General Relativity and the Universe at large

On a big scale, on the scale of the Universe, gravity, space and time obey the rules of General Relativity, as far as we can tell. Einstein introduced the theory over the few years prior to 1916 as a description of the interaction of matter via gravity. His theory was based on the precepts of special relativity, on the cosmological principle, which says that on a large scale the universe is isotropic and homogeneous, and on the principle of equivalence, which I'll introduce shortly. Einstein himself showed that the theory predicted effects that Newton's theory of gravity didn't. It is therefore a theory that is testably different from Newtonian gravitation and, pretty remarkably, so far it has stood up to every test thrown at it.

What is General Relativity all about?

It is a theory where matter messes with space and time, to put it crudely. In our 'normal science' matter exists **in** space and time but neither time nor space are affected by matter. Objects move from one place to another against a background of a frame of reference that exists whether the objects are present or not. Objects move **through** a pre-existing space, and **in** a pre-existing time. The distance between two points, for example, does not depend on whether we place any matter at either of the points, or indeed, in between them. It seems almost absurd to suggest it might. What kind of place would that make the world? Einstein's view is not simply that matter messes with space and time but that matter **defines** the measurement of space and time. I'll borrow here a quotation of Charles Lamb used by Stephen Hawking. Lamb wrote '*Nothing puzzles me like time and space. And yet nothing troubles me less than time and space, because I never think about them*'. If you want to understand the Universe, you need to think about time and space.

Einstein's definitive 1916 paper

Einstein didn't set out to change the fabric of space and time just for the sake of doing so. The story had begun over 10 years prior to 1916. Einstein turned his attention to what happens when you want to transform coordinates into a frame of reference that is accelerating with respect to you.

The result of Einstein's work was his theory of General Relativity with a set of equations relating space-time to the matter within it. *They are the most valuable discovery of my life* Einstein said to fellow theoretical physicist Arnold Sommefeld in 1915, and most people agree. Einstein became more convinced that he had discovered, or perhaps you'd prefer to say 'invented' a fundamental truth about nature. He later commented *Nature rarely surrenders one of her magnificent secrets* and he felt privileged to have found such a secret in the equations of General Relativity. Let's look first at accelerations.

Accelerations

Accelerations occur in a vehicle when you put your foot on the accelerator or the brake. They occur when a lift starts or stops. They also occur when you let something fall. In this last case the cause is gravity and Einstein could see no difference between the accelerations caused mechanically and those caused by gravity. He based his new theory on what is now called **the strong principle of equivalence**.

Principle of Equivalence

This states: *To an observer in free fall in a gravitational field the results of all local experiments are completely independent of the magnitude of the field.* General Relativity is based on a physical principle rather than any obscure mathematical postulates. One way of reading this is that it is a re-definition of the concept of an *inertial frame*. The principle applies not just to mechanical experiments but to those involving light. The net result is that General Relativity is a theory that intimately links together the concepts of space and time, the gravitational effects of matter and, as we'll see, the propagation of light.

Einstein's thought experiments – example 1

Einstein devised a number of 'thought experiments' carried out in lifts, or elevators as they are referred to in the US. The 'lift' is supposed to represent a small space where the observer can carry out experiments. Rockets, space-stations and the like weren't part of the currency of ideas in the early 20th century and so Einstein imagined a laboratory in a lift that could be accelerated or could drop freely.

Our first imaginary experiment is to project a ball in a lift that is accelerating upwards. The ball arcs forward in a parabolic curve, as can be shown from basic mechanics if air resistance is ignored. Now imagine the projection of the ball taking place in a stationary lift sitting in a gravitational field, such as on or near the surface of the Earth. The result will be exactly the same. Einstein concluded that gravity and accelerations in space are just the same thing, and in fact can't be distinguished.

