Musings on space elevators

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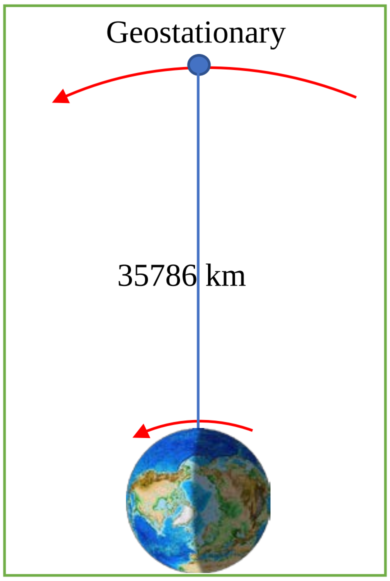


This is one of those pieces where I hope I'll understand a bit more by the time I'm finished than I do now. The space elevator concept starts off as a simple idea: a geostationary satellite ideally orbits the Earth in a circular orbit above the equator in the same time as a point on the equator rotates. Seen from the Earth, the satellite and the point below it are two fixed points. Why not drop a line from the satellite to the Earth so that objects can be lifted up, just as a line can be dropped from the branch of a tree to the ground below? A nice sketch can be drawn. The line represents the 'space elevator'. The 'satellite' is imagined as a space station, where space hardware can be assembled and rockets depart for other space destinations with less energy needed than from the surface of the Earth.

That's one beginning. A second is to take inspiration from the athletics event of 'hammer throwing'. The athlete whirls around, stretching taut the cable to a heavy ball, before releasing it. Well, the Earth is whirling round, albeit sedately compared with the hammer thrower. Shouldn't we be able to use this effect to stretch an elevator cable?

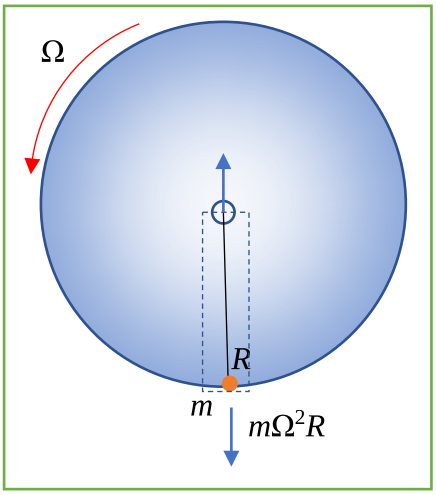
I've opened with two unanswered questions. Unfortunately, the realities of physics knock the simplicity out of the idea. I'll tread slowly, from first principles, which is how I like to tackle a problem. First, though, what is the point of having an elevator? Surely the energy required to raise 1 kg to the height of geostationary orbit against gravity is not reduced by having an elevator? Indeed, it's not. The gravitational potential energy (PE) of 1 kg at distance R from a body of mass M is -GM/R. Take $G = 6.674\times10^{-11}$ m^{3}kg^{-1}s^{-2}; $M = 5.97\times10^{24}$ kg; $R = 6.378\times10^{6}$ m which gives the potential energy of 1 kg at the surface of the Earth as -62.5 MJ. Raised to geostationary orbit it is -9.5 MJ. Every means of transporting that 1 kg mass must provide the difference in energy, namely 53 MJ. That's a lot of energy to shift 1 kg. Where the elevator wins is that no rocket is needed. The rocket needs fuel to create thrust to hold it upright, as well as fuel for motion. Getting 1 kg into geostationary orbit by rocket consumes very much more energy. The rocket, or parts of it, are raised to considerable heights using a lot of fuel whose products are exhausted and a carcass and engine that is allowed to fall back to Earth without recovering any of the energy used to raise it.

35786 km is an exceedingly long way, almost the circumference of the Earth. Travelling at a respectable 100 km h^{-1}, the journey will take a fortnight or so, without stops. [Rockets do it in less than a day. The need for a rocket to expend fuel simply to hold up its weight puts a high premium on making it go as fast as possible, so it uses as little fuel in simply supporting itself.] It's tempting to make the obvious assumption that elevator carriages could work in pairs, one going up as another is coming down. The simplest of scenarios doesn't work because gravity at 35786 km above the surface of the Earth is only 2.3% of its value on the surface so a carriage at the top weighs only 2.3% of one at the bottom and won't balance it. There need to be multiple carriages on the elevator, each linked to one coming down at a similar height.

