Cosmology 2 – Expanding horizons

Asking the big question

If you look back in history you'll see that people have always been keen to ask the big questions. Lack of facts hasn't been any impediment to providing an answer. Millennia ago, in the absence of much in the way of facts, people's imagination and speculation had a free reign to invent cosmologies. The most fantastic stories were concocted and, by constant repetition and assertion, ideas with no foundation in reality were raised to the status of fundamental truths and embedded in the rituals of society. Betrand Russell once advised his readers "*it is undesirable to believe in a proposition when there is no ground whatever for believing it to be true*". Wise advice from a wise man. In recent centuries, science has represented a quest to understand nature, using ideas that are firmly rooted in experiment and hence reality. Whether science has got it right or not, at least it has provided grounds for its beliefs. To put it crudely, science has asked nature questions rather than asking people questions. Science tackles understanding by working outwards from the minutiae of commonplace phenomena and building on that. One of the lessons of several centuries of science has been to recognise what are the big questions and which are the questions you are likely to make progress with.

Modern astronomy has developed for over four centuries now, building up a big body of knowledge about planets and stars, how they are born, how they work, how they are organised in space and develop in time, and about interstellar matter too. We know a huge amount more than our predecessors and are also more aware of what we don't know. Perhaps it's no surprise that today's cosmology is different from cosmology in the past because it is only in recent times that we've acquired a reasonable understanding of what's out there that needs explaining. You may think it's a bit arrogant of me to say this, since each generation probably thinks the same, only to be told by its successors that its understanding was primitive. However, it's certainly true that our knowledge of the universe is much greater now than it has ever been.

Why is our knowledge much better these days? Our new understanding has been based on observations in the 20th century and early 21st century using tools that simply weren't available to our predecessors. Tools such as big telescopes that can see further out into the Universe than ever before; the Hubble space telescope that can see without the impediment of the Earth's atmosphere; other satellites and space probes that can receive signals over a wide range of the electromagnetic spectrum that is absorbed by the Earth's atmosphere; modern photonic detectors like photomultipliers and CCD imagers that are far more sensitive than photographic plates; computer-based image processing that can isolate the images and signals we want from the ever present background noise in sensitive detectors, and other developments too.

Before you try to explain something, you need to know what it is you're trying to explain. It is only with the passing of the 20th century that we have accumulated enough observations to know what kind of universe we are trying to build a cosmology to explain. It's my view that philosophers, theologians, astrologers, astronomers, and others of earlier centuries who tried to devise cosmologies could not have got it right because they didn't know enough of the facts they were trying to explain. They also didn't know enough of the laws of nature to be able to apply them to the Universe as a whole. I'm not saying they shouldn't have tried. I am saying that cosmology as a discipline is littered with ideas that should now rightly be consigned to the wastepaper basket. Today's cosmology may in its turn be screwed up by our successors and thrown into the basket, but that's their perogative. What makes today's cosmology different is that it's based on many observations that earlier cosmologies were not. As the philosopher George Santanya put it as long ago as 1923 *It's a great advantage for a system of philosophy to be substantially true*.

Hubble's law

Hubble's law is an apparently simple law that says something very significant about the Universe as a whole. Hubble's law was found experimentally by Edwin Hubble, with some help from fellow astronomers of the time. The time was 1929. Hubble's law, as many of you may already know, links the distance away of a remote galaxy with the speed it is found to be moving away from us.

The law

This slide shows briefly the law we're talking about. How did Hubble arrive at his conclusion? It was the advent of big telescopes early in the 20th century that allowed Hubble and others to investigate distant galaxies. Hubble and his contemporaries in California were able to image very faint galaxies in the largest telescope then built, the 100 inch Mount Wilson telescope (whose main mirror is over 2.5 m across). To come up with his law he had to not only image distant galaxies but he needed to measure both the distance of a galaxy and its speed of motion independently. That is harder to do, as we'll see.

