Gravitation – if the Earth could see

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When we look at objects in the sky they appear at different magnitudes, partly because the objects themselves have intrinsically different brightnesses (the astronomers' *absolute magnitude*) and partly because the light from distant objects fades as the inverse square of the distances they are away. Gravity works in much the same way. The gravitational force experienced by the Earth depends on the gravitational strength of the influencing body (determined simply by its mass) and on the inverse square of the distance it is away. If the Earth could 'see' gravity in the same way we can see light, what kind of view would it get of our surroundings?

It's no surprise that the gravitationally brightest object in the sky is the Sun, for that holds us in our annual orbit. Different visual magnitudes in astronomy are represented on the logarithmic magnitude scale, where a factor of 100 in visual brightness converts to 5 on the magnitude scale. Hence a step of 2.512 in brightness corresponds to a step of 1 on the magnitude scale. Remember that smaller numbers correspond to brighter objects. The Sun has a visual magnitude -26.7; Venus at its brightest has a magnitude -4.4. This implies that light from Venus is weaker than sunlight by a factor of $2.512^{22.3} = 8.3\times10^{8}$. In comparison, Venus is gravitationally much stronger relative to the Sun's 'gravitational brightness', as can be seen from the following numbers.

The Sun is about 115 times the diameter of Venus and hence, other things being equal, one might expect it to be 115^{3} times more massive (since the mass of a sphere scales as the cube of its diameter). Other things aren't equal, though. Venus is about 3.7 times the density of the Sun and, moreover, at its closest to Earth it is only 0.28 the distance to the Sun. Hence gravitationally the Sun is stronger than Venus by $115^{3}\times0.28^{2}/3.7 = 3.22\times10^{4}$. This is not nearly as much as the Sun is brighter than Venus. In magnitude terms, this converts to a magnitude difference of 11.27. Visually, this is about the difference between full moon and Sirius and is a smaller difference than between the Sun and Moon.

Based on similar calculations, the adjacent table shows the 'gravitational brightness' compared with the Sun of the Moon and the planets at their nearest distances from Earth, and that of α Centauri, one of the nearest stars and one similar to our Sun, and the galactic centre. I've taken as the 'galactic innards' a mass of 4×10^{10} solar masses at a representative distance of 25000 light years which are ball-park figures causing the whole solar system to rotate around the galactic centre in about 240 million years. Remember that astronomical brightnesses have a larger number the less bright the object is. The black hole at the very centre of the galaxy in the constellation of Sagittarius is known as Sgr A* and has a mass of about 4.3×10^{9} solar masses.

Body	***Gravitational brightness relative to the Sun***
Moon	5.5
Mercury	15.6
Venus	11.3
Mars	14.2
Jupiter	10.5
Saturn	13.3
Uranus	17.1
Neptune	18.9
α Centauri	27.2
Galactic innards	19.5
Sgr A*	21.9

Visually, full Moon has a brightness relative to the Sun of 14.2. Gravitationally, Venus, Mars, Jupiter and Saturn are all brighter or as bright. The Moon itself exerts a gravitational force on the Earth almost 1% of the Sun's force. The 'picture' of the gravitational forces acting on the Earth is a very different one to the scene shown by light falling on the Earth.

Conclusions

It's tempting to write off all distant objects except the combined effect of all the other stars towards the galactic centre as having no effect at all on the Earth. Yet because of the huge mass of stars, even a pretty distant star does exert a force on the Earth that is quite a sensible number of Newtons. In fact more or less every star in the Milky Way galaxy exerts a force on the Earth at least as big as the force that you or I exert, namely our weight. Surely that can't be right? Get out your calculator and remember that Newton's basic formula for the mutual gravitational attractive force F between Earth and a mass M is:
$F = 6.67\times10^{-11}\times M\times M_{\mathrm{Earth}}/(\mathrm{separation})^2$. You can work out that a star of the same mass as the Sun at a distance of 1.34×10^5 light years exerts the same force on the Earth as a person of 50 kg mass on the surface of the Earth. Now that's a sobering thought!

Thinking about gravitational force, there is no reason to stop at the galactic centre. Take the Andromeda galaxy with, say, at least 10^{12} solar masses, including dark matter, at a distance of 2 million light years. The gravitational force it exerts on the Earth is 2.2×10^{12} N, about the same as the weight of all 6 billion people on the planet if they were to stand in one giant field 10 km square. It's a striking picture, perhaps verging on the silly, but it brings home that the gravitational force of the Andromeda galaxy is quite enough to have influenced the relative position of us and Andromeda over the 4.6 billion years of the Earth's existence, which is why the Andromeda galaxy is part of the 'local group' of galaxies. The local group of galaxies has identity because of the significant mutual gravitational influence of its members. In 'gravitational brightness' terms, the Andromeda galaxy is 25.5 magnitudes weaker than the Sun. In comparison, Sirius, the brightest star in the sky, is 25.3 visual magnitudes weaker than the Sun. Taking matters further, our local group is part of the Virgo galactic cluster, centred in the region of M87 at a distance of 50 million LY, for weaker but still valid reasons of the mutual gravitational influence of its members. Gravity truly connects us to stars almost unimaginable distances away.

Appendix

If you want to try estimating magnitudes, m, yourself on some other examples, then if the gravitational force exerted by an object on the Earth compared with the force exerted by the Sun is x, then $1/x = 2.512^m$. Hence $\log(1/x) = m \log 2.512$, giving $m = \log(1/x)/\log 2.512$.

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