A fortnight getting there and some turn-around time implies about 20 full journeys per year, not a lot. Suppose an elevator carriage can take up one tonne of goods and people (also not a lot. Proponents argue for much more). What energy source is needed? 1 tonne mass will be 1 tonne wt (say 9.8 kN) for the early part of the journey. Lifting it at 100 km h^{-1} (= 27.8 m s^{-1}) requires a power input of 2.72×10^5 KJ s^{-1} or 272 KW. Add on various inefficiencies in power conversion, etc. and 1 MW will be a ball-park figure for when the elevator carriage is close to the ground. Multiple carriages moving up at once (as suggested above) will take many MW but that is within the compass of an electrical power 'station' or network of renewable sources.

Can you hang a cable from a geostationary satellite? Orbital motion has its own unfamiliar dynamics. If an astronaut were to take a roll of cable outside a geostationary capsule and try to drop it, it wouldn't unroll like it would do if dropped from a tree. It would just keep orbiting with the capsule. We need to look into the physics of what goes on in a rotating frame of reference. My first picture was supposed to represent the equatorial plane, looking down from above the North Pole with the Earth rotating anticlockwise and a satellite rotating with it. The rotation speed Ω is 7.52655×10^{-5} rad s^{-1}; the rotation axis is pointing up out of the paper. That's the scene.

I'll begin by ignoring gravity. Imagine the situation is out in space. The blue circle is at a distance *R* in a frame of reference that is rotating at angular speed Ω. '*m*' is a mass at rest in the rotating frame at distance *R*. In that frame we can apply Newton's laws of motion provided we add an outward force from the centre of $m\Omega^2R$. This force is often called the centrifugal force. If you like the idea, you can think of a rotating frame as having its own 'quasi-gravity'. This quasi-gravity is zero at the centre and increases linearly with distance away from the centre - rather different from our own familiar gravity. I've suggested this by shading the circle. The quasi-gravity has strength $m\Omega^2R$ at distance *R*.



The mass *m* can be hung in quasi-gravity from the centre of rotation by a cable. If the cable is light (has negligible mass) then when you look at the forces acting on cable and mass (within the dashed rectangle) if the mass is at rest, there must be an equal and opposite force to the centrifugal force ($m\Omega^2R$) at the origin to hang up the mass. If the origin is something fixed and solid, then the mass *m* is just like a mass hanging from a tree branch, only in quasi-gravity. If the cable is not light (and an elevator cable in practice is going to be millions of tonnes), then the mass of all its elements will contribute quasi-weight and the total force needed to hold up the cable and the mass will be the sum of all the quasi-weights. If the cable has mass ρ per unit length, then for the mathematicians the extra force on the origin due to the cable mass will be $\int_0^R \rho\,\Omega^2 r\,dr = \rho\Omega^2R^2/2$. A real elevator cable is likely to have a mass per unit length that varies along its length, making the answer more complicated.

Now let's put the Earth at the centre of rotation and add Newtonian gravity, F_{grav}, centred on the axis of rotation, directed towards the centre and decreasing as the square of the distance from the centre. $F_{grav} = GmM_{earth}/r^2$. With no motion in the rotating frame of reference $0 = -F_{grav} + m\Omega^2 r$ for a mass *m* at distance *r* from the centre. The – sign represents Newtonian gravity directed towards the centre. The distance at which Newtonian gravity balances quasi-gravity and there is no force on an object in the rotating frame of reference is the geostationary distance, call it r_g. Here, $GmM_{earth}/r_g^2 = m\Omega^2 r_g$. It is the same distance for all masses. Putting in the numbers for the Earth at the centre gives r_g = 42160 km from the centre of the Earth. The figure in the first sketch is from the Earth's surface. At this distance the two 'gravities' just balance. You have to go a lot further out before quasi-gravity has increased to the strength of Newtonian gravity on the Earth, i.e. quasi-gravity equals the weight of something on Earth. This distance is about 1.84 million km, approaching 5 times as far away as the Moon.

