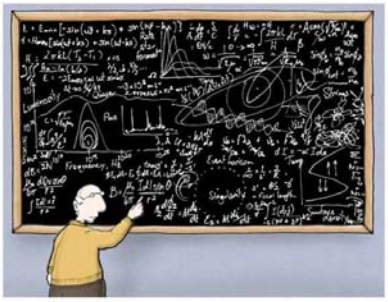
Special Relativity



Relativity made simple

Dr. Jan Skakle
University of Aberdeen

Einstein - 1905

Published '5' papers in *Annalen der Physik*

- Photoelectric effect (led to Nobel Prize in 1921)
- Brownian Motion (proved existence of atoms)
- Special Relativity
- Special Relativity 2 ($E=mc^2$)
- Molecular dimensions (PhD)

3 were considered to be "among the greatest in the history of physics"

But the greatest was yet to come...

Special?

The concept of relativity had been well know since the time of Galileo

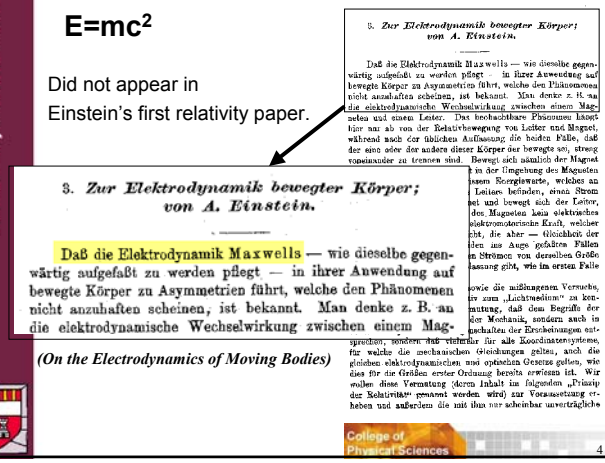
It was used by Newton, and ***Poincaré*** developed this idea.

Jules Poincaré
1854-1912

Einstein said that he thought of the idea whilst riding his bicycle

$E=mc^2$

Did not appear in Einstein's first relativity paper.



3. *Zur Elektrodynamik bewegter Körper;*
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Mag-

(On the Electrodynamics of Moving Bodies)

Postulates of Special Relativity

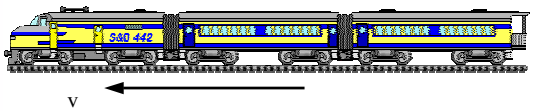
Einstein built the Special Theory of Relativity on two postulates:

- The Relativity Principle: The laws of motion are the same in every inertial frame of reference.
- Constancy of the Speed of Light: The speed of light in a vacuum is the same independent of the speed of the source or the observer.

Einstein in 1912

Inertial Reference Frames

An inertial Frame of Reference is one in which the basic laws of physics apply – e.g. a train moving at a *constant velocity*. In this, objects move "normally".



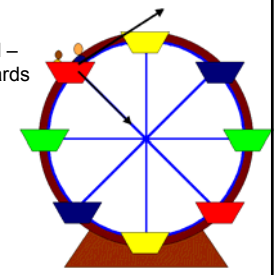
This relates to objects obeying Newton's first law.

Non-Inertial Frames

Compare this with an accelerating or decelerating object. During this period, you are in a *non-inertial frame.*

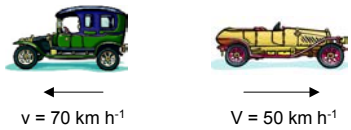
Same if you are in a Ferris wheel – you are always accelerating inwards so *non-inertial.*

Strictly speaking the Earth is a non-inertial frame, but it's close enough so we consider it *inertial.*



Can be intuitive...

Moving apart; what is the green car's velocity relative to your frame?



We know the answer intuitively (120 km h^{-1}) but this also works in terms of Lorentz transformations (of course!)

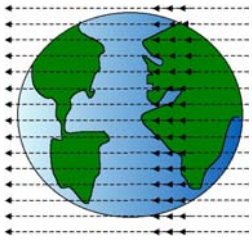
Same idea, but with light:

From the examples above, we would expect the relative velocity to be 2c = 6 x 10^8 ms^{-1}.

This is in fact wrong!

The luminiferous ether

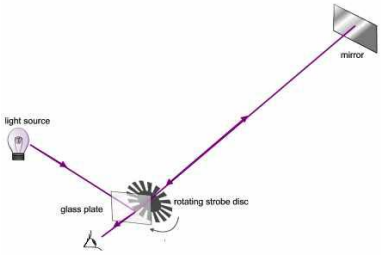
The ether, or æther, was the basis of understanding
After all, if the speed of light is c, what is that *relative to?*



The ether
- was tranparent
- had zero density
- was everywhere
- was the substance which allowed light to propagate.

