Light Science

A course by

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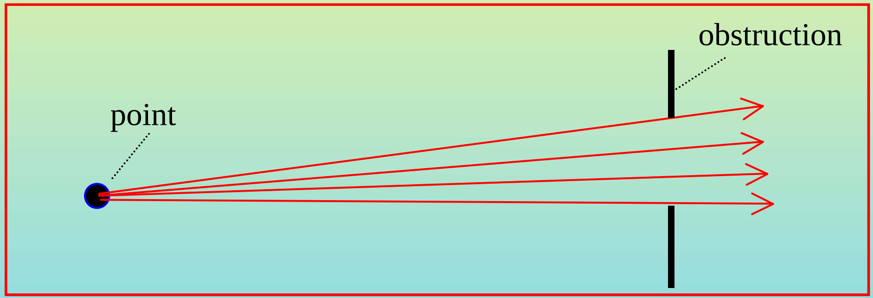
Light Science

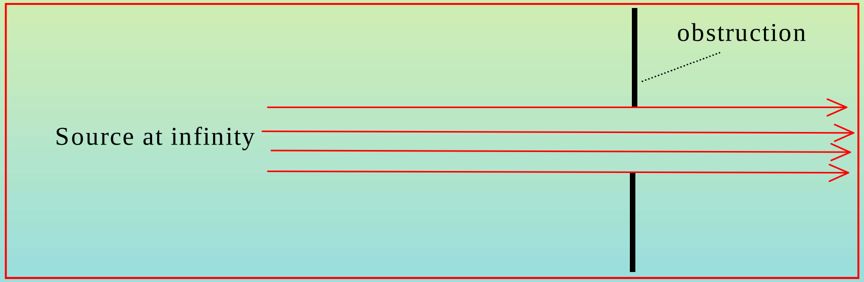
- Optics has seldom been more relevant than it is today
  - design of cameras, holograms, telescopes, spectacles, surveying instruments …
  - design of lab optical instruments: microscopes, spectrometers, …
  - fibre-optic communication and the new electronics
  - new laboratory techniques: confocal microscopy, fluorescent molecular marking, ….
  - optics of natural phenomena

Straight-line Propagation

- Definitions of **Rays, Pencils, Beams**
  - A **Ray** of light is the direction of propagation of light energy

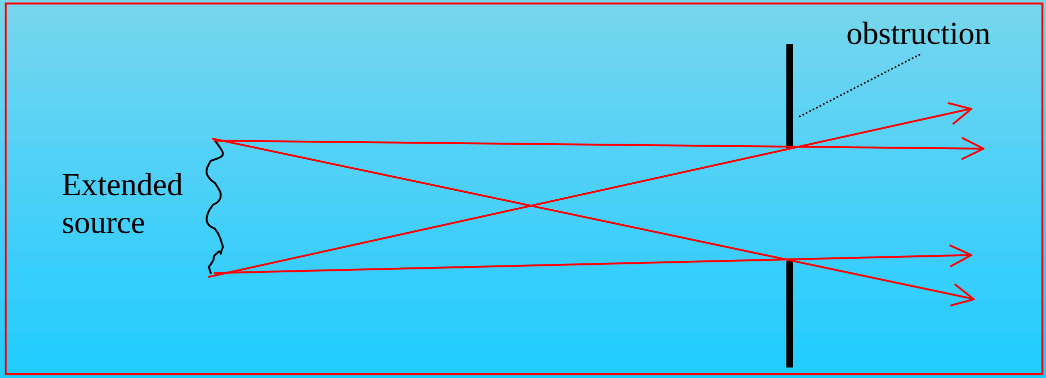


A **pencil** of light ↑



A **parallel pencil** ↑

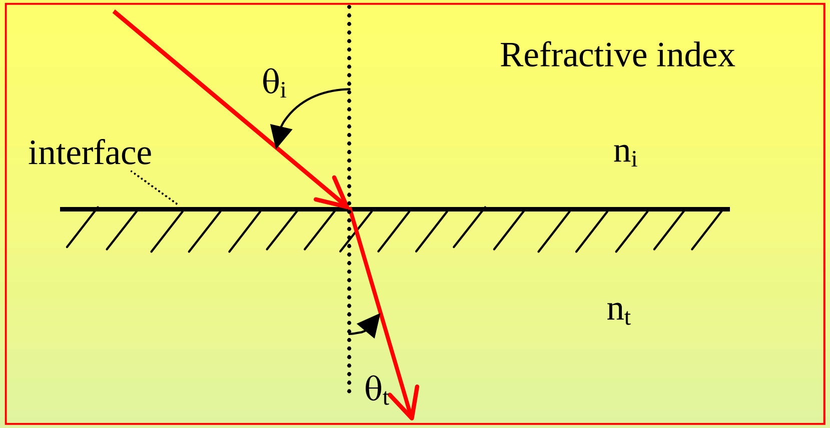
A **beam** of light →



Rays or Waves?

- The relationship between rays and waves in optics is fascinating
  - ray/particle view: Newton & Einstein
  - wave view: Hooke, Huygens, Fresnel, Maxwell
- We shall see that the fundamental properties of light can be described in both terms
- Light is light; the rest analogy

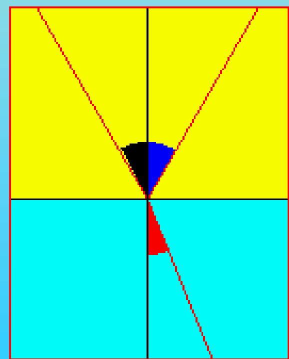
Refraction



- Snell's law
  - $n_i \sin\theta_i = n_t \sin\theta_t$
  - the refractive index, n_x, of the medium x is related to the speed of propagation $v_x = c/n_x$

    c is the speed of light in vacuum
    - e.g. $n_{air} = 1.0003$, $n_{glass} = 1.54$, $\theta_i = 45°$

      hence $\sin\theta_t = 0.4593$ and $\theta_t = 27.34°$
  - simulation of refraction

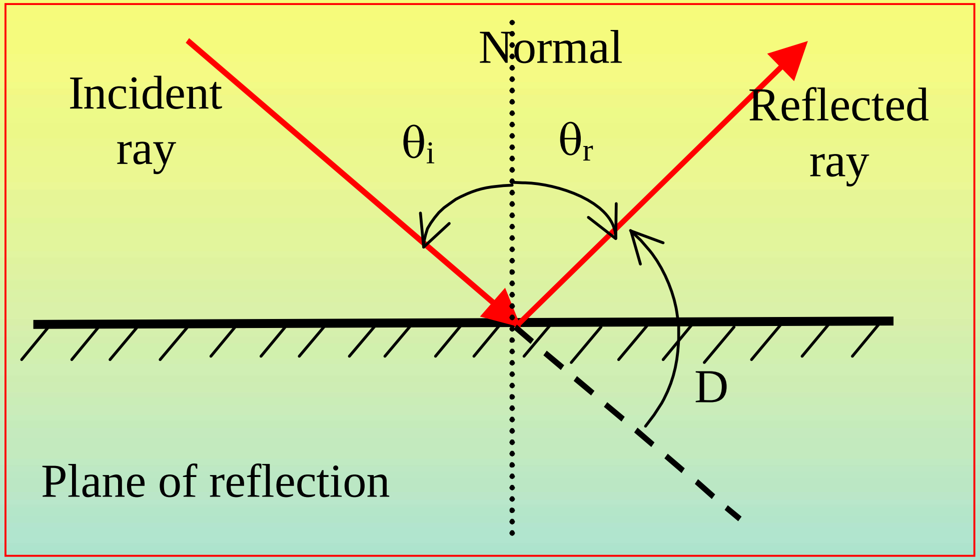


courtesy http : //home.a - city.de/walter.fendt/phe/refraction.htm

Examples of refraction in nature?

- What natural phenomena are caused in whole or in part by refraction?

Reflection

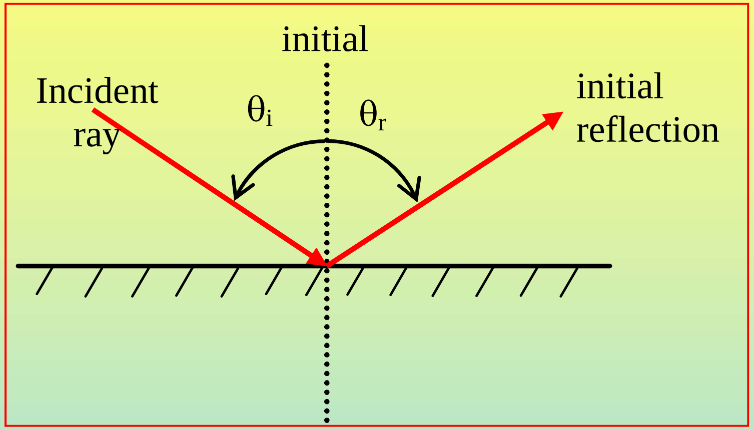


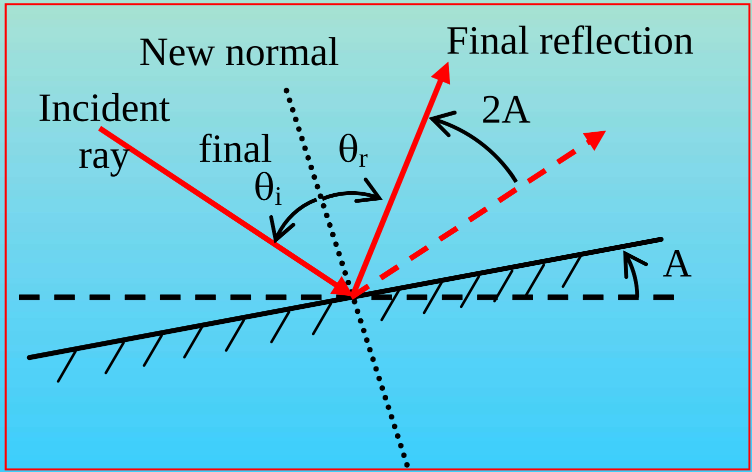
Courtesy: http://en.wikipedia.org/wiki/Image:Hand_with_Reflecting_Sphere.jpg

- The laws of reflection are
  - $\theta_r = -\theta_i$
  - the incident ray, surface normal and reflected ray are all in the same plane - the *plane of incidence*
- Deviation, D, of a reflected ray: $D = 180° - 2\theta_i$

