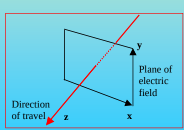
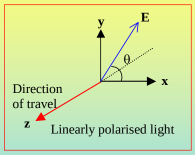
Polarisation of light

- The polarisation of light is scarcely discernable with our eyes
- Polarisation describes the behaviour of the electric field associated with light
  - types of polarisation are **linear**, **elliptical**, **circular**, **unpolarised**
- Remember that in isotropic materials, light is a transverse wave



Linear polarisation

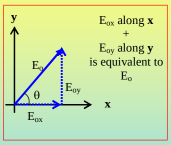
- The direction of the electric field at a point stays constant in time
  - its direction is the **direction of linear polarisation**
  - its components along the x and y axes must always stay in step
  - mathematically, the 2 components of **E** at point z along the wave can be written



$$E_x(z,t) = E_{ox}\cos(kz - \omega t)$$
$$E_y(z,t) = E_{oy}\cos(kz - \omega t)$$

A note on components of **E**

- **E**, the electric field, has a direction and a size
  - it is a **vector**, like a displacement



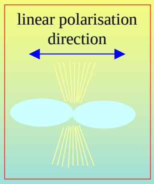
- Every electric field of magnitude E_o has **components,** E_{ox} and E_{oy}
  - the sizes of the components depend on the angle θ between E_o and the **x** axis

$$E_{ox} = E_o\cos(\theta)$$
$$E_{oy} = E_o\sin(\theta)$$

- Polaroid transmits the component of **E** along its axis (see later)

Haidinger's brush

- Some people can detect the direction of linear polarisation of light
- A very faint figure is visible in linearly polarised light a few degrees across in the centre of your field of view
  - if you rotate a piece of polaroid in front of your eye, this figure rotates with the polaroid
- The figure is called **Haidinger's brush**



Relationship between irradiance of light and electric field **E**

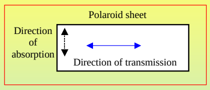
- Light meters measure irradiance, cameras and our eyes respond to irradiance
- The irradiance, I, is proportional the average square of the electric field:

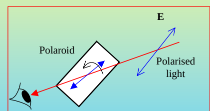
$$I \propto \langle E^2 \rangle$$

- Polarisation phenomena are about the direction and amplitude of the electric field wave, **E**

Polaroid sheet

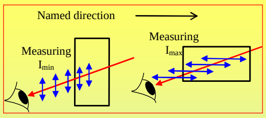
- Polaroid produces linear polarisation of light by transmitting the electric vector along the axis of the polaroid and absorbing the perpendicular electric vector





- Polaroid placed in front of polarised light transmits the most when its axis is rotated || to the direction of polarisation and least when ⊥

% of polarisation



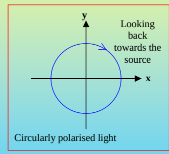
- Light can be partially polarised
- Measure the maximum intensity I_{max} and the minimum intensity I_{min}
- Calculate the % polarisation in the direction of maximum intensity

$$\%\ polarisation = \frac{(I_{max} - I_{min})}{(I_{max} + I_{min})} \times 100$$

- Example:
  - if $I_{max} = 2I_{min}$, then % polarisation = 100/3 = 33%

Circular polarisation

- With circular polarisation, the **x** and **y** amplitudes are both equal (call them E_o) but there is a phase difference of $\pi/2$ between them
- Circular polarisation comes in two flavours
  - **right circular** polarisation, in which **E** rotates clockwise looking back down along the direction of propagation

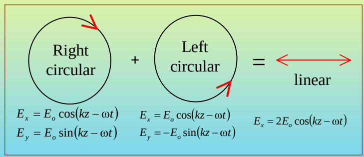


$$E_x = E_o \cos(kz - \omega t)$$
$$E_y = E_o \sin(kz - \omega t)$$

  - left-hand circular polarisation
    - circular polarisation can't be distinguished through a sheet of polaroid

Combination of opposite circular polarisations

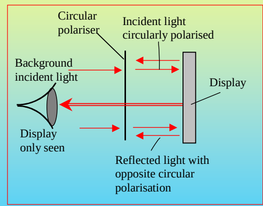
- If you combine right-handed and left-handed circular polarisation in equal amounts, you get linear polarisation