Speed of light - Fizeau

First successful "earthbound" measurement



A. H. L. Fizeau (1819-1896)

Leon Foucalt (1819-1868)

Obtained value of 3.1 x 10^8 ms^{-1}
Refined to 2.98 x 10^8 ms^{-1} by Foucalt

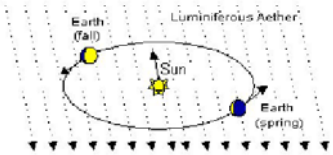
Maxwell

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

Had calculated that the speed of light was 3 x 10^8 ms^{-1}

This was assumed to be with respect to the ether

Maxwell in 1878 suggested how to make the measurement, but said that no method would be able to detect the subtle difference involved….



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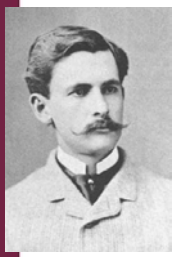
Enter Michelson...



Albert Michelson
1852-1931

- Born 1852 on German/Polish border
- Moved to US in 1854 - California
- Gained free place at US Naval Academy, learned Maths & Physics amongst other things
- Professor in Cleveland, Ohio
- Years of experiments to find speed of the ether

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Apparatus

"Might slightly more sophisticated apparatus help, Dr. Michelson?"

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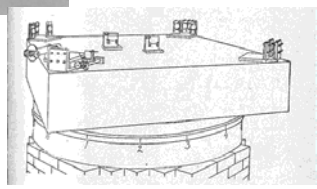
The interferometer



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Michelson-Morley Experiment

- Instrumentation by Michelson
- 1882 - 1887…including nervous breakdown
- Found speed of light (in a vacuum) is always the same

"Most famous negative result in the history of Physics" (William H. Cropper)

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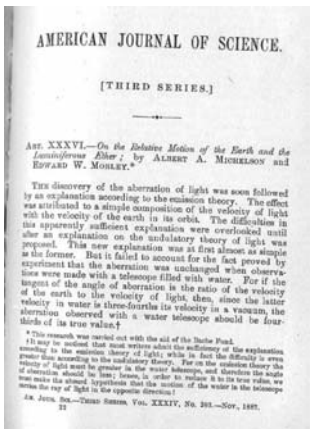

AMERICAN JOURNAL OF SCIENCE.

[THIRD SERIES.]

ART. XXXVI.—*On the Relative Motion of the Earth and the Luminiferous Ether*; by ALBERT A. MICHELSON and EDWARD W. MORLEY.*

THE discovery of the aberration of light was soon followed by an explanation according to the emission theory. The effect was attributed to a simple composition of the velocity of light with the velocity of the earth in its orbit. The difficulties in this apparently sufficient explanation were overlooked until after an explanation on the undulatory theory of light was proposed. This new explanation was at first almost as simple as the former. But it failed to account for the fact proved by experiment that the aberration was unchanged when observations were made with a telescope filled with water. For if the tangent of the angle of aberration is the ratio of the velocity of the earth to the velocity of light, then, since the latter velocity in water is three-fourths its velocity in a vacuum, the aberration observed with a water telescope should be four-thirds of its true value.†

* This research was carried out with the aid of the Bache Fund.

† It may be noticed that most writers admit the sufficiency of the explanation according to the emission theory of light; while in fact the difficulty is even greater than according to the undulatory theory. For on the emission theory the velocity of light must be greater in the water telescope, and therefore the angle of aberration should be less; hence, in order to reduce it to its true value, we must make the absurd hypothesis that the motion of the water in the telescope carries the ray of light in the opposite direction.]

AM. JOURN. SCI.—THIRD SERIES, VOL. XXXIV, NO. 203.—NOV., 1887.
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Back to Einstein...

Some questions...

- Can you catch up with light?
- What would happen if I rode a light beam?
- If you were travelling at the speed of light and looked in a mirror - would you see your reflection?

"Questions keep me awake at night. If you were in your vehicle travelling at the speed of light and you turn your headlights on, what would they do?"

"OK, what's the speed of dark?"

Stephen Wright

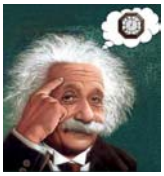
Second postulate

The speed of light in a vacuum is absolute

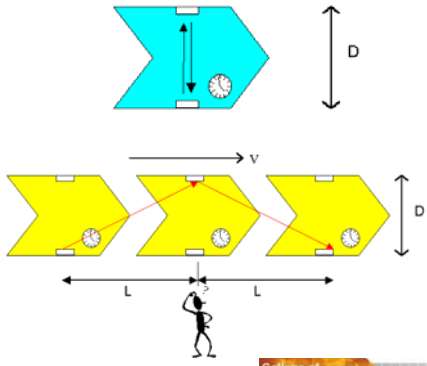
$c = 3 \times 10^8\ ms^{-1}$ no matter the speed of the observer.