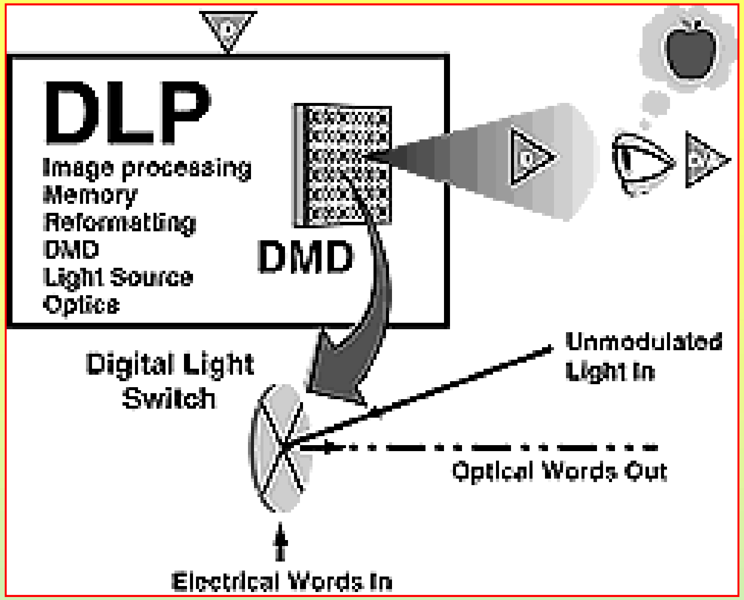
Optical Lever

- Tilt a mirror through angle 'A' about an axis perpendicular to the plane of reflection
  - the change in angle of incidence can be written $\delta\theta_i$
  - $\delta\theta_i = -A$
  - $\delta D = -2\times\delta\theta_i = 2A$
  - in words: the reflected beam twists through twice the twist of the mirror

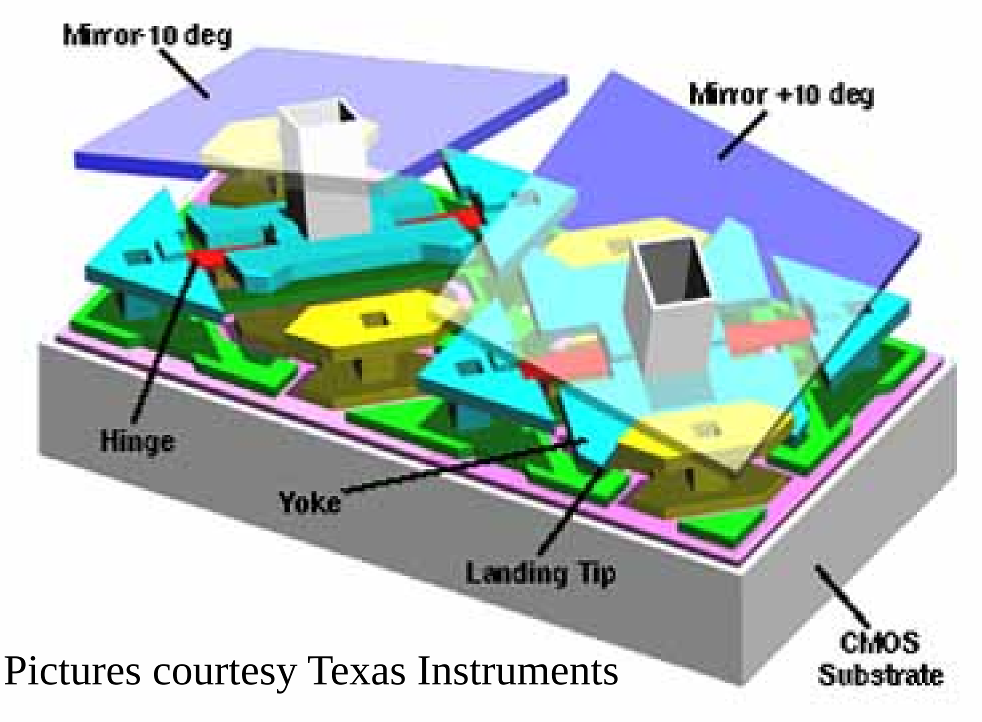




Optical lever example



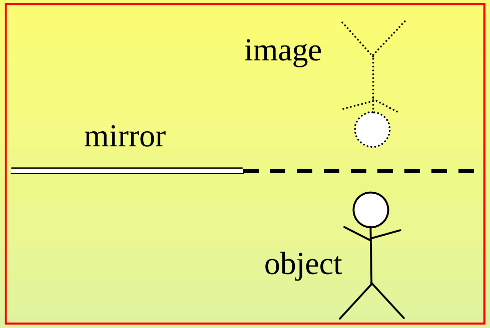
- The new generation of video projectors uses digital input to control the pixel illumination
- Each pixel is controlled by a moving mirror 16 μm square
  - resolution of 2048×1536 readily available
  - exceptional illumination

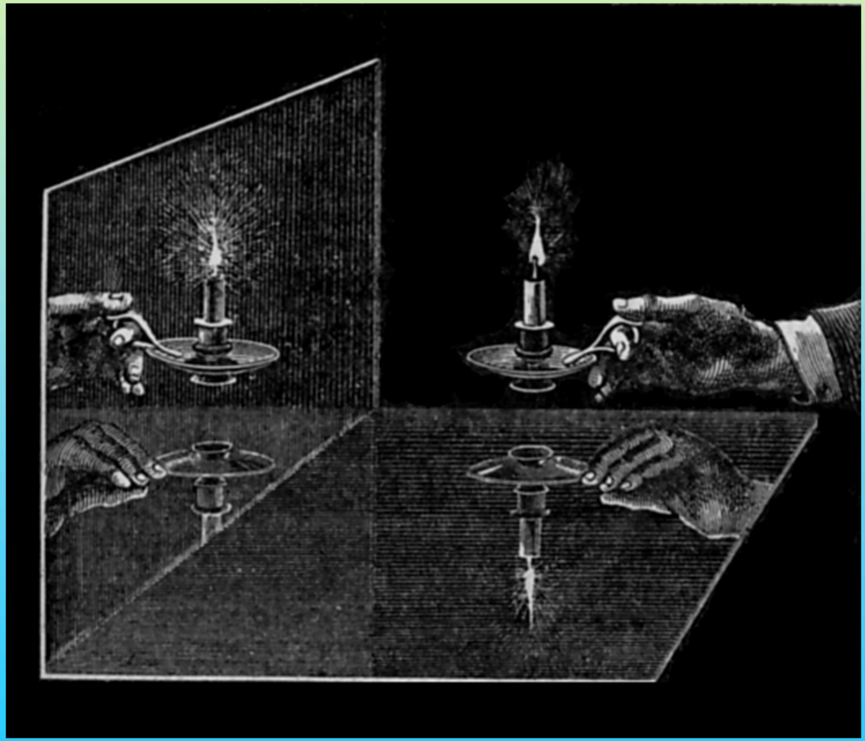


Pictures courtesy Texas Instruments

Plane Mirrors

- Where is the image?
  - as far behind the plane of the mirror as the object is in front





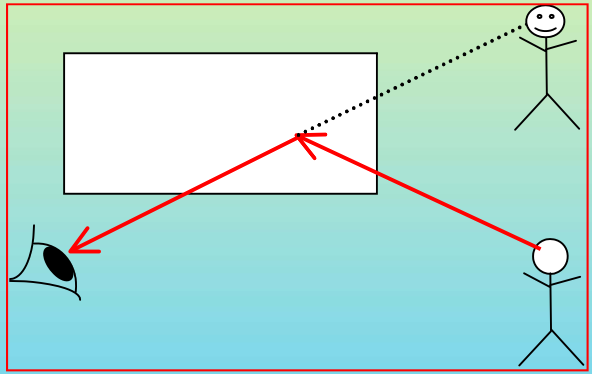
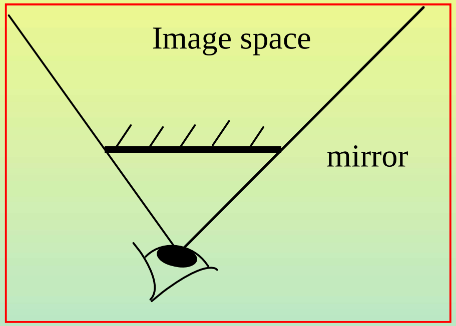
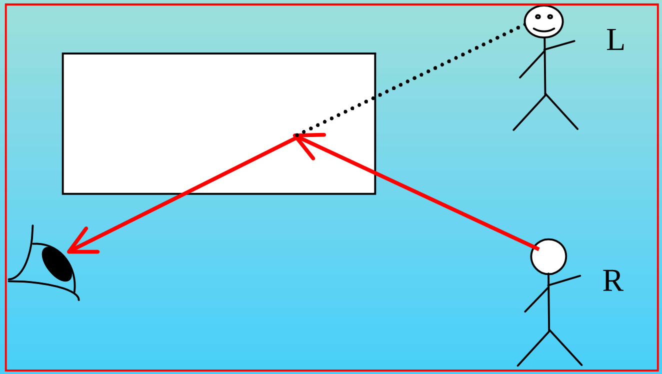


Image space and handedness

- How much is seen in **image space**?

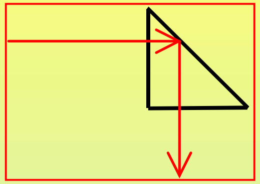


- Every reflection changes the handedness of the image

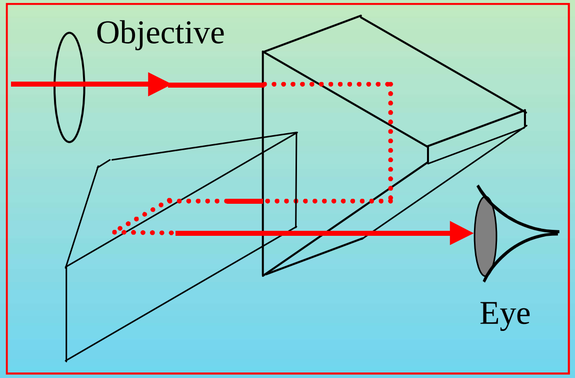


Examples

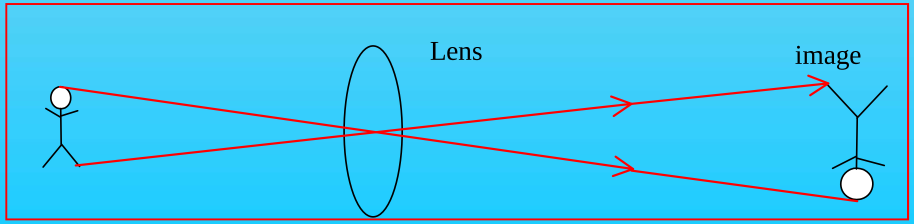
- A 90º prism - is there a change in handedness of the image?



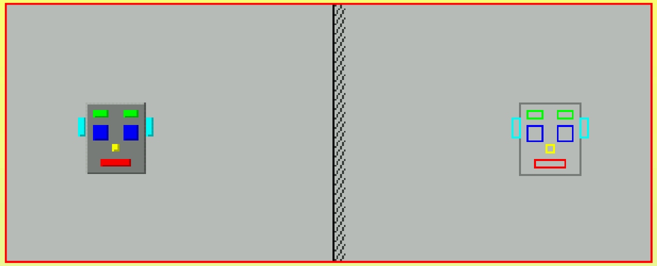
- How many reflections are there in the prisms of traditional binoculars?
- An overhead projector has only one mirror. Why do written overheads not appear as mirror reflected writing?