The evidence

The Milky Way is part of a cluster of galaxies called the **local group**. The local group is a few Mpc (megaparsecs) across. Analysing the motion of stars in our galaxy shows that the whole galaxy is rotating, though not all at the same angular rate. Stars at different distances from the centre are going at different speeds. The detail is a story for another day. The Andromeda galaxy is a comparably large galaxy to the Milky Way situated in our local group, at a distance of 770 kpc away. It, too, is rotating but on top of the rotation the galaxy is moving towards us. We know this because on average the spectral lines of light from the stars in the galaxy are Doppler shifted towards the blue by a measurable amount. Blue Doppler shift of a source of light means that the source is coming towards us. Red means it's receding. As we'll see in a minute or two, the light from most galaxies is observed to be red-shifted.

The red-shift

Red-shifted light would be quite hard to detect if it weren't for the characteristic spectral lines that are found on all starlight. Most of these lines are dark lines. They show very well on the Sun's spectrum, as you may have met in other courses. Dark lines are due to absorption of light in the outer regions of a star. They occur at wavelengths determined only by the element that is absorbing the light. Thus lines can be identified with particular elements present in a star, which is an enormously helpful piece of information for astronomers. The lines occur in patterns that are quite readily identified. Now, when a star, or indeed a galaxy of stars, is receding from us, all the spectral lines move towards the red end of the spectrum. Some already red lines may move off into the infra-red and some previously invisible ultraviolet lines may be shifted into the visible part of the spectrum. The slide shows what would happen to the four hydrogen lines in the visible part of the spectrum if their galactic source were moving away from us very quickly. That is how astronomers perceive the red shift. The parameter z is how it's measured.

Red-shift parameter z

For red-shifted light a spectral line emitted at a wavelength of λ_{em} is observed at a greater wavelength, λ_{obs}. A measure of the red-shift is given by the quantity z defined as:

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} . \tag{2.1}$$

In words, z is just the wavelength shift divided by the emitted wavelength. If the observed wavelength is longer than twice the emission wavelength, then z is greater than 1. The quantity z will come into our story later.

E.g. for one of the hydrogen lines, λ_{em} = 486 nm and λ_{obs} = 520 nm. Hence z = 34/486 = 0.07. λ_{em} is cyan in colour, whereas λ_{obs} appears in the green part of the spectrum. The symbol z is so closely associated with red-shift that a well-known catalogue of reliable red-shifts of galaxies is simply known as ZCAT and cosmologists all over the world know why. So do you, now!

Velocity of recession

Measuring z enables the velocity, v, of a galaxy to be deduced via the standard formula for the Doppler effect. For a velocity that is not too large the formula is simply:

$$v = cz, \tag{2.2}$$

where c is the velocity of light (= 3×10^5 km s^{-1}). For the example on the slide, z = 0.07 and this therefore corresponds to a recessional speed of 21,000 km s^{-1}, pretty fast but, looking ahead a bit, the speed expected for a galaxy about 300 Mpc distant from us.

[Since Special Relativity formed an earlier part of this course, I'll add here for background that the relativistic version of the formula above is $v = c\frac{(z+1)^2 - 1}{(z+1)^2 + 1}$. Using the relativistic formula for z = 0.07 gives a smaller factor to multiply c of 0.06756 instead of 0.07. The difference corresponds to a velocity that is smaller by 733 km s^{-1}, or 3.5%. For accurate work the relativistic formula needs to be used when $z > 0.01$ but to get a general picture of what's going on, just think of the recession velocity v = cz. Very distant objects in the Universe have z > 1. Since recession speeds of objects we can see can't be faster than the speed of light, there's no question that you need to use the relativistic formula for very distant galaxies.]

To come back to our story, the measurements on the Andromeda galaxy and other galaxies in our local group tell us that nearby galaxies are moving relative to each other. Such a motion of a galaxy is called its **peculiar motion** and can be as much as 600 km s^{-1}, which is fast by terrestrial standards but the vastness of space is such that even at this speed we can never see any change in the separation between galaxies, either year on year or even decade upon decade.