Consistent with Michelson-Morley

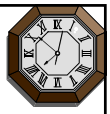
Reconciles Maxwell's equations with mechanics.



Gedanken



What time dilation means...



$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Usually written $\Delta t = \gamma\, \Delta t_0$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

Δt_0 is called the *proper time interval*

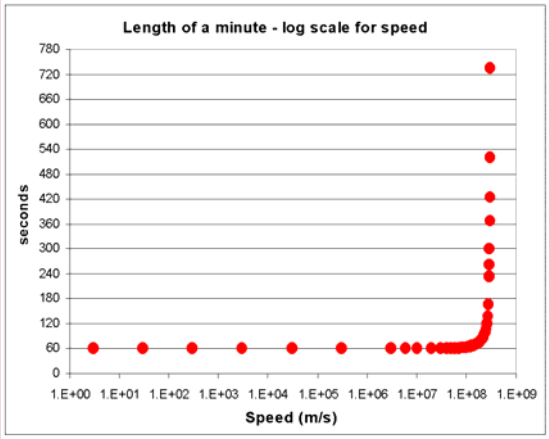
"Moving clocks *are measured* to run slowly".

Valid when proper time refers to 2 events occuring at same point…

"Time is an illusion. Lunchtime doubly so."

Hitch-hikers Guide to the Galaxy

Douglas Adams
1952-2001



Evidence?

Atomic clocks

- flown on commercial jets for about 45 hours then compared with ground-based clocks.

Elementary particles

- measurement of *muons* at the earth's surface.
- should have decayed in the atmosphere

Example….

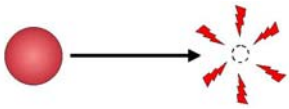
Example

A *muon* at rest has lifetime 2.2×10^{-6}s.
In a particle accelerator, it moves at 0.99c. Find:
(a) how long it lasts at this speed and
(b) how far it travels before it decays.

(a) $\Delta t = \Delta t_o / \sqrt{[1 - (v^2/c^2)]}$

$= (2.2 \times 10^{-6}) / \sqrt{[1 - 0.99^2]} = 16 \times 10^{-6}$ s

(b) Distance travelled $= v\, \Delta t = 4.6 \times 10^3$ m



The Twin Paradox

One twin travels to Alpha Centauri (4.3 light years away) at a speed of 0.96c. The other stays on Earth. Compare the time intervals.

Light takes 4.3 years to get to Alpha Centauri.

At 0.96c it takes 4.5 years (4.3/0.96).

There and back thus take 9 year - this is Δt_o.

The Twin Paradox

As before $\Delta t = \Delta t_o / \sqrt{[1 - (v^2/c^2)]} = 9/\sqrt{[1 - (0.96)^2]}$

$\Delta t = 9/\sqrt{[0.08]} = 9/0.28 = 32$ years

So one twin ages by 9 years,
the other by 32 years!

The Paradox

The above seems fine EXCEPT…the special theory states that no reference frame is better than another. Who is moving?

If we consider the spaceship to be stationary – Earth moves by 0.96c in opposite direction

This time it is the opposite twin who ages more!

?

If the spaceship comes back, it will have had to decelerate & accelerate and this makes it *non-inertial* – general relativity….

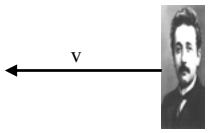
Length contraction

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

So this time the proper length is contracted as the speed increases.

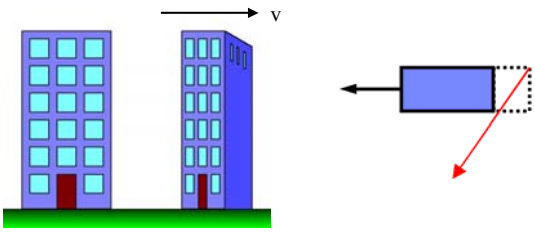
Contraction in direction of motion

Einstein at rest



Einstein at v = 2.6 x 10^6 ms^{-1}

Distortion



If we move past building at v to the right, then consider ourselves at rest – the building moves to the left at v

The building "gets out of the way" of the light from the side.

Mass

- Mass is also relative….

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

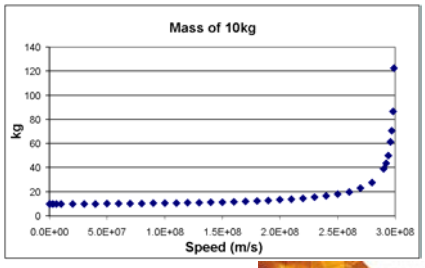
This has been observed for elementary particles

However we should beware of visualising anything getting bigger.