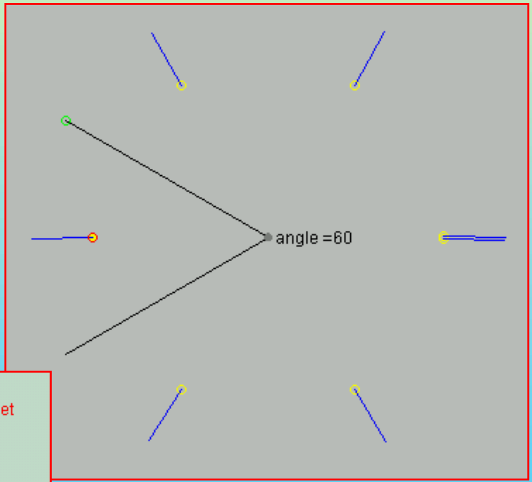
- Is the image in a lens a different handedness from the object?



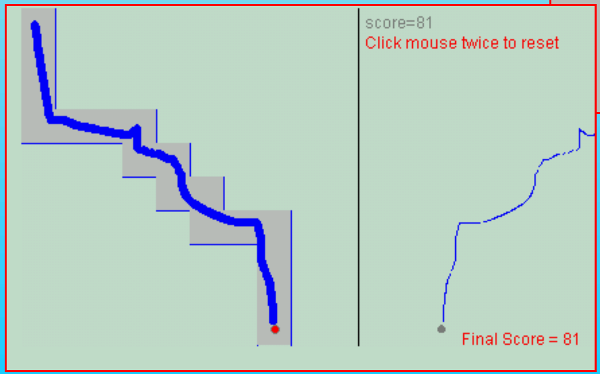
Java applet Simulations



- Mirror reflection
  - shows the location of an image in a plane mirror and handedness change upon reflection
- Inclined mirrors
  - shows the creation of multiple reflections around a circle centred on the intersection of the 2 inclined mirrors



- Mirror game

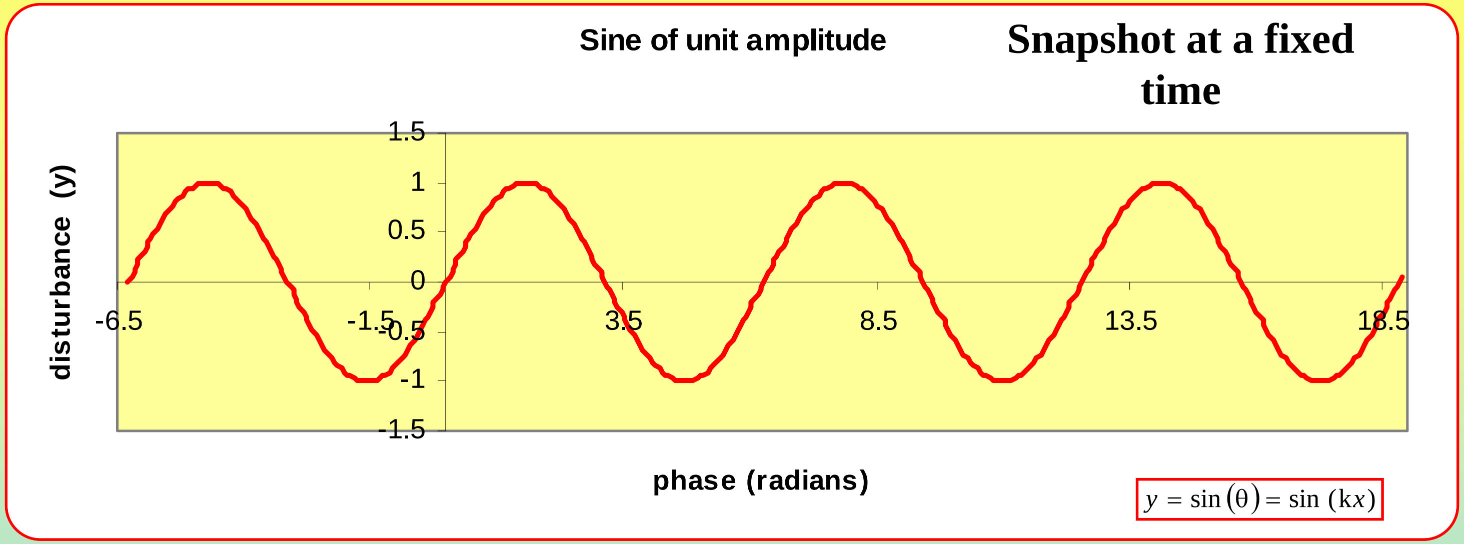


Waves

Joseph Fourier

- The phenomena of **interference**, **diffraction**, and **polarisation** are very naturally described in terms of waves
- Very common phenomena such as **straight-line propagation**, **refraction** and **reflection** can also be described in terms of waves
- **Fourier** (1768 - 1830) first realised that all complex wave forms could be described in terms of a sum of sine waves

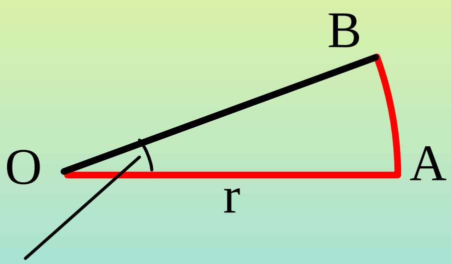
Snapshot of a sine wave



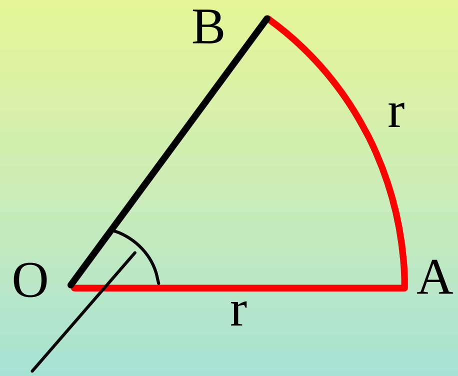
- A wave disturbance (y) propagates in one direction (x)
  - **amplitude**: midline - peak disturbance, A
  - **wavelength**: repeat distance, λ
  - **angular wavenumber**: 2π/λ, k measured in (rad) m^{-1}
  - **phase**: argument of the sine term, measured in radians. i.e. θ or (kx) above

Digression on radians

- Radians are the natural unit to use for measuring angles



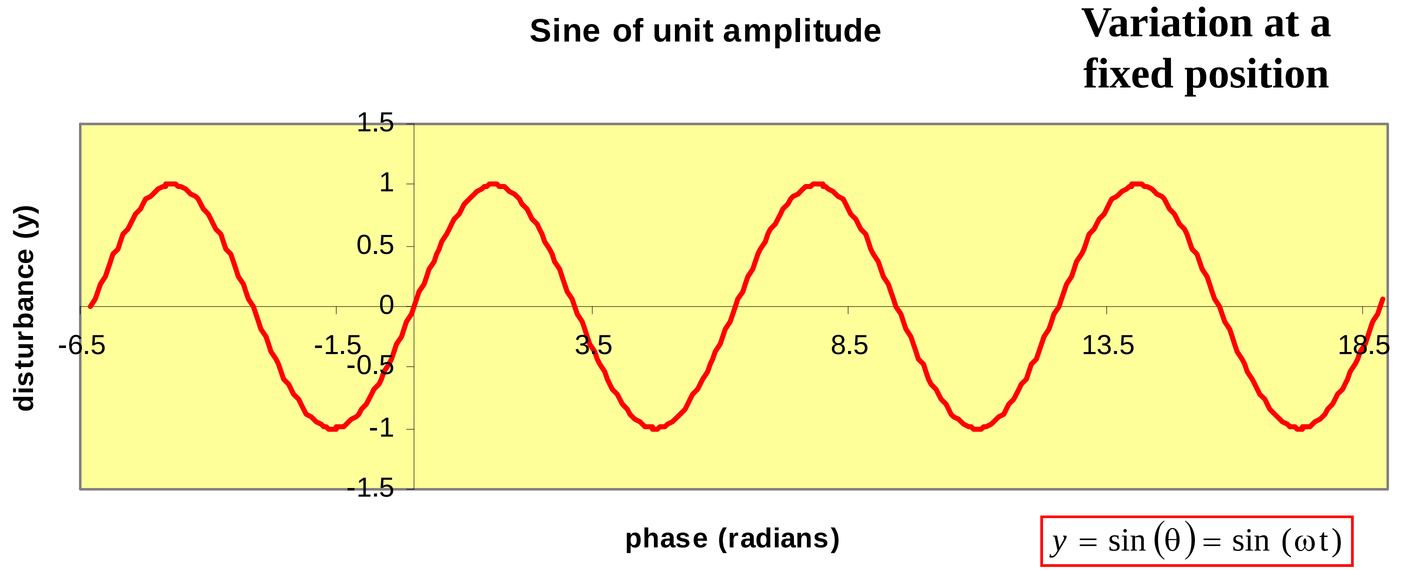
general angle



1 radian

- For a complete circle, 2π radians $\equiv 360^{o}$

Disturbance of a passing sine wave



$y = \sin(\theta) = \sin(\omega t)$

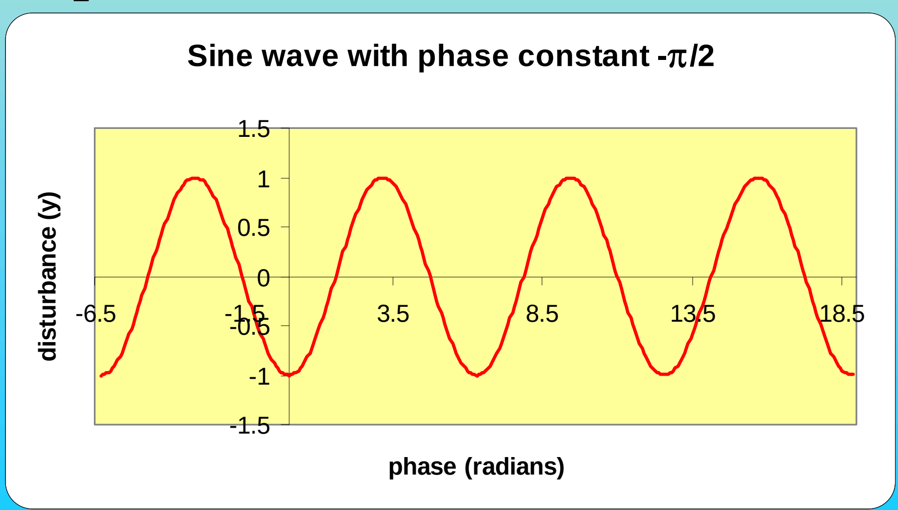
- Periodic displacement produced by a wave
  - ▶**period**: repeat time, T, measured in s
  - ▶**frequency**: no. of repetitions s^{-1}, f or ν in Hz
  - ▶**angular frequency**: 2πν, ω in rad s^{-1}

Working with sine waves

- Putting together the variations in space and time for a sine wave gives the relationship:

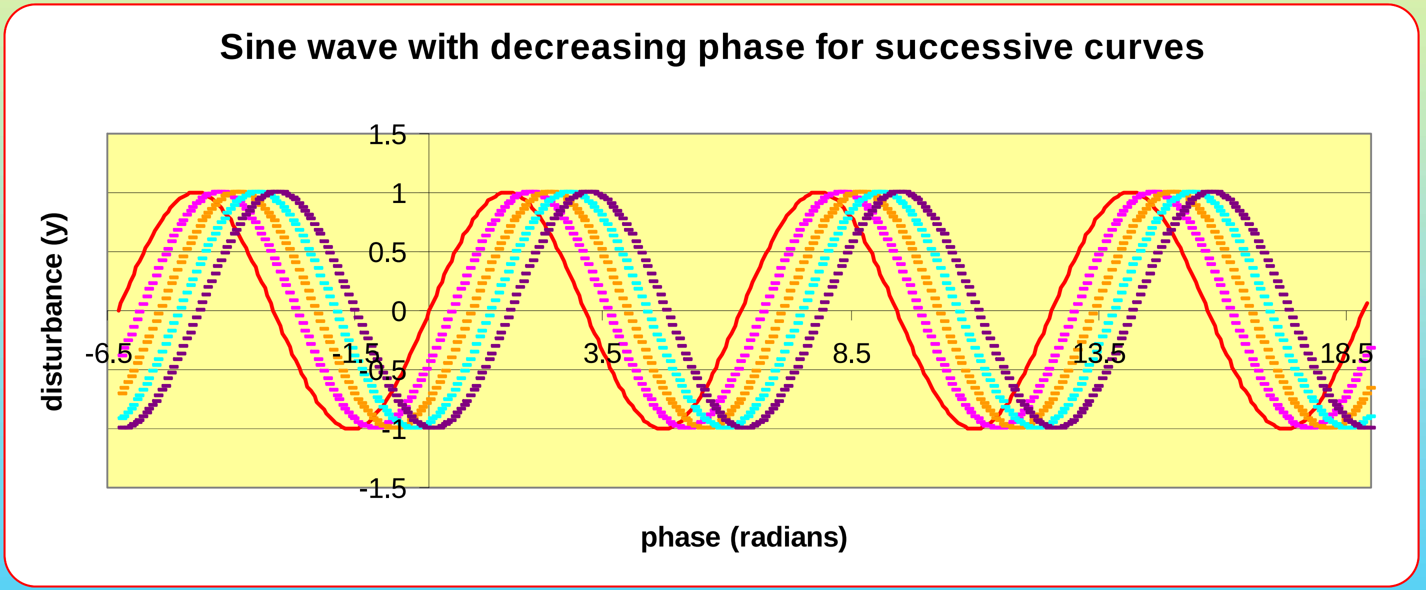
$$y = A\sin(kx - \omega t)\ .$$

- At a **fixed time**, t_1, this looks like $y = \sin(kx - \phi)$, where the constant $\phi = \omega t_1$
  - example plot:
    - $y = \sin(\theta - \pi/2)$
    - compared with $y = \sin(\theta)$, the trace has moved to the right



Successive sine waves of decreasing phase

- The phase of $y = \sin(kx - \omega t)$ decreases as time goes on



- Snapshots of the wave starting with the red curve show it moving to the right (in the +x direction)

The speed of a wave

- The speed of a wave is determined by the motion of a point of constant phase
  - represent the speed by v:

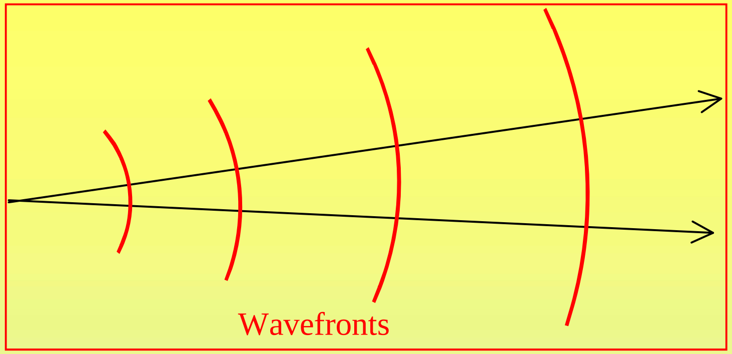
$$\mathrm{v} = \frac{\omega}{k} = \lambda f$$

.

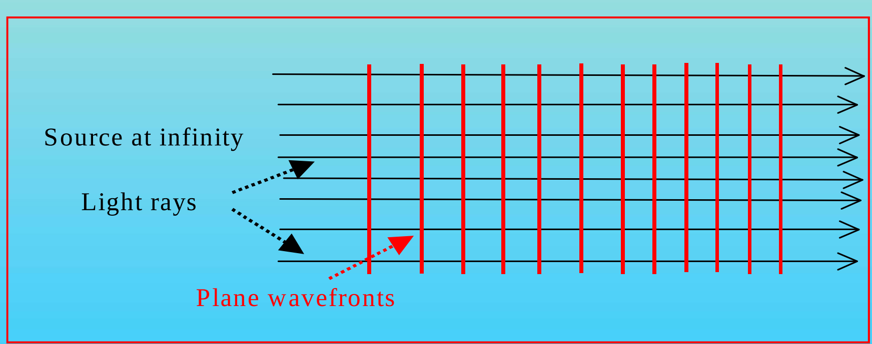
- The wavelength in vacuum: $$\lambda_{vac} = \frac{c}{f}$$
- The wavelength in a medium of refractive index *n* is less than the wavelength in vacuum

$$\lambda_{med} = \frac{\mathrm{v}}{f} = \frac{c}{nf} = \frac{\lambda_{vac}}{n}$$

Wavefronts



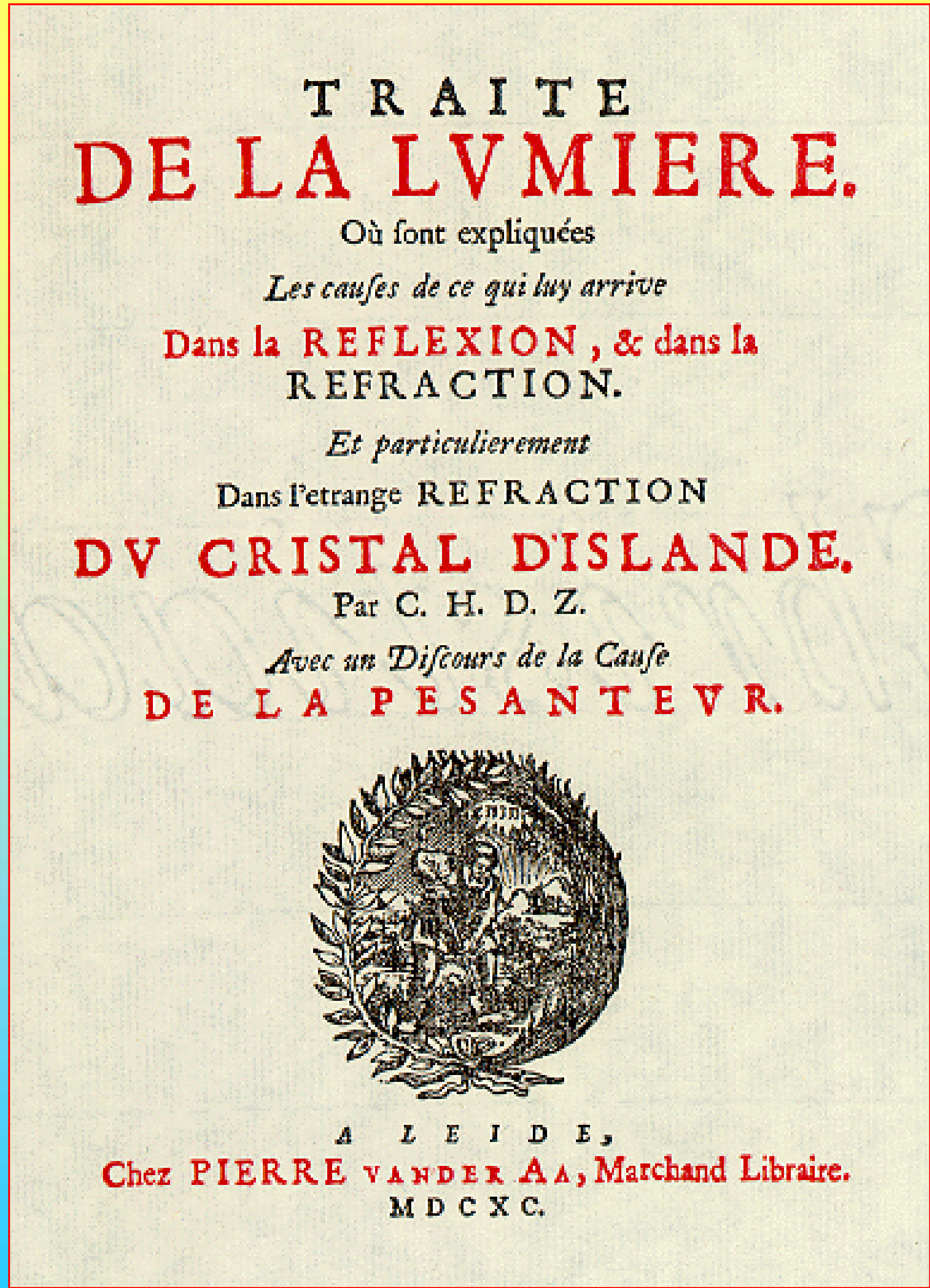
- Wavefronts are surfaces of constant phase
  - wavefronts show successive crests or troughs of a propagating wave
  - wavefronts from a point source expand as spheres
    - from a distant source, they are 'plane waves'



- Wavefronts are perpendicular to rays

Huygens' Principle

- Christiaan Huygens was able to explain how waves propagate in his far-sighted book *Treatise on Light*, published in 1690



TRAITE
DE LA LVMIERE.
Où font expliquées
Les caufes de ce qui luy arrive
Dans la REFLEXION, & dans la
REFRACTION.
Et particulierement
Dans l'etrange REFRACTION
DV CRISTAL D'ISLANDE.
Par C. H. D. Z.
Avec un Difcours de la Caufe
DE LA PESANTEVR.

A LEIDE,
Chez PIERRE VANDER AA, Marchand Libraire.
MDCXC.

Huygens' Principle

Christiaan Huygens

1629 – 1695

1) Take the wavefront at some time.

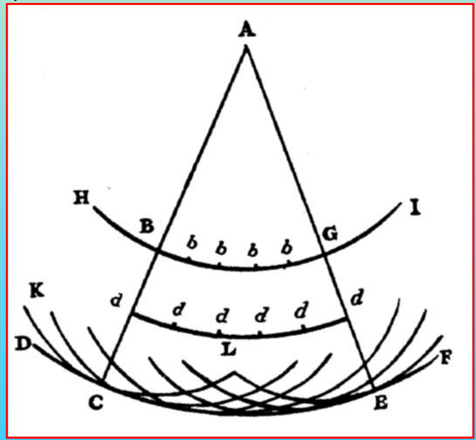
2) Treat each point on the wavefront as the origin of the subsequent disturbance.