- The polarisation angle (i.e. the direction of the linear polarisation) depends on the phase difference between one component (e.g. *x* component) of the two hands
  - relevant to interpreting other polarisation phenomena

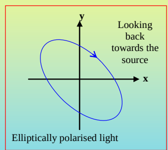
Application of circular polarisation

- Circular polarisers are used to enhance the contrast of LED displays
- Background light is circularly polarised before it reaches the reflecting front of the display
- The handedness of the polarisation is changed by the reflection and it fails to get back through the polariser
- The direct light from the display does pass through the polariser



Elliptically polarised light

- With elliptical polarisation, the amplitudes of **x** and **y** components are generally not equal and neither are phases between the components anything special

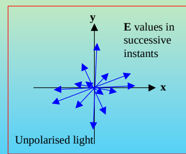


$$E_x = E_{ox} \cos(kz - \omega t)$$
$$E_y = E_{oy} \cos(kz - \omega t + \varepsilon)$$

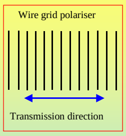
- Elliptical polarisation is the most general case
  - $\varepsilon = 0$ is the special case of linearly polarised light
  - $\varepsilon = \pm\pi/2$ **and** $E_{oy} = E_{ox}$ gives circularly polarised light

Unpolarised light

- Unpolarised light consists of light where the direction of **E** varies at random between successive measurements at one point
  - any direction is equally likely
- Unpolarised light can be considered as a combination of equal amounts of linear polarisation in two directions at right angles, where the **two components are incoherent**

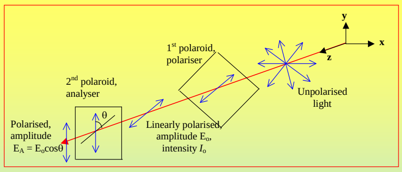


Producing linear polarisation



- Polaroid sheet
- Transmission through a wire grid
  - the distance between wires < λ/4
    - modern polaroid sheet works in a similar way
- Scattering of sunlight by the atmosphere
  - bees and other insects use polarised light to navigate
- Reflecting light
  - reflections can be reduced by looking though polaroid sunglasses oriented to cut out the strongest polarisation
- Transmission through birefringent materials
  - used in the petrological microscope
  - analysis of strain in transparent materials

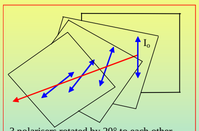
Malus' law



- Malus' law gives the irradiance transmitted by an analysing polariser, I_A, set at angle θ to the direction of polarised light of irradiance I_o
- The irradiance of the light transmitted varies as $\cos^2\theta$
  - this is just what you'd expect from our earlier section on the relationship between irradiance and amplitude $I_A = I_0 \cos^2\theta$
  - e.g. a polariser is set at 30° to the direction of polarised light, *how much is transmitted by the polariser?*
    - fraction transmitted = 0.75 $I_A = I_o \cos^2(30°) = 0.75 I_o$

Rotating the direction of polarisation

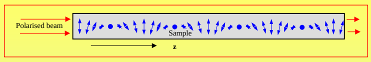
- Several sheets of polaroid in succession will rotate the direction of polarisation of light



3 polarisers rotated by 20° to each other

- Some molecules, such as sugar solutions and quartz, can do the same only more efficiently. This ability is called **optical activity**, or sometimes rotary polarisation

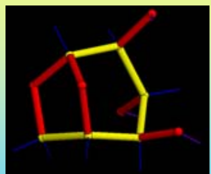
Optical activity



- Optically active materials rotate the direction of polarisation as the light propagates through
  - **dextro-rotatory**; **levo-rotatory**
  - measured by **specific rotation**, in ° mm^{-1} for solids
- Cause is that left and right circularly polarised light have different refractive indices n_R and n_L.
  - linearly polarised light travels through as two circularly polarised rays, at slightly different speeds
    - as their phase difference varies, so the direction of linear polarisation alters

Chiral molecules

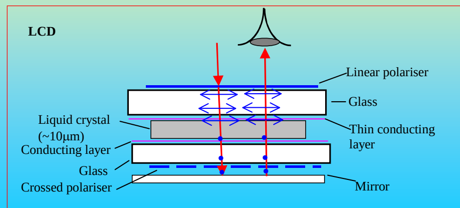
- Optical activity is caused by molecules that have a helical twist, called **chiral** molecules
- All chiral amino acids are l-rotatory – why?
- Natural sugars like dextrose are d-rotatory
- (Some optical activity can be caused by twisted molecular arrangements)