Mass increase

So, as speed increases, the mass increases, until at the speed of light….?

We cannot accelerate anything up to the speed of light.

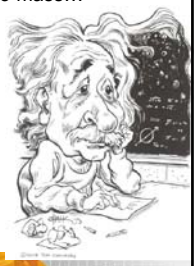


Consequences

If we try to accelerate something close to c then the mass increases but we can't speed it up any more.

Energy (the work we are doing) converts to mass…

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$



For a particle at rest, v=0 so

E_o (rest energy) = mc^2

E=mc² ??

13. *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*
von A. Einstein.

Die Resultate einer jüngst in diesen Annalen von mir publizierten elektrodynamischen Untersuchung[1]) führen zu einer sehr interessanten Folgerung, die hier abgeleitet werden soll.

Ich legte dort die Maxwell-Hertzschen Gleichungen für den leeren Raum nebst dem Maxwellschen Ausdruck für die elektromagnetische Energie des Raumes zugrunde und außerdem das Prinzip:

Die Gesetze, nach denen sich d
kalischen Systeme ändern, sind unabhä
von zwei relativ zueinander in gleich
lationsbewegung befindlichen Koordin
standsänderungen bezogen werden (Rel

Gestützt auf diese Grundlagen[2]) l
das nachfolgende Resultat ab (l. c. §

Ein System von ebenen Lichtwell
ordinatensystem (x, y, z) bezogen, die
richtung (Wellennormale) bilde den W
des Systems. Führt man ein neues,
in gleichförmiger Paralleltranslation
system (ξ, η, ζ) ein, dessen Ursprung si
keit v längs der x-Achse bewegt, so besitzt die genannte Lichtmenge — im System (ξ, η, ζ) gemessen — die Energie:

$$l^* = l\frac{1-\frac{v}{V}\cos\varphi}{\sqrt{1-\left(\frac{v}{V}\right)^2}},$$

wobei V die Lichtgeschwindigkeit bedeutet. Von diesem Resultat machen wir im folgenden Gebrauch.

1) A. Einstein, Ann. d. Phys. 17. p. 891. 1905.
2) Das dort benutzte Prinzip der Konstanz der Lichtgeschwindigkeit ist natürlich in den Maxwellschen Gleichungen enthalten.

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Unter Vernachlässigung von Größen vierter und höherer Ordnung können wir setzen:

$$K_0 - K_1 = \frac{L}{V^2}\frac{v^2}{2}.$$

Aus dieser Gleichung folgt unmittelbar:

Gibt ein Körper die Energie L in Form von Strahlung ab, so verkleinert sich seine Masse um L/V^2. Hierbei ist es offenbar unwesentlich, daß die dem Körper entzogene Energie gerade in Energie der Strahlung übergeht, so daß wir zu der allgemeineren Folgerung geführt werden:

$$M = L / V^2$$

Examples

- Atomic bomb
- Particle physics – decay of particles
- The sun

The mass of the sun is 1.99 x 10^{30} kg. If it were all converted to energy this would give

1.99 x 10^{30} x (3 x 10^8)^2 = 1.79 x 10^{49} J.

We can also do some other calculations…

The sun

The sun radiates energy at 3.92 x 10^{26} W
1 W = 1 J/s so sun loses 3.92 x 10^{26} J every second

This means it will last 1.79 x 10^{47} / 3.92 x 10^{26}

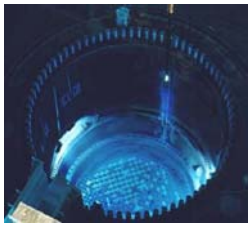
= 4.57 x 10^{20} seconds = 1.5 x 10^{13} years!
(15 million million years)

Also if it loses 3.92 x 10^{26} J every second:

m = E/c^2 = 3.92 x 10^{26} / (3 x 10^8)^2
= 4.4 x 10^9 kg every second!!

Čerenkov radiation

- Sonic boom happens when you pass through the sound barrier (330 ms^{-1})
- In dense media, light can be overtaken too
- Best described as a "sonic boom" for light



Pavel Čerenkov
1904-1990

Slowing down light

- 1999 - Lene Hau (Harvard)
- Tiny cloud of ultracold sodium atoms (0.9 μK)
- Very low pressure
- Light slowed to 28 ms^{-1}
- Stopped for 0.001 sec

Divisions

1) Things that travel below the speed of light (*tardions*). These are "normal" things known to us with mass.

2) Things that travel *at* the speed of light. Particles with zero mass, e.g. photons.

3) Things that travel above the speed of light (*tachyons*) *Imaginary mass*