With the cable mass per unit length ρ, then at distance r along the cable, the combination of Newtonian gravity and quasi-gravity gives unit length of cable an 'outward weight' (i.e. like the pull on the hammer thrower) of $\rho\Omega^2 r - G\rho M_{earth}/r^2$. This must be counteracted by the anchor on the Earth. If the result of adding up the pull from the whole cable is negative, the Earth must push back.

I'll include a digression here that may or may not help! Imagine yourself standing on bathroom scales, on the equator for simplicity. The scales push back on your feet and you might think that the amount they have to push back is equal to your weight. It's actually not quite your weight. The difference between your weight (mg) and the reaction of the scales is a force turning you around once in a sidereal day. If I do this with a mass of 90 kg, and I am standing on the equator, the force needed to rotate me on a radius of about 6.38×10^6 m once in 86,164 seconds is about 3 N or 0.35% of my weight. At the risk of getting seriously distracted, this isn't the only reason why the scales may not show true weight. The buoyancy of the air space I occupy, say 0.1 m^3, provides an upward force of about 1 N. The atmosphere hasn't yet come into our story.

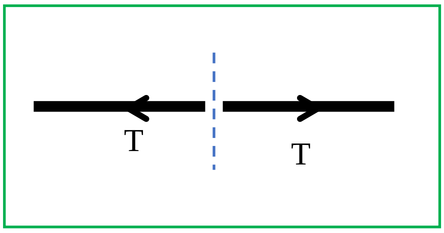
Returning to the theme of the cable, even though the weight of a section of the cable decreases with increasing distance from Earth, for many thousands of km this weight will still be much larger than its quasi-weight. The quasi-weight won't support the cable. If the cable is extremely long, then quasi-gravity, which increases with distance away, will finally become dominant as ordinary gravity weakens and ultimately will be enough to support the whole cable. The Earth will then have to pull, like the hammer thrower, to maintain the cable. How long a cable is needed so that the Earth's push turns into pull?

Let the cable stretch from a point S on the equator of the Earth (distance 6378000 m) to a point at distance X. Again for the mathematicians, the total pull for a cable from S to X is $\int_S^X \left(\rho\Omega^2 r - \frac{G\rho Mearth}{r2}\right) dr = \rho\Omega^2(X^2 - S^2)/2 - G\rho M_{earth}(1/S - 1/X)$. This clearly depends on the mass of the cable per unit length. Setting the pull as zero gives a cubic equation to find X that does not involve the density of the cable. If you want to try it yourself, $\Omega^2 = 5.3175\times10^{-9}$; $GM_{earth} = 3.9844\times10^{14}$. My answer is near 144000 km from the surface. This is well beyond the geosynchronous distance. If we take a ball-park figure of one tonne per metre (1000 kg m^{-1}) for the cable density, our cable stretching this far will have a mass of 144 megatonnes. Even at 100 kg m^{-1} the cable will have a mass of about 14.4 Mt. Wow! The figure is so large I thought at first I must have made a mistake. If so, I can't spot it. A well-publicised suggestion is to terminate the cable on Earth on a floating platform which can exert neither push nor pull. For this to work the cable of uniform thickness needs to be 143750 km long. Alternatively, it could be loaded with megatonnes of mass nearer the Earth. Tapered cables will give different figures. Speak of the tail wagging the dog! It will never work, not least because reality isn't as stable as theory.