Dextrose
red - O
yellow - C
blue - H bonds

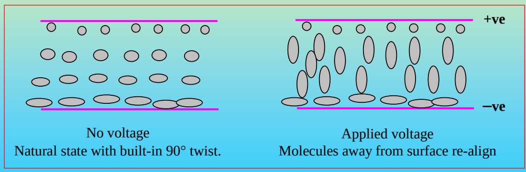
Liquid crystal displays

- An LCD pixel uses crossed polarisers to produce the dark state and an electrically induced change of polarisation to produce the bright state
- The popular **twisted nematic** LCD:



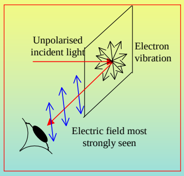
Molecular orientations with an LCD

- The alignment of molecules is induced by a surfactant to produce a highly optically active cell
- A small voltage is sufficient to re-align the molecules



Polarisation by scattering

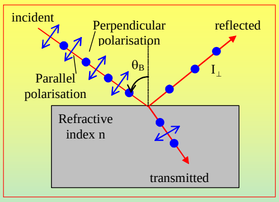
- Vibrating electrons emit light asymmetrically
  - most light is emitted ⊥ to their vibration direction
  - no light is emitted along their vibration direction



- Light scattered through 90° is strongly polarised
- The blue sky is polarised, particularly at 90° from the sun
  - use is made of this by insects, particularly bees, for navigating

The Brewster angle

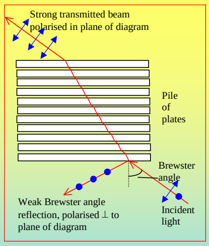
$$\tan\theta_B = n$$

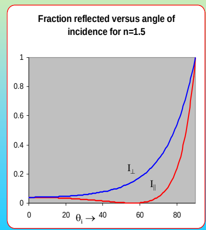


- The Brewster angle, θ_B, is the angle of incidence at which the reflected light is 100% polarised, ⊥ to the plane of incidence
- The reflected and transmitted rays are at 90°
- Example: for n = 1.5, θ_B = 56.3°

Polarisation by reflection

- Fraction of light reflected at different angles of incidence depends on its linear pol'n



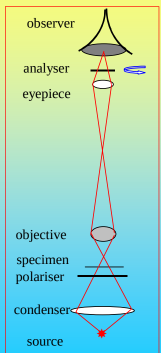


- Observation in nature

- 'Pile of plates' polariser

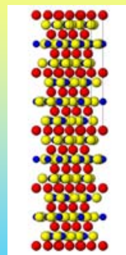
The polarising microscope

- The polarising microscope incorporates a 'polariser'
  - the sample is illuminated by linearly polarised light
- An 'analyser' allows the polarisation of the image to be investigated
  - the analyser is often set at 90° to the polariser
  - the geologists version is the **petrological microscope**



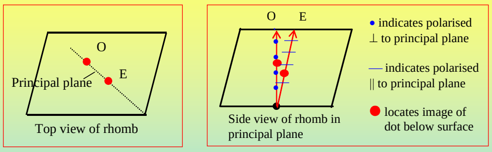
Birefringence

- Birefringence is a new range of phenomena opened up by the **anisotropy** of materials to the propagation of light
- These materials usually transmit light as **two** rays, even when one is incident
- $CaCO_3$ (calcite, Iceland spar) is the archetypical solid