The result isn't obvious, because the effect of gravity is controlled by the 'gravitational mass' found in the equation $F = mg$ while the inertial mass relevant to an accelerating body is the mass relevant to the use of Newton's law $F = ma$. What Einstein was saying is that gravitational mass and inertial mass are identical. They could have been quite different quantities but of course by using the same symbol there is an implication that they are the same. Einstein drew some deep conclusions from the fact that these masses were the same, as we'll see. Einstein wasn't working here with new results. Anyone during the past two centuries could have made the same deduction. No-one did.

Various experiments have been done to try to detect the difference between these two kinds of masses, starting with the famous balance experiment of the Hungarian Loránd (Roland) Eötvös who began his experiments in 1890. You can look this up on the web. The experiment has been repeated on several occasions and, like the Michelson Morley experiment, tries to measure a difference that relativity says will be zero. Experimentally, any difference in the two different kinds of mass is less than 1 part in a million million, which is zero in my money.

Gravity curves space and time: Example 2

One consequence of the General Relativistic view is that matter, through its gravitational influence, is seen as curving space, and indeed space-time. Imagine two small masses given a small initial velocity in the same direction in empty space. They will travel along two parallel straight paths. Now imagine the same experiment made just above the Earth. The diagram on the slide shows that the masses will freely fall along converging paths toward the centre of the Earth. The principle of equivalence, which sounded innocent enough, says that the freely falling masses will behave independently of any gravitational field that may be present. The parallel lines of travel in the absence of the Earth are now converging towards the centre of the Earth. The General Relativistic view is that the gravitational field of the Earth has locally curved space (and time) so that the previously parallel lines converge. This is really what General Relativity is about. General Relativity eliminates gravitational forces by saying that the gravitational effect of masses is to alter the geometry of space and time. By describing the geometry of curved space and time with suitable curvature induced by the presence of masses, you have effectively described the influence of gravity.

Well, you may say, I'll take your word that this produces the same result as the usual view that gravity provides a force on the masses and that this force attracts them to the centre of the Earth. Surely Einstein's approach is a bizarre way of stating what's happening. You may not want to believe it but Einstein's way of looking at gravity is actually a better way. It produces results that are in fact **not** quite the same as the usual view and, when properly formulated, the laws satisfy the cosmological principle. This is a statement that the laws of physics should turn out to be independent of the frame of reference used to determine them. Newton's law of gravity in fact doesn't satisfy this principle. Newton's law of gravity has gravity propagating instantaneously, which is quite inconsistent with Special Relativity. General Relativity is quite consistent with Special Relativity and gravity propagates at the speed of light.

Another casualty of General Relativity is the strict application of the inverse square law of gravity. This only comes out of Einstein's viewpoint as a good approximation in 'usual circumstances'. Nature, in fine detail, is more subtle. The difference is seen in two ways. First, orbits in General Relativity don't strictly repeat, as an ellipse does, but turn around in space, or precess to use the proper word. Secondly, orbits are accompanied by a loss of energy due to the radiation of gravitational waves. This loss is usually very small but in some circumstances is substantial.

There was a young man who observed
I confess I am somewhat unnerved
I had never before
Seen the truth of the lore
That, where matter is, space must be curved

(Anon)

The rubber sheet model

I hesitate to show this illustration but pictures like this well-known one try to show how distances between points change in the region of a mass. The illustration shows how curvature looks when seen from outside. In General Relativity, curvature is a property of space-time and is related to how unit distance changes in different regions. You don't have to get 'outside' space-time to measure curvature. In reality, large distortions like the one idealised here would only because visible very close to highly concentrated masses. I'll talk about black-holes soon.

What the picture does hint at is that if you imagine a ball rolling around the depression in the sheet, it can orbit quite happily not because of any gravity concentrated at the centre of the depression but because of the curvature of the sheet. Curved space does the job of gravity. That is the basic message of General Relativity.