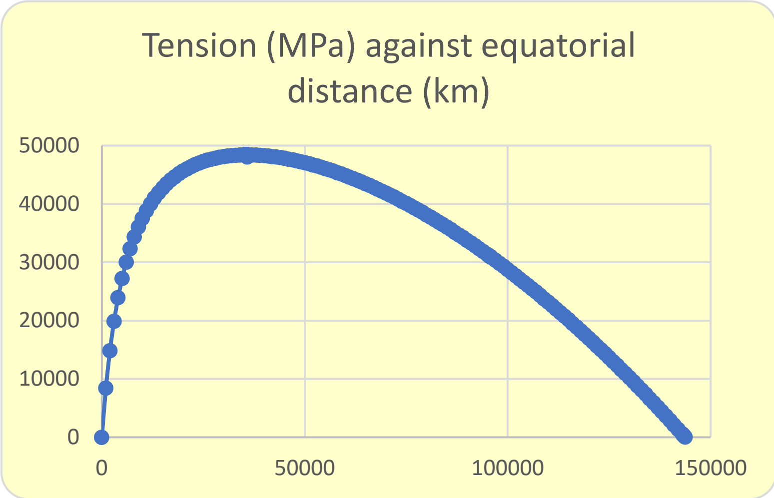
If the cable were to be terminated at the geosynchronous distance of 35786 km, at 100 kg m^{-1} its mass will be 3.5786 Mt. Its total Newtonian weight will be 5.405×10^5 tonnes weight but some of this will be cancelled by its quasi-weight. Nevertheless it would still exert a weight on the ground of 4.840×10^9 N, which converts to 493900 tonnes weight. The slow rotation speed of the Earth means that the centrifugal force, or quasi-weight, hasn't had much cancelling effect on Newtonian weight over that length.

So what's special about the geostationary distance? What it has going for it is that attaching a space station at this distance doesn't load the cable. However, to relieve the weight of the cable on the Earth it needs to be loaded well past this distance. Many web pages talk about 'anchoring the cable' with a distant mass. An anchor is what holds a boat firmly to the seabed, or what holds the cables of a suspension bridge to the adjacent rock. No way is a cable with a substantial mass at the end 'anchored' in space. Since mass is needed at the end of a cable, then it makes sense to me to put the space station further out than the geostationary distance. The figures just given suggest that the mass needed is huge, well beyond anything that mankind can launch this century.

Even the theoretical figures given so far send the message that the Earth is not the ideal place for a space elevator! It gets worse when you think about some practicalities. What's the cable to be made of? How long a cable can you hang up before the top is stretched to breaking point by the hanging weight below? The relevant concept is 'tensile strength' usually measure in MPa. If you can stop it unrolling, you may have difficulty hanging up a 30 m roll of cheap toilet paper by the first sheet, though in fairness to the paper it has been perforated to allow it to tear. Hanging a cable of length 'x' and density ρ, its weight will be ρgx per unit cross-section and if this exceeds the tensile strength then the cable will snap. Selecting some tensile strengths, steel rope might just support a 25 km length, spider silk 75 km, polypropylene rope 5 km (rounded figures). Is it possible to develop a bulk material with the necessary tensile strength? This is a serious challenge.



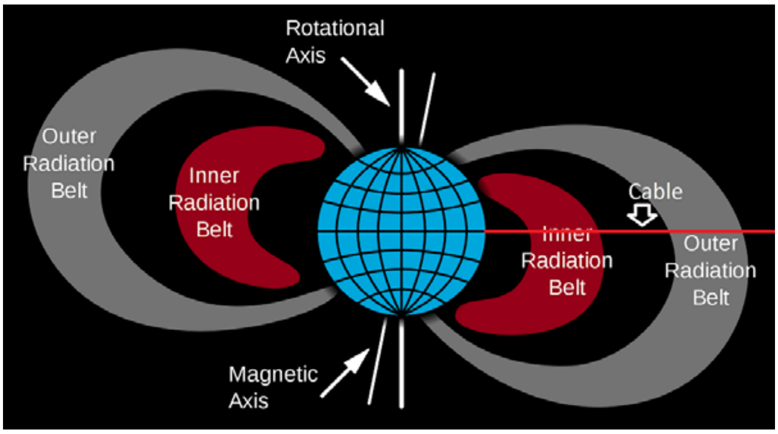
If the cable is long enough so that it just stretches out, exerting no force on its anchor on the ground (144,000 km as calculated above for a uniform cross section) then what is the tension along the cable? The tension 'T' is the force needed at any point to hold it together across an imaginary cut. The force on any side of the cut is the sum of the Newtonian gravity and the quasi gravity if the Earth itself is exerting no pull or push. At distance *r* from the centre of the Earth, this will be $\rho\Omega^2(r^2 - S^2)/2 - G\rho M_{earth}\,(1/S - 1/r)$ in magnitude. To convert to Pascals (or MPa), we need to divide by the cross-sectional area. That will be another figure taken out of the air but if we have a linear density of 100 kg m^{-1} and a supposed volume density of 1000 kg m^{-3} (a bit light for a solid but it may have a fancy construction) then the cross section will be 0.1 m^2. The next figure shows how the tension changes along the cable, reaching a maximum at the geostationary distance. With the figures above, this maximum is 48400 MPa. Given that any design needs to have a considerable safety margin to allow for manufacturing flaws, degradation and unexpected stress, then at the moment we're out of options.