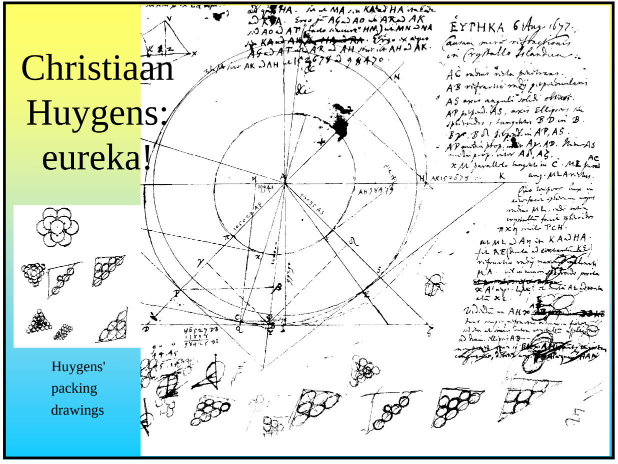
$CaCO_3$ viewed up hexagonal axis

Ordinary & extraordinary rays



- The **ordinary ray** obeys Snell's law
- The **extraordinary ray** deviates in a plane containing the optic axis direction of the crystal
  - such a plane is called a **principal plane**
- Both rays are linearly polarised at right angles to each other

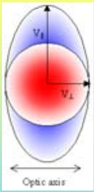
Christiaan Huygens: eureka!

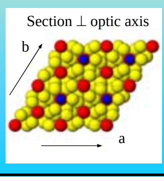


Huygens' packing drawings

Waves in a uniaxial crystal

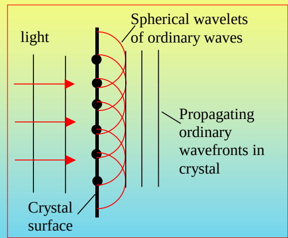
- Calcite **optic axis** || 3-fold axis
- Ordinary rays are propagated by an expanding spherical wave
  - the electric vector is ⊥ optic axis
  - refractive index $n_o = c/v_\perp$
- Extraordinary ray is propagated by an expanding ellipsoidal wave
  - the electric vector is || princ. plane
  - smallest refractive index $n_e = c/v_\parallel$





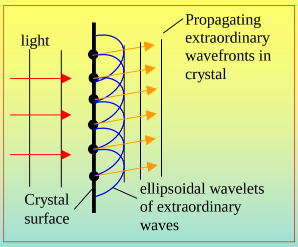
Propagating ordinary waves

- Ordinary waves propagate as you would expect from Huygens' principle
- The refractive index n_o for calcite is 1.658
- n_e for calcite is 1.486
  - calcite is an example of a **negative uniaxial** crystal, because $n_e < n_o$



Propagation of extraordinary waves

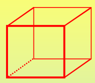
- Remember that extraordinary wavelets propagate as ellipsoidal wavefronts

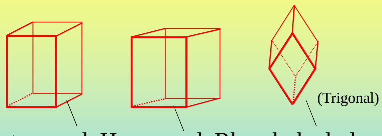


- The axes of the ellipsoids are inclined to the surface
- The common tangent cuts the ellipsoids off to the side
- The direction of the propagating ray is therefore not perpendicular to the surface
  - inside an anisotropic crystal, the extraordinary light is generally not a purely transverse wave
  - **Biaxial** crystals have 2 extraordinary rays; they are complicated

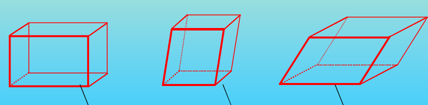
Birefringence is related to crystal class

- Cubic – **isotropic**





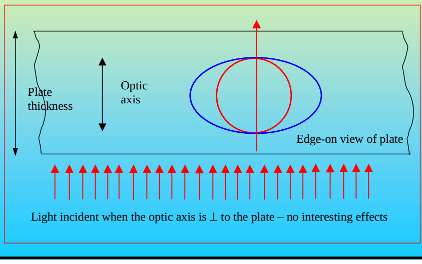
- Tetragonal, Hexagonal, Rhombohedral – **uniaxial**



- Orthorhombic, Monoclinic, Triclinic - **biaxial**

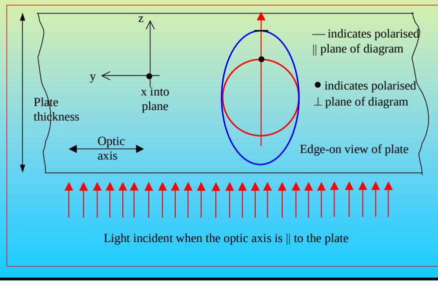
Light incident || optic axis

- Both rays travel together, producing no special effects



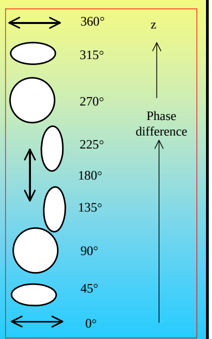
Light incident ⊥ optic axis

- The 2 polarisations travel at speeds c/n_o and c/n_e, acquiring a phase difference