3) Construct a sphere (circle) centred on each point to represent possible propagation of the disturbance in all directions in a little time.

4) Where the confusion of spheres (circles) overlap, the possible disturbances all come to nought

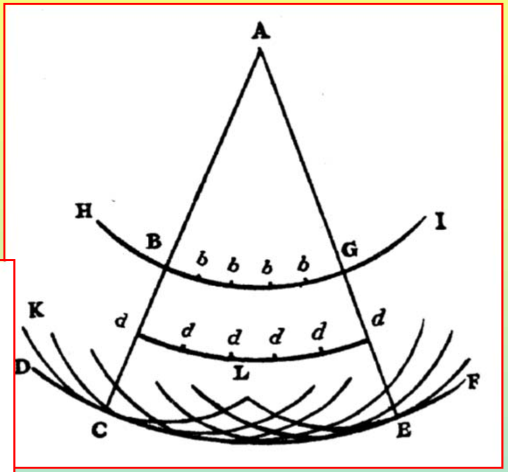
5) The common tangent of the system of spheres (circles) defines the new wavefront a little time later

6) Starting with the new wavefront, the construction goes back to step 2 to see where the wavefront reaches a little later on; and so on..



Prediction of Snell's law and law of reflection

- Huygens' own diagrams from his *Traité de la lumière*



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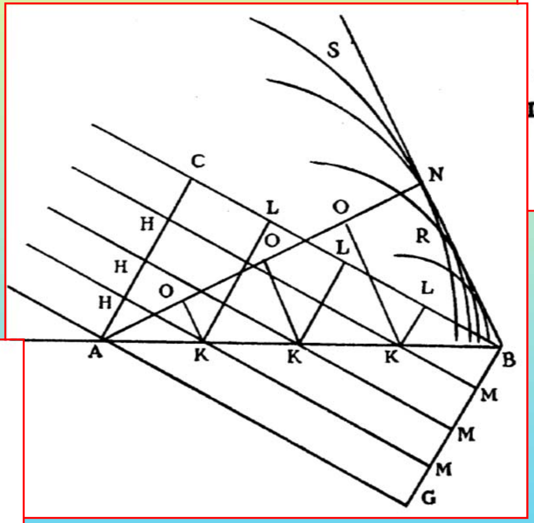
Straightline propagation



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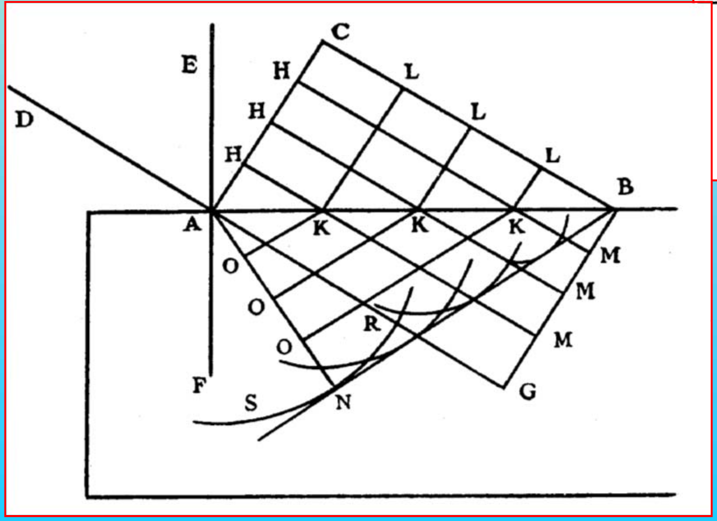
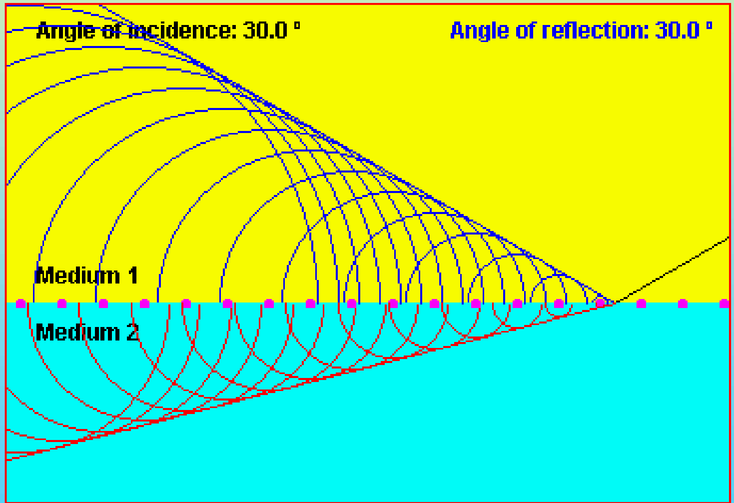
Reflection



← Refraction

Simulations of Huygens' principle

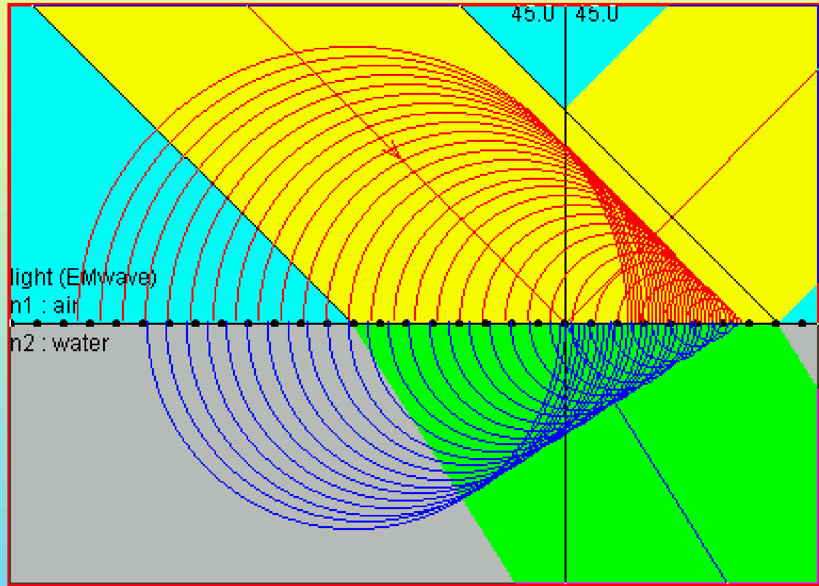
- Advancing waves
  both reflected and refracted



java courtesy :

http : home.a - city.de/walter.fendt/phe/huygenspr.htm

- Alternative view

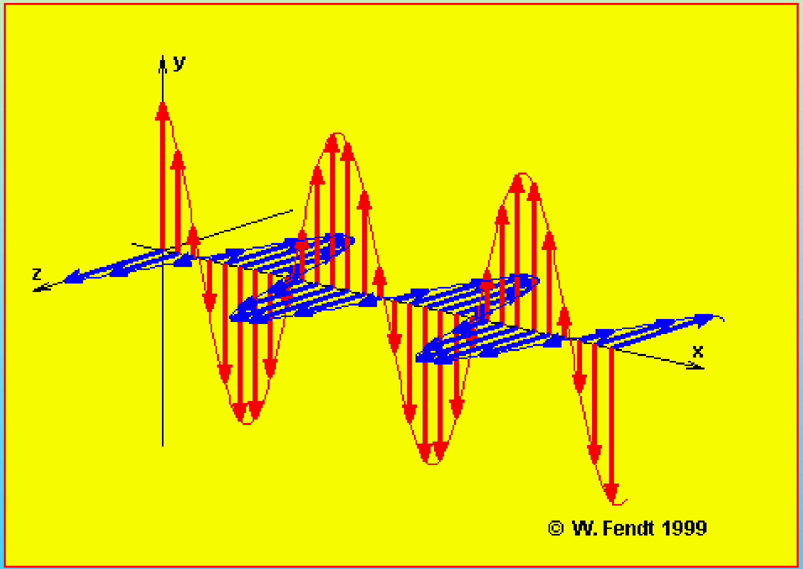


java courtesy :

http : //www.abdn.ac.uk/ntnujava/propagation/propagation.html

Electromagnetic waves

- Light consists of electromagnetic waves
- EM waves consist of periodic variations of electric field and corresponding variations of an accompanying magnetic field
  - in most ordinary materials, the electric field is at right angles to the direction of propagation
    - such waves are called *transverse*
  - the magnetic field is usually at right angles to the electric field, and is also transverse
- See the simulation

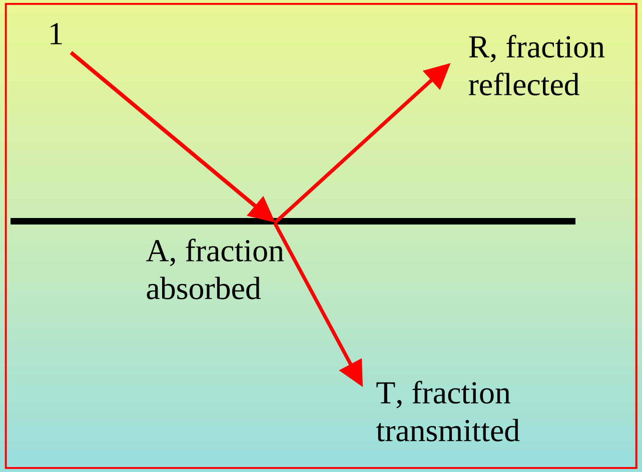


java courtesy :

http : //home.a - city.de/walter.fendt/emwave.htm

Fraction of light reflected & transmitted

- Conservation of energy tells us that all the incident energy goes into **reflection**, **absorption** or **transmission**

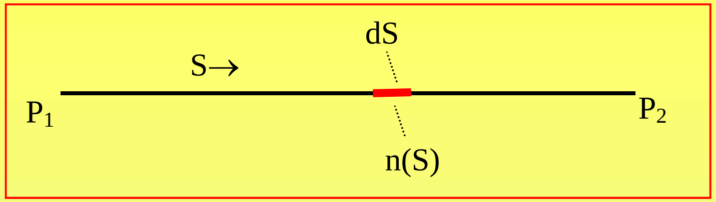


$$R + A + T = 1$$

- The fractions of light reflected and transmitted from a transparent surface were predicted by Fresnel in the early 19th century

Augustin Fresnel 1788 - 1827

The optical path length



$$d\,(OPL) = n(s)\,dS$$

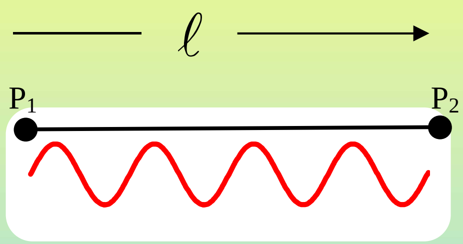
$$\therefore OPL = \int_{P_1}^{P_2} n(s)\,dS$$