We have to work hard to see individual bright stars in galaxies further away than the Andromeda galaxy. From most galaxies further away we see the collective light from millions of stars in each pixel of our images. It becomes easier in fact to deduce whether the whole galaxy is moving towards or away from us. Observing beyond our local group, Hubble found that the dimmer a galaxy is, the greater its red-shift. That was the observation. He surmised that dim galaxies were, by and large, dim because they were further away and hence that the red-shift of a galaxy increased with its distance from us. Attributing the red-shift to the Doppler effect he deduced that the further a galaxy is from us the faster it is receding. In symbols:

$$v = \mathrm{H}_0 r \,, \tag{2.2}$$

where v is the speed of recession of a galaxy and r is its distance away. The constant of proportionality I've written as H_0 to be consistent with symbols we'll introduce later. H_0 is **Hubble's constant**. Hubble's constant is a central feature of modern cosmology. Hubble found the value for his constant as 550 km s^{-1} Mpc^{-1}. We now know that he got it wrong by a factor of some seven-and-a-half, for today's value is about 7.6 times smaller.

If you got the wrong answer in a physics lab by a factor of 7.5 you wouldn't get many marks for your result. Hubble, though, was no blunderer, though he was a bit of an optimist in announcing such a general law from the very modest amount of data that he had. In retrospect, there was a good reason for his mistake. The reason was that it is very difficult to determine the distance of far-off galaxies. You need to find the distance independently of the red-shift and that's not easy. How we find the distance of very far off objects is not a simple story. The bottom line is that we work outwards from terrestrial distances, which are defined in terms of the metre, to solar system distances, to astronomical distances of nearby stars and then outwards again to further stars, then to nearby galaxies and finally to distant galaxies. Hubble got his figure wrong because the errors in pushing out the distance scale this way had accumulated so that in his day the distances of far off objects were underestimated. Astronomers' maps in Hubble's time were a bit like those ancient maps you sometimes see of Europe. The Mediterranean regions are quite accurately drawn but the shapes and distances of Britain and other places in Northern Europe are all wrong. Hubble's data turned out to be even further wrong than the detail in ancient maps. Galaxies he deduced were 10 Mpc away are, by our current reckoning, more like 76 Mpc away. As recently as the 1990s, astronomical distances were revised outwards following on from the hugely successful Hipparcos distance survey of nearby stars. That is just another small example of how today's knowledge is better than the knowledge of the past.

Since Hubble announced his results in 1929, a lot of effort has been put in to trying to determine Hubble's constant. It's proved pretty difficult, just because of the problem of finding the distance of galaxies independently from their red-shift and for nearer galaxies, where we can find their distances more accurately, of allowing for any peculiar velocities they may have in their own group. In the 1990s, two completely separate approaches were made to try to firm up on the Hubble constant. One reason for taking the trouble is that the age of the Universe is given pretty directly by the Hubble constant, as we'll see soon. Everyone wants to know the age of the Universe. One approach was the Hubble Space Telescope Key Project that used a variety of methods to determine how intrinsically bright a galaxy is. Then, by comparing its observed brightness to its intrinsic brightness, its distance can be deduced independently of its red-shift.

Cepheid variables

The first distance measuring technique uses stars called Cepheid variables. I introduced Cepheid variables in the level 1 astronomy course. They are bright variable stars with characteristic light curves that vary over several days. Astronomers believe that once they have identified a Cepheid variable from its light curve, then they know its intrinsic brightness, its absolute brightness. The star becomes what's known as a 'standard candle', because candles were the traditional standard for defining visible illumination. You'll have heard of lights being measured in 'candle power', a unit of the illumination sent out by a light source in a specific direction. The modern version of the unit is called the 'candela'. Hence by comparing the observed brightness of a Cepheid variable with its standard brightness deduced from its light curve, then its possible to work out how far away it is, using little more than the inverse-square law of falling light intensity with increasing distance. This is one way of obtaining the crucial distances needed for Hubble's formula. Of course you need to know how far away a nearby Cepheid variable is for this technique. The nearest is the pole star, Polaris, whose distance has been determined by the Hipparcos survey to be 431 light years.