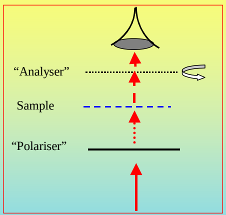


Polarisation change during propagation

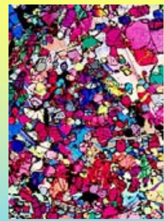
- The phase change between the 2 rays is $z(n_o\text{-}n_e)2\pi/\lambda_{vac}$
- If the 2 rays start off with equal amplitude, then the diagram shows how the polarisation changes with z, the distance travelled
  - the sequence happens every 3 μm in calcite
  - 100 μm is more typical of minerals



Minerals and the microscope



Appearance of Moon rock in the polarising microscope

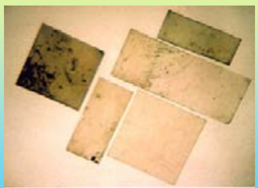


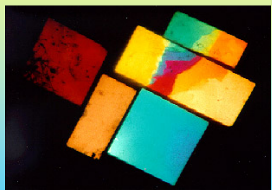
picture courtesy: micro.magnet.fsu.edu

- Isotropic material appears black; birefringent material appears with **polarisation colours**
  - the most intense colours are when the optic axis is at 45°
  - **extinction** occurs when the optic axis is || or ⊥ to the polariser
  - additional colouring is provided by **pleochroism**, selective polarisation dependent absorption of some colours

Demonstration example

- The first picture shows several sheets of mica of different thicknesses seen in ordinary light

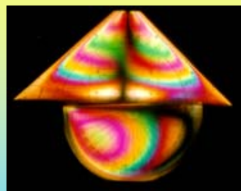


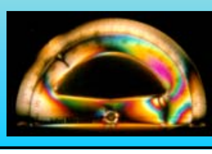


- The second picture, the same sheets between crossed polaroids

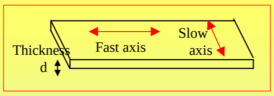
Strain in transparent materials

- Colours are caused by strain induced birefringence
  - also by variations of thickness
  - for a 1 mm thick material, 360° phase shift is caused when $(n_o - n_e) \approx 5\times10^{-4}$





Retarders

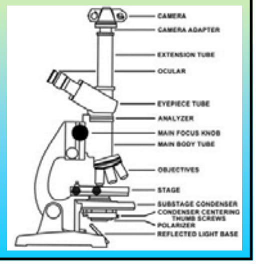


- A **retarder** is a uniform plate of birefringent material whose optic axis lies in the plane of the plate. Retarders can be used to
  - make circularly polarised light
  - analyse elliptically polarised light
  - interpret colours in the polarising microscope
- **Slow axis** is optic axis for calcite
  - **fast axis** is ⊥ slow axis
- Phase retardation Δϕ, in radians

$$\Delta\varphi = k_{vac} d \left(n_{slow} - n_{fast}\right)$$

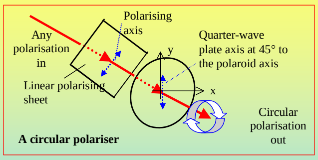
Retardance

- A **full-wave plate** retards the slow wave relative to the fast wave by 2π radians
- A **quarter-wave plate** retards by π/2
  - in terms of phase, the retardance is **chromatic**
  - the **retardance** may be measured in wavelength
    - e.g. a retardance of 250 nm, which is $d(n_{slow} - n_{fast})$
- Why bother?
  - e.g. in the polarising microscope, sliding in a retarding plate between sample and analyser enables a microscopist to decide how birefringent the sample is, helping identification of the sample



Making circularly polarised light

- Circular polarisation is made by shining linearly polarised light at 45° onto a quarter-wave retarder



- The output looks like:

$$E_x = E_o \cos(kz - \omega t)$$
$$E_y = \pm E_o \sin(kz - \omega t)$$

  - the + sign occurs if the slow axis is || y direction, giving right circularly polarised output
    - -ve sign for slow axis ⊥ to y axis, giving left circularly polarised light