- Definition
  - ▶the optical path length (OPL) in any small region is the physical path length multiplied by the refractive index
- In a medium, generally use the optical path length instead of the actual path length
  - ▶e.g. time of propagation, t

$$dt = \frac{dS}{\mathrm{v}(s)} = \frac{n(S)dS}{c} = \frac{d(OPL)}{c}$$

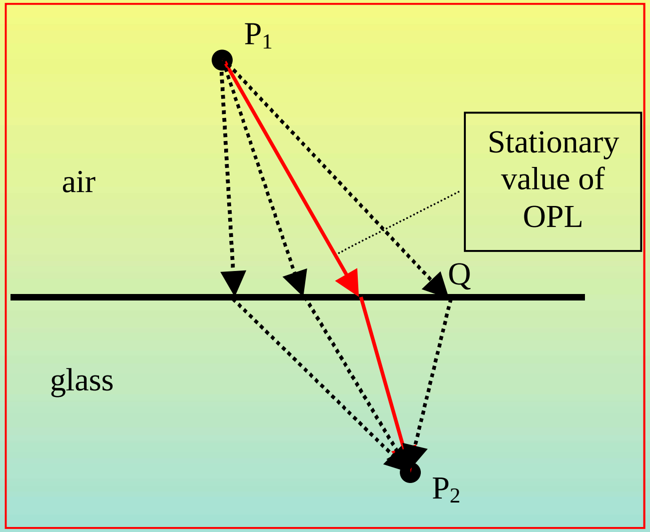
$$\therefore t = \frac{OPL}{c}$$

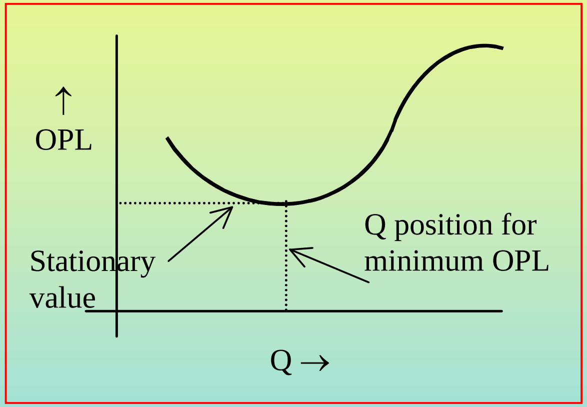
The number of wavelengths in a given path $P_1 \rightarrow P_2$



- If the path is in vacuum, then the number of wavelengths in the length P_1P_2 is ℓ/λ_{vac}
- If the path is in a medium, then the no. of wavelengths is: $\ell/\lambda_{medium} = OPL/\lambda_{vac}$
- The phase change along the path is therefore $2\pi \times OPL/\lambda_{vac} = OPL \times k_{vac}$
- These results will be useful later

Fermat's Principle





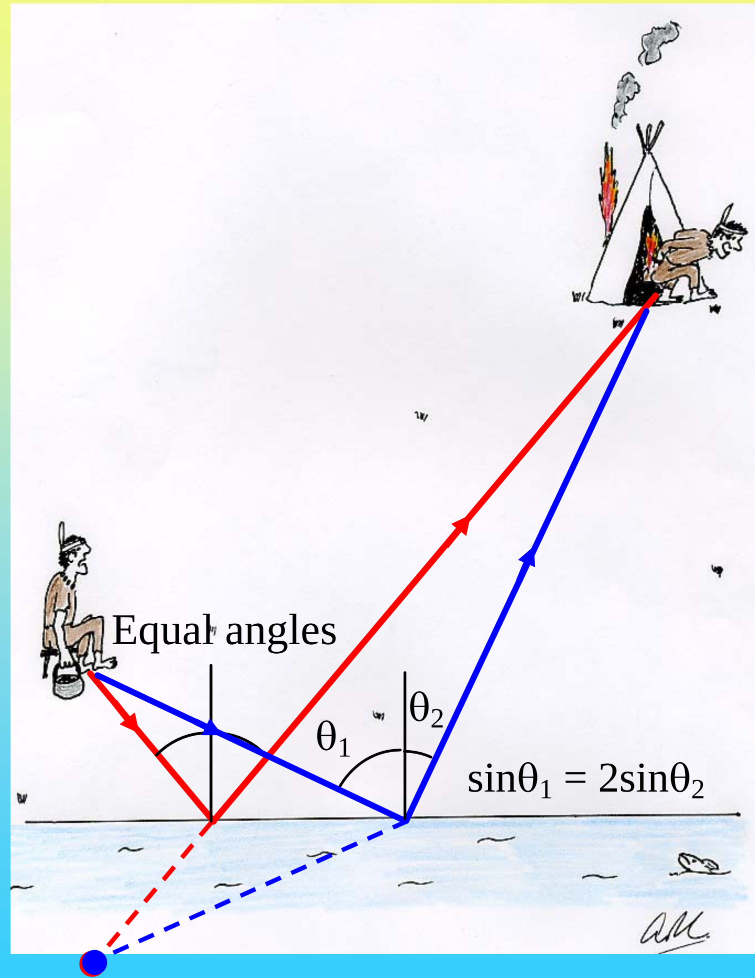
Pierre de Fermat

1601–1665

- Of all the geometrically possible paths that light could take between point P_1 and P_2, the actual path has a stationary value of the OPL
- Simulation 1; simulation 2

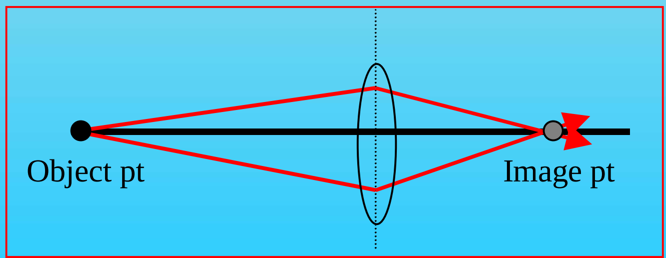
Digression

- The burning tepee problem
  - a brave working 20 m from a river sees his tepee on fire. It is 60 m downstream and 60 m from the river. What is his shortest path to take a bucket of water to the tepee?
    - Fermat's principle!
  - if he can only run at half speed carrying the bucket of water, is this the fastest path?
    - no!

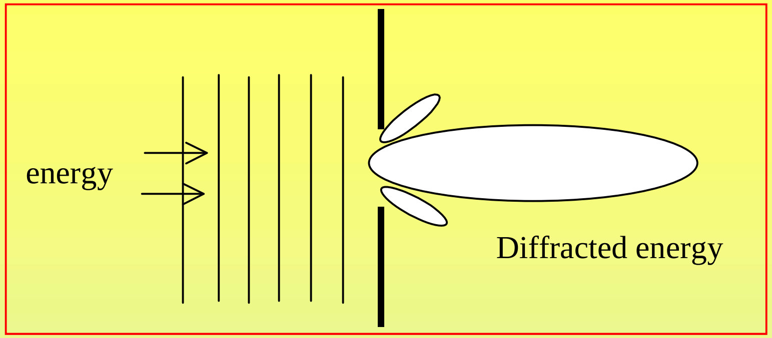


Implications of Fermat's Principle

- Fermat's principle can be used to deduce straight-line propagation, Snell's law and the law of reflection
- The reversibility of light rays
  - if a ray propagates from P_1 to P_2 along a particular path, then light goes from P_2 to P_1 along the reverse path
- All paths through a lens from object point to image point have the same OPL



Departures from Geometrical Optics



- **Diffraction**: the propagation of light around obstacles and the spreading out of light through apertures
- **Interference**: the cancellation or addition of light waves
- **Quantisation of illumination**: Light energy arrives in bundles called *photons*

Photons

- Photons are the central concept in **quantum optics**

Max Planck
1858 - 1947

- Photons have energy, E, that depends on the light's frequency, through Planck's constant, h

$$E = h\nu$$

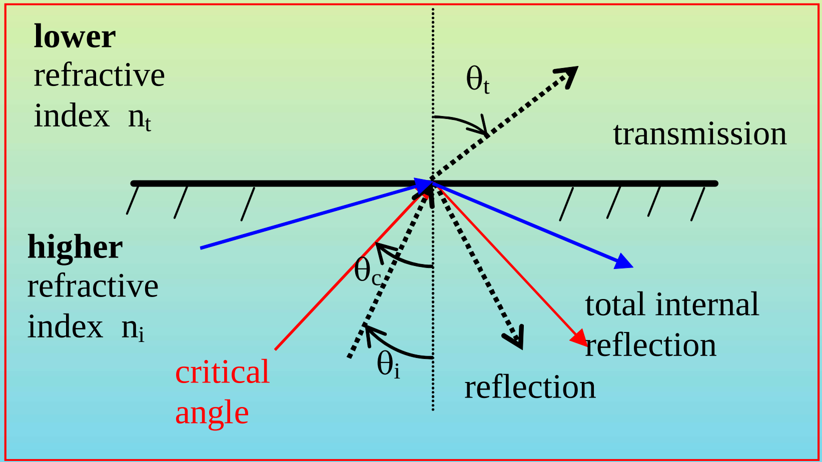
Louis de Broglie
1892 - 1987

- Photons have momentum, p, that depends on the wavelength of light

$$p = h/\lambda$$

Total internal reflection

- There is a progressive rise in the intensity of internal reflection with increasing angle of incidence θ_i
  - limit occurs when $\theta_t = 90^o$, *i.e.* $\sin \theta_t = 1.0$
  - the corresponding angle of incidence is known as the *critical angle* θ_c



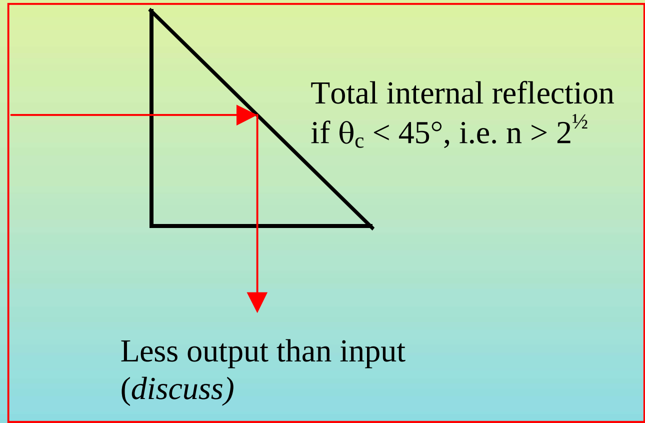
$$n_i \sin \theta_c = n_t \sin 90^o \quad \text{Snell's law}$$

$$\therefore \sin \theta_c = \frac{n_t}{n_i} = \frac{1}{n} \quad if \ \ n_t = 1$$

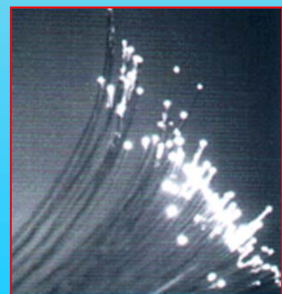
$$\therefore \quad \theta_c = \sin^{-1}(1/n) \quad n \text{ is the refractive index of the incident light medium}$$

Total internal reflection - 2

- Total internal reflection occurs for all angles of incidence $\geq \theta_c$

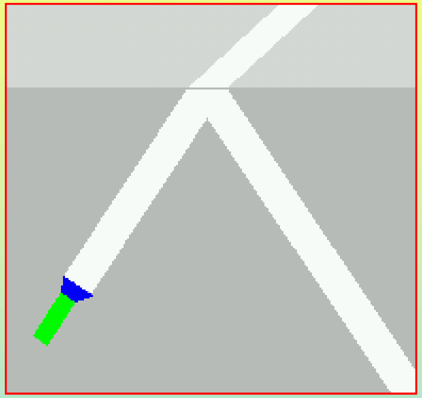


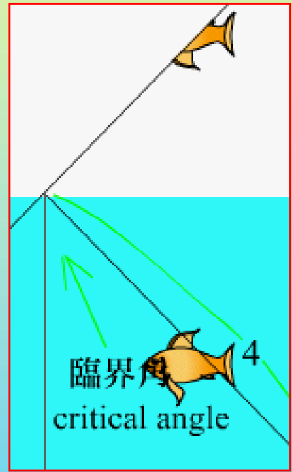
- Examples
  - reflecting prisms
  - fibre optics
  - light guides (illuminated fountains, motorway signs, etc.).