Gravity affects light: example 3

Let me take a third example. Imagine yourself in a lift that is rapidly accelerating. In reality that would be a pretty dire situation so it's fortunately only a 'thought experiment' and not one to try at home. Now arrange that a light beam is shone in through a hole in the side of the lift. Because the lift is accelerating, the path of the light beam, if you could follow it showing in some haze, would appear bent. By the principle of equivalence, namely that gravitational fields are equivalent to accelerated frames of reference, the gravitational field must do the same thing. In other words, the light beam behaves as if it were a projectile with some mass following a curved path in space. Einstein showed that the mass *m* was the same as the mass you need to associate with light in the theory of Special Relativity, namely for each photon E/c^2, where E is the energy of a photon of light. So out of General Relativity comes the concept that gravity acts on light, too, and can bend light paths. This was one of the new views from General Relativity that Newton's law of gravity said nothing about.

Gravity red-shifts light

A related thought experiment compares the light emitted from a source on the surface of a mass *M* to the light emitted from a source that is accelerating by an amount *g*, the strength of the gravitational field at the surface of the mass *M*. Again, the light photons behave as if they have a mass E/c^2 (= hf/c^2, *h* is Planck's constant and *f* the light frequency) and lose energy as they 'escape' from the gravitational field of *M*. The result is a gravitational red-shift predicted by General Relativity. Applied to light from a massive star, this red-shift is over and above any Hubble red-shift. This red-shift increases the more massive the star and the smaller is its radius. If the mass to radius ratio is big enough, the object becomes a black-hole with infinite red-shift and no light can escape from it. Thinking through the matter even further, you can see that light coming towards our Earth-based telescopes will be blue-shifted a tiny amount as it slides down the gravitational field we're sitting in on Earth. The ratio of M/r for a star is greater, usually much greater, than the ratio of M/r for the Earth so in fact the stellar red-shift will win.

Pound & Rebka experiment

A convincing experimental demonstration of the gravitational red-shift was made by Pound and Rbeka at Harvard in 1960. They used the newly discovered Mössbauer effect (for which Rudolf Mössbauer won the Nobel Prize for Physics in 1961) that certain radioactive elements emit and absorb gamma rays at extremely precise energies. Placing a detector at the top of a 22 m high tower at Harvard, they found that it would not absorb these precisely defined gamma rays as it would when placed next to the source kept at the bottom of the tower. The reason was that the very small red-shift of the gamma rays between the foot of the tower and the top changed their wavelength enough to spoil the absorption. To measure the effect they moved the source slowly upward at a speed where the Doppler shift should just compensate for the gravitational red-shift, according to Einstein's equation. This effectively measured the gravitational red-shift and found it in agreement with Einstein's prediction to within the accuracy of about 1%. In the picture, Prof. Robert V Pound is on the right and Glen A Rebka on the left, with the Mössbauer source.

Black holes

Black holes were a predicted outcome of General Relativity, though people pointed out afterwards that a similar idea was suggested by Newtonian gravity. The black-hole idea came via Karl Schwarzschild (1873 – 1916), a brilliant astronomer and mathematician who in 1916 produced the first solution to Einstein's field equations of general relativity, showing how much space is curved around a point mass. To solve the equations under any conditions is impressive but to do so in comparatively short order while you are in the German army in Russia was even more so. The unfortunate Schwarzschild contracted a fatal disease in Russia and was sent home, but died in mid 1916. He had pointed out that there was a radius around a point mass, now called the Schwarzschild radius, within which light cannot escape. The Schwarzschild radius (r in the next expression) can be calculated from the Newtonian argument of setting the escape velocity equal to the speed of light. i.e.

$$c = (2GM/r)^{1/2}, \text{ giving } r = 2GM/c^2 . \qquad (3.1)$$

In so much as Schwarzschild had time to think about the implications of his solution, he believed that his radius was just a mathematical abstraction and not a physically reality.

