian line. Fig. 4 shows its relationship with the equatorial circle and illustrates its use. All stars which can be seen through the horizon ellipses are potentially visible if it is night time.
Fig. 4. The solid lines represent the horizon 'ellipses' and meridian line which Ferguson suggested should be inscribed on the dial cover glass. The position of these lines can be seen relative to the celestial equatorial circle that is engraved on the star plate. Also shown on this figure, though not marked on the clock, is the zenith and the mapping of the four quadrants of the sky. In spite of the quadrants' distorted shape the constellations themselves, which are fairly local star groups, appear on the plate much as they do in the sky.
The stars close to the left hand segment are rising, the stars approaching the right hand segment are setting and the stars crossing the meridian line are at their highest point in the sky. The transit of a star across the meridian line was in practice the astronomical observation used in Ferguson's day to establish true local time. As is well-known, the difference in true local time between different places on the earth was the key used in the 18th century to solve the problem of accurate terrestrial longitude determination. It seems odd nowadays, but was completely logical, that it was by observation of the star plate and not the solar time, that one regulated the length of the pendulum in order to set up the clock as an accurate timekeeper.

This clock might be described by some as a horological curiosity, particularly as the Aberdeen version may be the only working one in existence. However there is no doubt that it illustrates very succinctly the apparent motions of sun, moon and stars around the earth and makes a true functional use of the analogue nature of a clock dial in a way which cannot be replaced by a digital display. It is also a fine example of workmanship by craftsmen who were not clockmakers by trade.

The author would particularly like to thank the contemporary craftsmen at Aberdeen University who, following the tradition in which the clock was made, restored it with great care and patience to its original glory.

REFERENCES

1. "Select Mechanical Exercises: Showing how to construct different clocks, orreries and sundials, on plain and easy principles". By James Ferguson, F.R.S., London.


4. "Tables and Tracts, Relative to several Arts and Sciences". By James Ferguson, F.R.S., London 1767.
In general. Great circles tilted at an angle \( \lambda \) (degrees) to the equator intersect the equator at two points F and H. Any point P on the great circle is at a maximum angular distance \( \lambda \) away from the equator at points E and G. Figure 1 shows that a curve representing the great circle on the star plate must pass through EFGH. Measured from the centre of the plate N, a point P on the curve can be given co-ordinates (r, \( \theta \)), as shown in Fig. 1b. Let the circle representing the equator have radius a.

The approximate ellipse. This is the ellipse which passes through EFGH. It is centred on the point half way along EGH and has eccentricity \( \lambda /90 \).

Fig. 1 (a) The celestial sphere showing the poles (N and S), the equator and a great circle inclined at an angle \( \lambda \) to the equator; (b) The mapping of the points in (a) onto the star plate. The equatorial circle has radius a. A point P on the great circle curve has co-ordinates (r, \( \theta \)) as shown, with a relationship between r and \( \theta \) discussed in the text.

It is not often that an error survives in the professional literature for over 200 years. In a recent article in the Journal about a James Ferguson clock I described his mapping of the heavens onto a dial plate where the North Pole is at the centre and lines of constant declination from the pole are represented by concentric circles, with equispaced circles for equal steps of declination. Subsequent correspondence has mentioned that my remark that the ecliptic, the horizon line and other great circles are not ellipses is contrary to established opinion in horological works (e.g. Geared to the Stars), including Ferguson's own account. The remark was, however, true and arises from the fact that Ferguson's star plate is a mapping of the celestial sphere and not a projection. To expand on this point for those readers interested in astronomical clocks I offer an outline derivation of the equation of the true curves with a diagram showing their form and a graph of the error in the established elliptical approximation. The results are also relevant to modern star charts that are drawn in the same way.
Fig. 2. Showing the true shape of curves on the map of the celestial sphere corresponding to great circles inclined to the equator at the angles ($\lambda$) shown. The ecliptic has $\lambda = 23\frac{1}{2}^\circ$, the horizon plane at a latitude of $52^\circ$ has $\lambda = 38^\circ$ and the plane containing the directions East, West and the zenith at a latitude of $52^\circ$ has $\lambda = 52^\circ$. A curve must be correctly oriented in azimuth ($\theta$) around the central (north) pole for a particular application.
Fig. 3. The percentage error in the radius $r$ upon using the established elliptical approximation for different inclinations ($\lambda$). The error is noticeably asymmetric as a function of azimuth $\theta$, being much larger towards the south. For even larger inclinations than shown, the error becomes extremely large south of the equator but actually becomes smaller for most of the region north of the equator.
The true curve. A trigonometric argument giving the true curve can be constructed by considering the great circle as the intersection of a sphere centred on the origin and a plane passing through the origin. Mapping the resulting curve according to the rules of the star plate gives the equation of the true curve. If the great circle is chosen to be inclined at angle $\lambda$ to the equator and to cut the equator along the $y$ axis, then the true curve becomes

$$r = a \tan^{-1}\left(\frac{\cot \lambda}{\sec \theta} \right) / 90$$

In this form, azimuths are measured from the $x$ axis. All other great circles at the same inclination $\lambda$ are found simply by changing the origin of $\theta$.

An alternative expression. Using the geometry of spherical triangles one can find the declination of the point $P$ from the north pole as $P$ moves around the inclined great circle. The resulting expression looks different from (1) but is in fact equivalent.

$$r = a \left(1 + \sin^{-1}\left(\frac{\cos \theta \sin \lambda}{(1 - \sin \lambda \sin \theta)}\right)\right) / 90$$

The consequences. Figure 2 shows a set of true curves for inclinations $\lambda$ ranging from $9^\circ$ to $89^\circ$ in steps of $10^\circ$. They begin at small $\lambda$ almost circular, change to almost elliptical and finally, for $\lambda$ greater than about $45^\circ$, become noticeable asymmetric. Figure 3 illustrates the percentage error in the radial distance $r$ if the ellipse approximation is used. Both curves were drawn by computer controlled graph plotter, a device which Ferguson undoubtedly would have appreciated in spite of his considerable drawing skills. In conclusion it can be seen that as far as inscribing a clock face is concerned the elliptical approximation is a quite satisfactory practical solution for moderate inclinations of a great circle (such as the ecliptic), even though it is not exact.

I would like to thank John Millburn of Aylesbury for his relevant remarks and for his suggestion that the topic may be of wider interest than I had realised.

REFERENCES