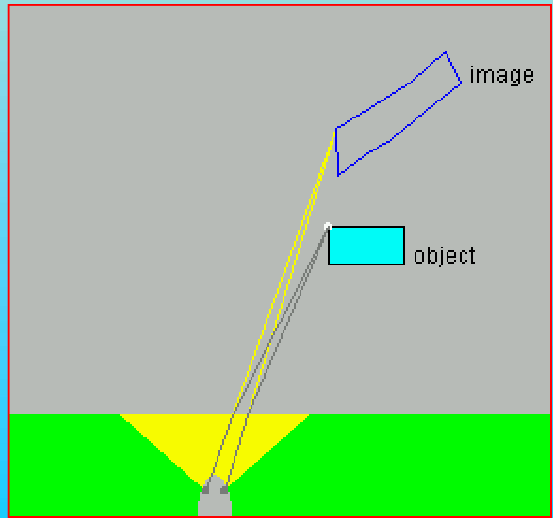
Simulations including total internal reflection

- Torchlight under water

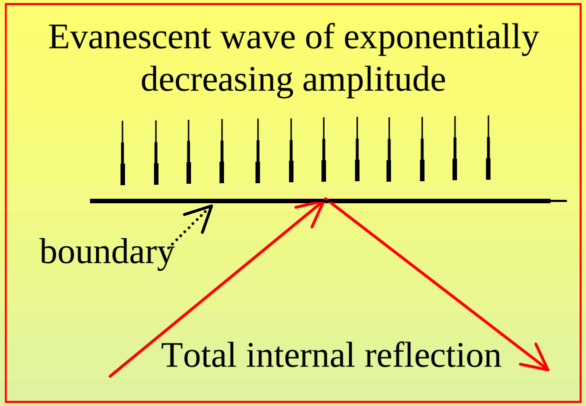




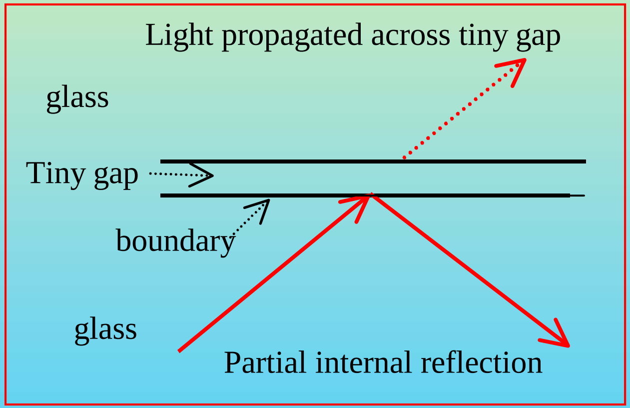
- Reflection of a fish
- Image seen by a fish



The Evanescent wave



- A phenomenon of ever increasing application
- Must the light wave be zero in the low refractive index medium?
  - not for insulating materials
- By creating a tiny gap between 2 media, you can *frustrate* total internal reflection and obtain a controlled amount of transmission into an adjacent material

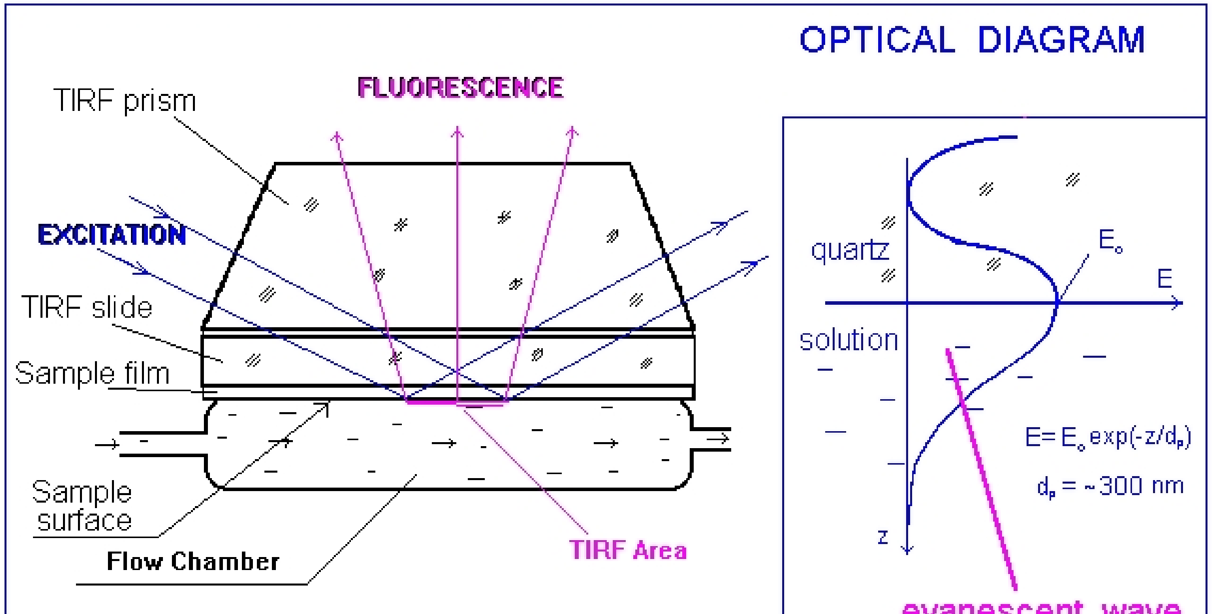


Evanescent wave application

- Total internal reflection fluorescence
- Detects very small concentrations of specific proteins, drugs, DNA etc.

- A sensor molecule binds with a protein coating the internal optically flat surface of flow tube
- Fluorescence of bound protein excited by evanesc. wave and detected

Total Internal Reflection Fluorescence Flow Cell



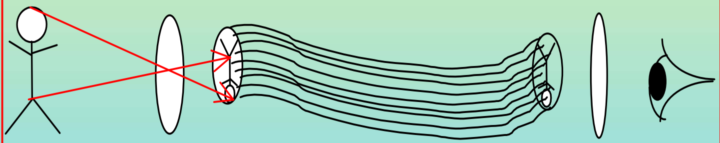
courtesy : www.bioelectrospec.com

Fibre optics

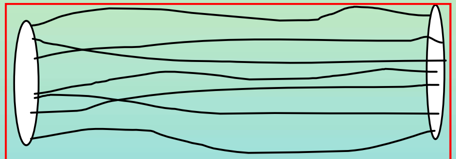
John Logie Baird

1888 - 1946

- Original patents to John Logie Baird in 1930s
  - fibre bundles can be coherent or incoherent

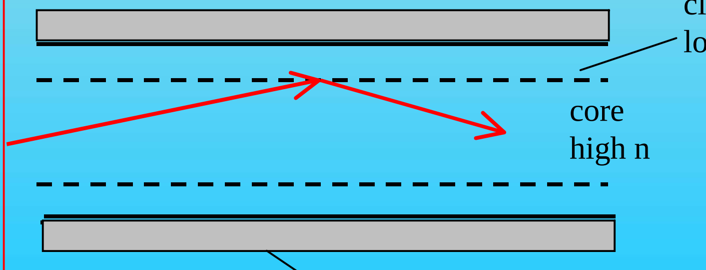


Coherent bundle



Incoherent bundle

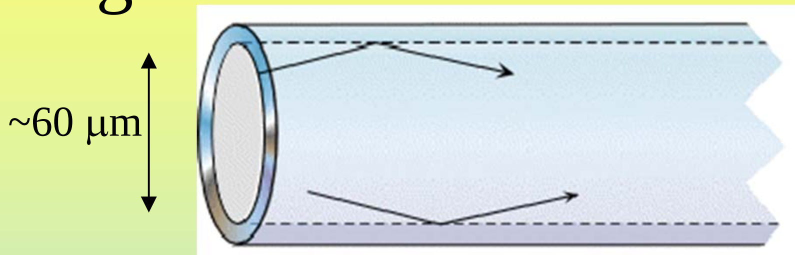
  - individual fibres have a structure like this



Single fibre

Fibre optic advantages

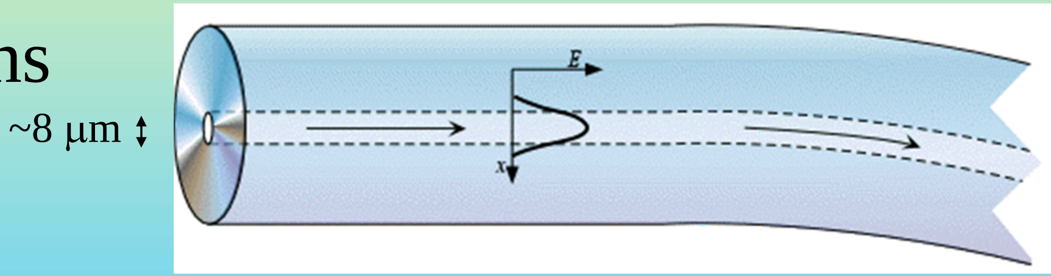
- Bundle for transmission of images
  - flexible
  - long
  - little loss
  - simple construction



Multimode fibre

Figs courtesy : www.cirl.com

- For communications
  - closed circuit
  - long-life
  - not subject to electrical interference
  - very high bandwidth (subject to refractive dispersion and propagation dispersion)
  - *disadvantage*: repeaters may be needed