Astronomy in the second half of the twentieth century has confirmed for us that black holes are an astronomical reality, though clearly you have to infer their existence out there rather than see them directly. The density of matter at the Schwarzschild radius decreases with increasing mass involved. We believe that there is a black hole at the centre of many galaxies, including our own, of mass at least a million times the mass of our Sun. Say the mass is $10^6\ M_\odot = 2\times10^{36}$ kg. Taking $G = 6.67\times10^{-11}$ m^3 kg^{-1} s^{-2} and $c = 3\times10^8$ m s^{-1}, gives the density as 1.8×10^7 kg m^{-3}, less than the density of a white dwarf. The Schwarzschild radius for such a mass is 2.96×10^9 m, or about 4.75 times the radius of our Sun.

Black holes are formed in astronomy by the self gravitational collapse of mass. That collapse doesn't stop when the mass is within the Schwarzschild radius but what happens is complicated by other properties a black hole can have, namely spin and charge. A black-hole with 'only' the mass of the Earth has a Schwarzschild radius of about 9 mm, much more like the popular picture of a black hole.

There was a young fellow named Cole
Who ventured too near a black hole
His dv by dt
Was quite wondrous to see
But now all that's left is his soul

(A. P. French)

I'll mention a few more experimental tests of General Relativity. Experiment is the supreme court of appeal, as Eddington once said.

Starlight is bent by the Sun

The bending of light by a star was predicted by Einstein in his 1916 paper. This paper was published by a German-born scientist, in German in a German periodical during the first world war. When you've been fighting a country for 2 years in a singularly bloody and dirty conflict trying to belittle the status of your enemy there is good reason for ignoring everything they have to say. However, in Cambridge Arthur Eddington (1882 – 1944) was Professor of

Astronomy, Director of the Cambridge Observatory and a singularly bright spark, known for his mathematical brilliance and physical insight. He was also a pacifist and hence somewhat immune to wartime propaganda and the painting of stereotypes. Earlier papers by Einstein had alerted him to the thrust of General Relativity. He saw how to mount an experimental check of the predicted bending of starlight. He would exploit the forthcoming total solar eclipse of May 1919.

With the Moon temporarily screening out the disc of the Sun, this eclipse provided an opportunity to see a star whose light came to us along a path that went close by the Sun. If the starlight grazing the Sun was bent, then the position of the star should be altered relative to stars more distant from the Sun. If there were no bending, then the stars should be in the same relative positions as if the Sun wasn't there. Stars near and further from the Sun could be recorded at once on a photographic plate. Actually, the experiment wasn't quite that simple, for a hand-waiving argument using classical physics and assigning a mass to photons predicted a bending half that given by General Relativity. The actual experiment was therefore to try to distinguish the larger General Relativistic prediction from the non-relativistic prediction of half the amount.

Eddington organised two expeditions under the auspices of the Royal Society of London, one to Brazil and one that he went with to the island of Principe off West Africa. As everyone knows who has tried to see an eclipse of the Sun, any attempt is at the mercy of the weather and hence mounting two expeditions was a prudent strategy. The deflection of the starlight looked for was very small, about 1.75" arc. In Principe the weather almost clouded out the event but Eddington managed to get one successful photographic plate. In Brazil, the observations were better. The average of the two results confirmed the predictions of General Relativity within reasonable limits and certainly excluded the pseudo-Newtonian result. The successful outcome of this experiment was highly influential in raising Einstein to celebrity status. *Lights all askew in the Heavens. Einstein theory triumphs* as The New York Times trumpeted. *Revolution in Science. New Theory of the Universe. Newtonian ideas overthrown* was how the London Times put it. The results also put Eddington into the public eye as an expositor of General Relativity.

'*One thing is certain and the rest debate*
Light rays, when near the Sun, do not go straight'

as Eddington wrote on one occasion.

Gravitational lensing

Subsequent experiments have confirmed the effect more accurately. Indeed the bending of starlight by the gravity of other stars located almost directly between us and the source of light is an effect that is now in the forefront of astronomy. If distant starlight can be bent, then gravity has the effect of providing a refractive index gradient around stars. Such a refractive index can provide imaging, just as in optics. The effect is known as gravitational lensing.