All the above just refers to a naked cable, without any loading of carriages, engines to propel them vertically at 100 km h^{-1} (or whatever speed), and goods. To put it in perspective, my 'lightweight' option of 100 kg m^{-1} (or 100 tonnes per km, if you prefer) is the same mass as a typical pair of railway tracks (without the sleepers). This still requires launching megatonnes of mass and I suspect it is too little.

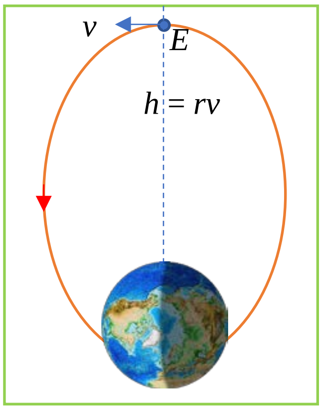
What can possibly go wrong?

There are a raft of issues not covered in the basic physics I've looked at. The Earth is orbited by the Moon and that will produce an additional variable gravitational effect depending on where it is in relation to the cable. Anyone who has even hung up a rope knows that it takes little for the free end to wobble around. The effect of wave disturbances on the cable needs to be taken into account. Will they introduce any instability in its position? Another physical issue of a different kind is that the cable will go right through the Van Allen belts. The inner belt extends to about 6000 km, the outer one to 60,000 km so the adjacent diagram is not to scale. Both belts are home to highly energetic ionising radiation, produced ultimately by interaction between the solar wind and the Earth's magnetic field. Spending days trundling through them in a carriage is not a healthy option for would-be astronauts. The radiation damage for the cable and components immersed in these belts for decades will be potentially serious.



A cable 100,000 km long and 0.5 m wide has a cross-sectional area of 50 million square metres, 50 square km if you prefer. That's a lot of area to be hit by space debris, satellites, meteorites or even planes. A satellite or debris in low earth orbit (say 1000 km high) circles the Earth in about 90 minutes; the cable takes almost 24 hours, so the low earth orbiting object is travelling about 16 times faster, over 8 km s^{-1}. If you like comparisons, a one kg piece of debris travelling at 8 km s^{-1} has the same kinetic energy as a 5 tonne elephant travelling at over 100 m s^{-1}. I suspect the cable wouldn't stand a chance. The lower part of the cable also has to stand up to the very worst the weather can throw at it.

What will happen if the cable snaps? The hammer thrower's ball flies off when released, carrying the comparatively weightless chain initially in a straight line before its trajectory curves downwards due to gravity. The situation of the cable is different. In fact the question is worthy of an undergraduate exam paper. First, an easier question to answer, if you want to try your hand, is what will happen to a carriage that becomes detached? If the carriage is on the portion of the cable nearer the Earth, it will be travelling slower than it would be in circular orbit. Looked at from the Earth, at distance r from the centre it is circulating with a speed of $v = \Omega r$. Assuming it misses the cable then it will fall into an elliptical orbit whose apogee is the starting point. Its initial velocity v determines its angular momentum h about the centre of the Earth and its initial distance and speed determine its energy E. From E and h the orbital parameters of eccentricity 'e' and semi-major axis 'a' can be found (see my piece on *Orbits of Satellites, Moons, Planets and Spacecraft*). In particular, the closest point to the Earth is the perigee distance of the orbit, namely $a(1 - e)$. If this is less than the radius of the Earth plus 100 km or so for atmospheric clearance, then the carriage will become a blazing meteorite in the sky. This will happen if the carriage is less than 25,300 km from the surface. At this distance the orbit has an eccentricity of 0.644 and a semi-major axis of 18,174 km.