Single mode fibre

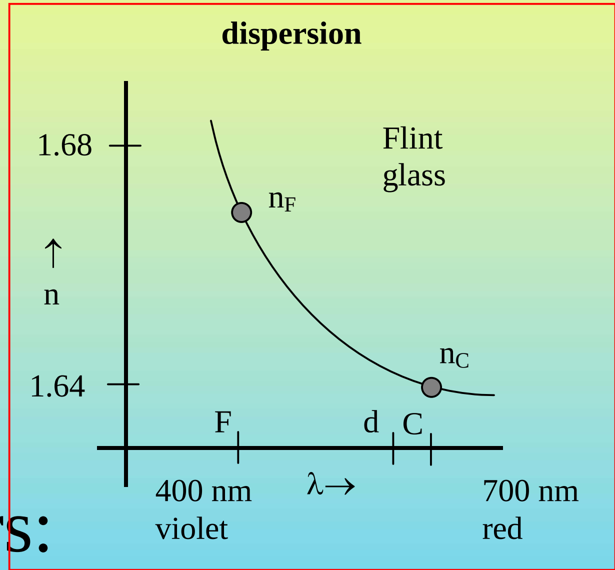
Dispersion

Augustin - Louis
Cauchy
1789 - 1857

- Variation of refractive index with wa
  - Cauchy's empirical formula

$$n_\lambda = n_0 + \frac{A}{\lambda^2}\left(+ \frac{B}{\lambda^4} + \ldots. \right)$$

  - there is not one universal curve for all materials
  - standard wavelengths are denoted by Fraunhofer's letters:



Fraunhofer letter	Origin	Wavelength nm
C	Red hydrogen	656.27
D	Na yellow	589.4
d	He yellow	587.56
F	Blue hydrogen	486.13

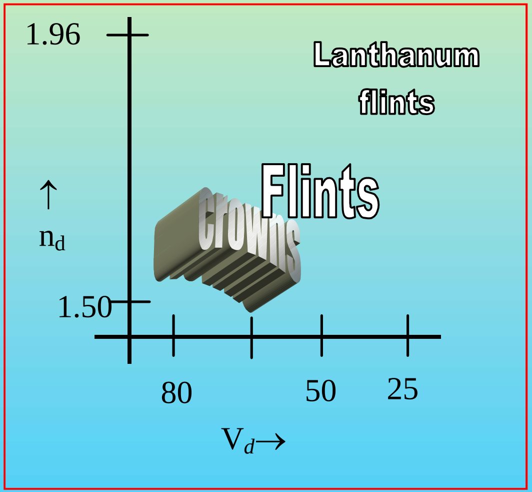
The Abbe number, V_d

- A single parameter to measure dispersion
  - the larger the dispersion, the smaller the Abbe number
  - optical glasses are displayed on an n_d/V_d graph
    - note the naming of glasses:

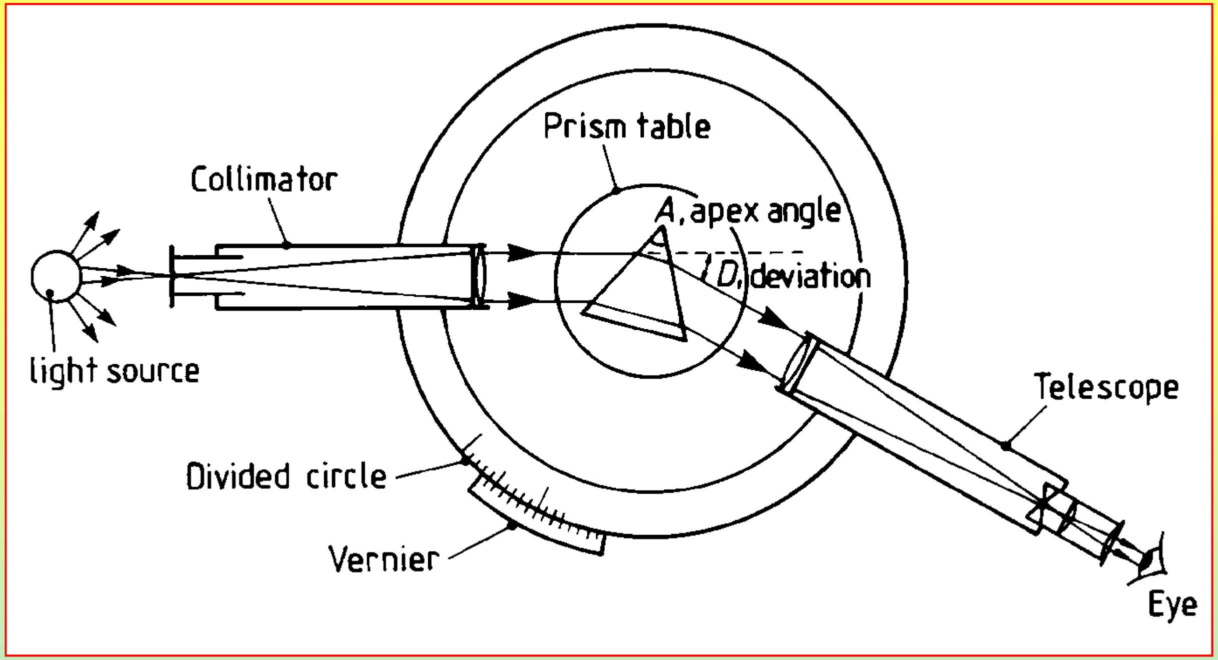
      e.g. BK7 517642 means

      $n_d = 1.517$; $V_d = 64.2$
  - from n_d and V_d you can calculate n_λ at all wavelengths
  - phenomena that depend on dispersion

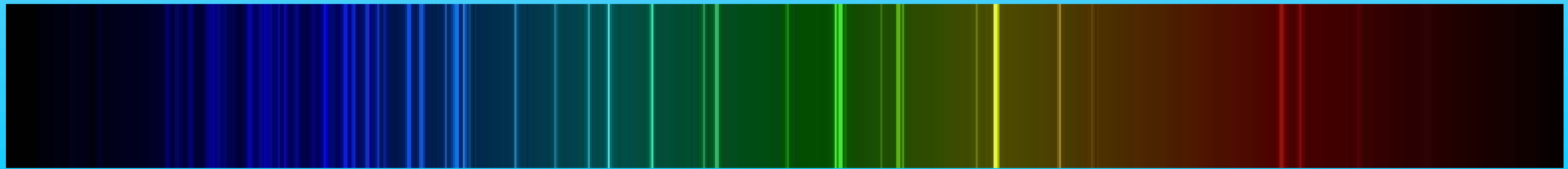
$$V_d = \frac{n_d - 1}{n_F - n_C}$$



The Spectrometer

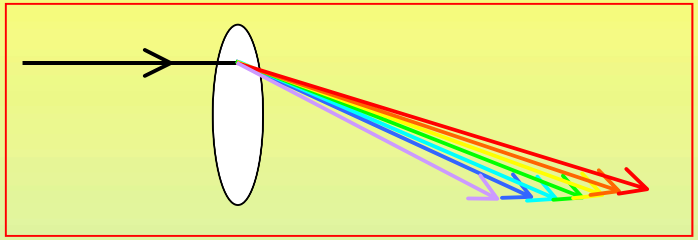


- Uses dispersion to show the spectrum of a light source
- Components are: the **slit**, **collimator**, **prism**, **telescope**, with various adjustments and scales
- Each frequency component of the spectrum appears as a **spectral line**

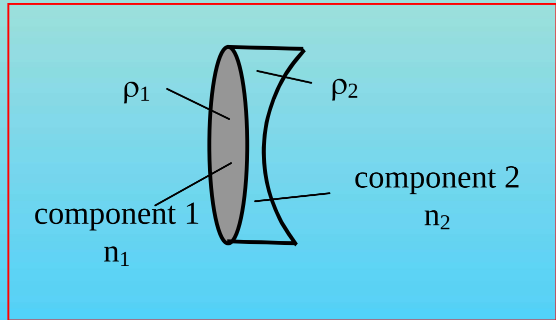


The achromatic doublet

- Unchecked dispersion will kill the performance of all lens based optical instruments

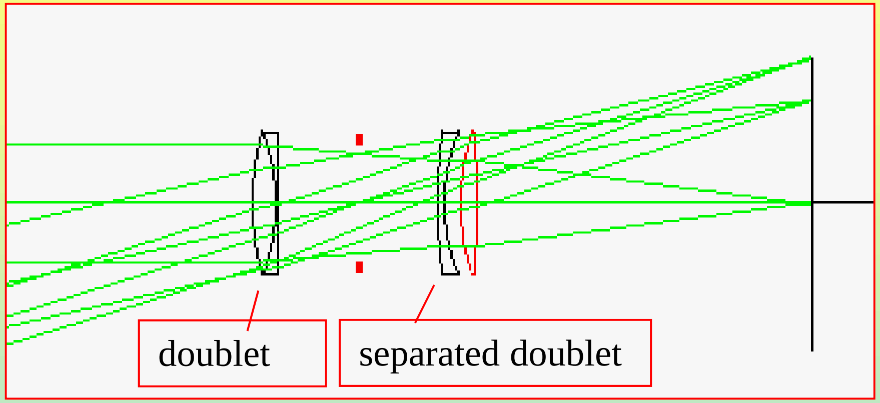


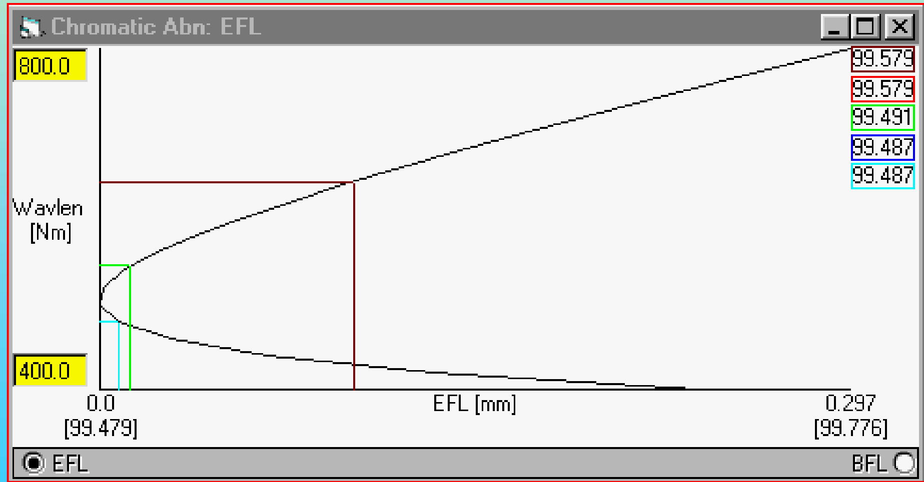
- The key to controlling the effect was found by John Dollond in 1758 - the **achromatic doublet**
  - the diverging component is made from a glass of higher dispersion
  - a weaker diverging component is able to cancel out the dispersion of the positive component without cancelling out its power



The achromatic doublet at work

- A 4-element camera lens looking at an object at ∞ off to left
- Calculated focal length of the lens for the spectrum of colours, shown vertically from 400 nm (violet) to 800 nm (near infra-red)





Diagrams using 'Winlens'