The effect is quite widely seen. Multiple images of very distant galaxies situated almost behind nearer stars or galaxies are seen on image plates. One result is that we see galaxies we wouldn't otherwise have done because a bigger range of light reaches our telescopes than would do so in the absence of lensing. It's a bit like the increase in brightness of an image that you get by using a lens in a camera instead of a having a pin-hole. So called weak gravitational lensing is already used and being widely developed as a tool to detect whether distant stars have planets circling them. If the lensing star has a planet, then this planet alters the characteristics of the light received by us from the distant star in a distinctive way that allows the presence of the planet to be detected. In short, the influence of gravity on light is now considered a well-known effect.

Precession of Mercury's orbit

I'll be even briefer about two other tests. Einstein also predicted that the orbit of the planet Mercury would twist around in space, *precess* is the technical word I used already, in the gravitational field of the Sun at a rate not predicted by Newtonian gravitation. The orbit in fact precesses quite fast, about 5600" arc per century, and most of this can be explained by the influence of the other planets on Mercury. Einstein predicted an additional contribution over and above the other known contributions. The additional rate is not very much (43" arc per century) but such an additional precession had already been seen and had been left unexplained. Further experiments have completely confirmed the General Relativity value, against different values offered by other theories attempting to account for it.

Gravity probe B

The Gravity Probe B is an experiment with a satellite launched last year after a 30 year development. You should know of its existence because the Physics Department here under the steering of Prof Mike Player has had an input into this experiment. The probe has an extremely technically challenging experiment on board that is attempting to measure the precession of a gyroscope in the gravitational field about 700 km above the Earth's surface, which is slightly different from the field at the surface of the Earth. The experiment aims to test not one but two predictions of General Relativity that have not been tested before. One involves the very slight effect on space-time produced by the rotation of the Earth. I'll leave those interested to read more about this on the Gravity Probe B web-page.

The stronger the gravity, the slower the clock

There are indeed other reasons to believe the predictions of General Relativity. The gravitational red-shift is an example of another effect of General Relativity, namely that clocks run slow in a gravitational field. The stronger the gravity, the slower the clock. You can look on the gravitational red-shift as this effect. The 'clock' that marks the frequency of emission of light runs slow if the emission takes place in a strong gravitational field. The effect is very small on the surface of the Earth. It is much bigger on a white dwarf star and has been verified to better than 1%.

One consequence of the gravitational effect on clocks is the need to correct the atomic clock time of the GPS satellites by the amount predicted by General Relativity because they are orbiting in a slightly different gravitational field from us on the ground. Without this correction, the positions given by the GPS would be systematically wrong. With the corrections, they are not.

Another very convincing test turns out to result from the observation of certain pulsars in binary systems. Pulsars are rapidly rotating neutron stars, usually going round faster than once per second with a regularity that matches the regularity of our best atomic clocks. If a pulsar is part of a close binary system, the rate of change of the regular pulses emitted can be measured, the changes being the influence of the very strong gravitational field of a companion neutron star. Observing such a system and related ones over many years not only provided experimental evidence for the effect of gravity on the rate at which a clock runs in a strong gravitational field but gave direct evidence of the rate of loss of energy of this binary system through radiation of gravitational waves exactly as predicted by General Relativity. For this work Russell Hulse and Joseph Taylor won the 1993 Nobel Prize in Physics.

One anecdote about Einstein relates that when a doctoral student Ilse Rosenthal-Schneider asked him in 1919 how he would have felt if Eddington hadn't confirmed the prediction of General Relativity, he is said to have replied "*I would have felt sorry for the good Lord. The theory is correct anyway*." It was probably said in humour but coming from anyone else would have been interpreted as evidence of a swollen head. The fact is that almost a century later and with more than 6 billion people now in the world, no-one has yet proved Einstein wrong.

The geometry of the Universe

In General Relativity, space can be curved. Curved space is not a new idea. What is the difference between a 2-dimensional coordinate system covering a plane and one covering a sphere? The plane is infinite and flat. The sphere is finite and curved. Flatlanders that lived in a 2D world could tell whether their world was a plane or a sphere. They would do this by applying tests to the geometry of the figures they drew.