At the geosynchronous distance, the carriage will have a circular orbit and keep up with the cable. Beyond that the detached carriage will go into an elliptical orbit with its perigee at the point of departure, for it is 'launched' with a faster speed than a circular orbit at that distance. There will come a distance r when this equals the escape velocity $(2GM_{Earth}/r)^{½}$. The distance is found when $r^3 = 2GM_{Earth}/\Omega^2$. Putting in our earlier figures gives a value of r of 53116 km. A carriage coming adrift beyond that distance will go into orbit around the Sun.

What about a break in the cable? Broadly speaking, the part attached to the Earth will stay attached and hurtle towards the ground. Will cable originally above 100 km burn up in the atmosphere as it re-enters? I guess that depends on what it, and the carriages it has on it, are made of. Will the remaining part stay in orbit? I guess that depends on where the break is. If it's beyond 53116 km from the centre of the Earth then the top part will leave Earth orbit. Any part of the cable remaining in some orbit will be a gigantic piece of space junk that has no useful purpose. I can't see any cable operator guaranteeing to remove a broken cable by any means. Even burning up megatonnes of mass in the upper atmosphere is not an acceptable disposal option.

The musing above hints that there is far more to a space elevator project than simply a cable in the sky. There needs to be an operations control structure for using it; there need to be servicing, monitoring and inspection routines and a means of repairing failure and damage to both cable and carriages; there need to be safety protocols and a means of rescuing stranded people, if people are going to be transported; there needs to be an end of life disposal plan for when it can not be maintained or fails irreparably. As I pen this, media coverage is talking about a possible space elevator on Earth in mid 21st century. I'll be surprised if it happens at all in the 21st century. It's a shorter challenge on Mars, though people haven't even got there yet and Mars has two close-in moons. Elsewhere in the solar system, Mercury and Venus rotate too slowly, amongst other impediments; asteroids generally have sufficiently weak gravity to make it not worthwhile, planets from Jupiter through to Neptune have no solid surfaces to stand on. The Moon rotates slowly but at least the far side of the Moon is always on the other side from the Earth. Most other moons are too much influenced by their planet to make a simple system work and there won't be any human colonies there needing an elevator in the 21st century.

The whole rationale for a space elevator is to save launching rockets to reach space. Yet to even set up a space cable, megatonnes have to be launched a great distance into space. The concept of a space elevator is easy to grasp and its usefulness isn't in doubt. Yet that is not enough to make it a reality. It is the numbers involved that make reality an awesome challenge – numbers for length and strength, numbers for the stability of the structure, numbers for operations, safety and emergency response. My take is that even if the technology could be developed in the coming decades, building a functioning space elevator is a capital-intensive project beyond the willingness of mankind to undertake at this time in history. It makes more sense to me to wait until we have the technology to build a space elevator mainly from ingredients already in space. That's unlikely to be in the 21st century.

Supplement on rotating frames of reference

Newton's laws of motion work fine in a steadily rotating frame of reference provided you include additional 'forces'. If an object is in equilibrium, as we hope a cable will be, then the only extra force needed is centrifugal force. If an object is moving with velocity $\boldsymbol{v}$, then an additional force comes into play, the Coriolis force. This force is proportional to the speed of the object in the frame and the rotational speed of the frame, Ω. It is directed at right-angles to the object's velocity and is in a plane perpendicular to the rotation axis. Textbooks give the full details. For example, the Coriolis force on a carriage travelling up or down the cable at 100 km h^{-1} will exert a small force at right angles to the cable. Another example of where the Coriolis force would come into play is if the end of a spooled cable in orbit is projected towards the centre of the Earth, it will move sideways because of the Coriolis force.

JSR

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