HUDF images galaxies with $z \approx 6$

Nowadays we can image galaxies far further away than Edwin Hubble could (more due to better light detectors than to larger telescopes). The Hubble Ultra Deep Field Image was an extension of the Hubble Deep Field project aimed at seeing how far back in time the Hubble telescope could record. An image of a tiny segment of sky where there appeared to be no obviously visible objects was imaged for a million seconds. The result, released in 2004, was a picture with several thousand galaxies many of which were earlier in the history of the Universe than any galaxies that had been seen before. Some of the most distant galaxies appeared only as red images, all that is left in the visible part of the spectrum of light originally emitted as ultra-violet.

Today's value of H_0

A second approach to finding Hubble's constant is completely different. It involves analysing the minute spatial fluctuations in the cosmic background radiation that you may have heard about but which I haven't yet mentioned in this course. Detail in these fluctuations is related to the age of the Universe and hence to the Hubble constant. Both results have converged to a value of:

$$H_0 = 71\ (+4, -3)\ \text{km s}^{-1}\ \text{Mpc}^{-1}\ . \qquad (2.3)$$

Because cosmologist have lived for much of the 20th century without having a really good idea of the size of Hubble's constant, most cosmology books introduce the parameter h defined as:

$$H_0 = h \times 100\ \text{km s}^{-1}\ \text{Mpc}^{-1}\ . \qquad (2.4)$$

h is a simple (dimensionless) number whose currently accepted value is close to 0.71. We'll meet this constant h several times in later lectures.

How far does a galaxy have to be away before its red-shift is mainly due to the Hubble flow, the word used to describe the increasing recessional velocity of a galaxy with distance? A cluster of about 1000 galaxies beyond our local cluster is in the constellation of Virgo, unsurprisingly called the Virgo cluster. It is about 17 Mpc away. The galaxies in this cluster have their peculiar motions but on average the whole cluster has a red-shift as if it were receding at about 1200 km s^{-1}. A yet further cluster is the Coma cluster, about 100 Mpc away. It appears to be receding at a speed of 7200 km s^{-1}. So by 100 Mpc almost all the red-shift is due to the Hubble flow. In addition, there are some 10,000 galaxies in the Coma cluster so it is not hard these days to measure enough to average out the effects of peculiar velocities.

Expansion of the Universe

Hubble's law implies that the Universe is expanding. Hubble's evidence was so clear that it persuaded Einstein to change his mind on the topic. If the only significant force on a large scale in the Universe is the attractive force of gravity, how come galaxies are all fleeing from each other? It certainly isn't an obvious state of affairs. Prior to the work of Hubble and a few of his contemporaries, astronomers favoured a steady universe for aesthetic reasons, or at least by application of the principle of adequate simplicity. The theorists tried to build this in to their conceptual models. It's actually quite hard to produce a steady Universe theoretically and even Einstein had to adjust his equations of General Relativity, the theory of gravity, space and time that he had published in several stages up to 1916, to make this possible. I'll say more about this later. The theoretical possibility of an expanding Universe was pointed out very soon after Einstein published his complete theory of relativity by a Dutchman Willem de Sitter (1872 – 1934). In the 1920s a whole range of expanding Universe possibilities were uncovered by the Russian mathematician Alexander Friedmann (1888 - 1925). Friedmann realised that if any of these corresponded to reality then there was a time in the past when the Universe must have begun. The words Big Bang hadn't been thought of then but in a sense Friedmann originated the idea. Einstein didn't like this concept and is said to have used his very considerable reputation to belittle the work of Friedmann. He was sufficiently effective on this count that Friedmann's theoretical work wasn't followed up until much later.

The Hubble time and Hubble radius

Before we look in more detail at the expansion of the Universe, notice that Hubble's law suggests that the Universe has a finite age and the observable universe is a finite width.