Everyone knows that if you draw a triangle on a flat sheet then the sum of the angles within the triangle is 180º. You may not know that if you draw a triangle on a sphere, then the sum of the angles exceeds 180º.

Curved spaces have another feature that the circumference of a circle drawn on them is less than the circumference of a circle with the same radius drawn on a flat plane. If you drew a big circle in a curved space, then you'd find the circumference was not 2π times the radius and you could deduce the curvature of your space.

I hope you can see where this argument is going. Even the flatlanders who live in a 2D world have a means of telling what kind of space they live in, whether it is flat or what's called a 2-sphere. We live in a 3D spatial world and we can't see what it looks like from 'outside'. So do we live in a flat space, or a 3-sphere, or indeed a world that's curved in the opposite way to a sphere, namely one with negative curvature? A 2-surface with negative curvature looks hyperbolic when viewed from the third dimension. See the diagram.

Our Universe

Curved spaces are an integral part of General Relativity. The $64,000 question, or perhaps nowadays it is the 15th question for £1,000,000, is "what is the intrinsic curvature of the Universe as a whole?" This has a direct relevance to the ultimate fate of the Universe and is dictated by the density of matter in the Universe, because in General Relativity matter controls the curvature of space. If there is a high density of matter in the Universe then that will provide enough gravitational force to bring the expansion of the Universe to a halt and to reverse it. In this scenario, our descendants long into the future, if we have any, will see galaxies coming towards them in what will end in a big crunch. This state of affairs is described as a **closed Universe**. Such a Universe has positive curvature. If the density of matter in the Universe is very low, then the expansion of the Universe will never be halted. Such a situation is described as an **open universe**. Such a Universe has negative curvature. In a **flat universe**, which has zero curvature, then the expansion of the Universe will come to a halt asymptotically. That means it will get slower and slower but in any finite time never quite stop. For that to happen the density of the Universe must be finely balanced at a critical value, denoted by cosmologists as ρ_c. General Relativity tell us that the **critical density** ρ_c (Greek rho, subscript c) is given by:

$$\rho_c = \frac{3H^2}{8\pi G}. \tag{3.1}$$

H is Hubble's constant and G the gravitational constant.

If you put in the current value of H, and the value for G, then ρ_c works out at about 10^{-26} kg m^{-3}, a very small number in terms of day-to-day densities. Just how small is it? The mass of a proton is 1.7×10^{-27} kg and hence this corresponds to about 5 hydrogen atoms per m^3. That all you need, spread over the Universe, to provide in total enough gravitational attraction to stop the Universe from expanding in the very, very long term. Compare 5 atoms per m^3 with the number of atoms in the air around us per m^3, which is about 3×10^{25}.

Cosmologists define the density parameter Ω as the ratio of the average density of the Universe $\rho(t)$ to the critical density, i.e.

$$\Omega = \frac{\rho(t)}{\rho_c}. \tag{3.2}$$

The most accurate answer we have to the question of the geometry of the Universe is that Ω_0, the value of Ω now, is close to 1 and hence **the Universe is flat**. This value is predicted by the Big Bang theory of the origin of the Universe along with the concept of **inflation**. The flatness of the Universe is deduced from a study of the fluctuations in the microwave background that permeates the Universe and is an experimental deduction.

In summary, the density of the Universe controls its geometry and is measured by the cosmological parameter Ω_0. The best measurements give $\Omega_0 = 1$ within a few % uncertainty, showing that the geometry of the Universe is flat.

There are a good number of other parameters that characterise our Universe and that can be measured. The Hubble constant is one and the acceleration of the expansion of the Universe is another. Cosmology today is looking much more like a science than just a philosophy, with characteristics that can be defined, measured and checked against the predictions of competing theories. All the deductions, though, are based on the assumption that General Relativity describes the space-time of our Universe. This space-time is determined by the matter within it.

JSR