If all the galaxies have been flying apart since the Universe began then a simple question to ask is *how long would it take for a point to recede from us by 1 Mpc if it were moving at 71 km s^{-1}*? Your calculator will soon produce the answer: 13.8×10^9 yr. This of course assumes a constant recession speed. You can put the question more generally and ask for the time T_H a galaxy moving at speed v takes to reach a distance r. It's the same question and simple maths says that T_H is given by:

$$T_H = r/v = 1/H_0 = 9.78\, h^{-1} \text{ billion years} = 13.8 \times 10^9 \text{ yr} . \qquad (2.5)$$

T_H is called **the Hubble time** and isn't far from the age of the Universe. It's not exactly so because even today's simple cosmology doesn't predict a constant recession speed independent of time. The calculation is over-simplified but it does give the idea of how directly Hubble's constant and the age of the Universe are connected. In fact due to a cancellation of different influences the answer is very close to today's figure of 13.7 billion years for the age of the Universe.

How far can light travel in the Hubble time? The answer is clearly c/H_0 and is given the symbol r_H. r_H is known as **the Hubble radius**. Of course it is just 13.8×10^9 light years, which is equivalent to 4200 Mpc. Moreover, using Hubble's law stated above, the distance at which a Galaxy will be moving at the speed of light is just r_H. Hubble's law doesn't include any restriction on a galaxy's speed and hence isn't consistent with the Theory of Special Relativity, which tells us that no massive body can move faster than the speed of light. However, just putting $v = c$, the speed of light, in Hubble's law gives:

$$r_H = c/H_0 = 3000\ h^{-1}\ \text{Mpc} = 4200\ \text{Mpc}\ . \qquad (2.6)$$

The Hubble radius is a natural scale factor for the universe. You can ask some questions that can be answered by modern cosmology in terms of the Hubble radius. The first is *How far are the most distant objects we could see now if technology were no impediment?* This is called the **particle horizon**. Because of the expanding universe, objects that we see now a very long way from us weren't so far away when their light was emitted. Hence to answer this question you need to know how the size of the Universe has changed with time. This in turn depends on such factors as the density of matter and radiation that are actually in the Universe. Interestingly enough, we have only just found reasonable values for these quantities, as I'll say in a few lectures time, and hence the answer I'm about to give is a very recent figure. In addition, we can't see anything further back in time than the Big Bang, 13.7 billion years ago. So what is the consequence of these two ideas? The answer is $3.38r_H$, or 14,300 Mpc. That's it. The visible universe doesn't extend beyond about 14300 Mpc (14.3 Gpc) as measured in our epoch. At this distance we would be looking back to the Big Bang, or near enough, looking back at particles created when the Universe as we know it was created – or would be looking back if we had the technology to see this far. This distance is more than 13.7 billion light years because it represents how far the most distant objects are away now. What we might see of them is light that left 13.7 billion years ago and the Universe has expanded since then so they are a different distance away from us now than they were then. We can't see back this far in time or distance but in practice our modern technology does quite well, for the most distant astrophysical objects we can see are at about 8 Gpc, or 8000 Mpc. If modern cosmology is right, even the best technology in future won't let our descendants see nearer the Big Bang than about 300,000 years, for the early universe was opaque. I'm coming to this later. The microwave background radiation that we measure today and which I'll discuss later in some detail is effectively the universe at 14 Gpc.

It's hard to grasp how enormous the Hubble radius of about 4000 Mpc is. I'm reminded of the description in the 'Hitchhikers Guide to the Galaxy', where the author is rightly struggling for words: *Space is big. Really big. You just won't believe how vastly, hugely, mind bogglingly big it is*. Indeed.

Here are two other questions that can now be answered. *Will we be able to see further in the future?* to which the answer is 'yes, but never more than 4.5 r_H or 19 Gpc. Secondly, *given that distant galaxies are receding from us ever faster, is there a distance beyond which we can never communicate because our signals will never get there?* The answer to this is also 'yes' and the distance is surprisingly close, just $1.2r_H$ or 4.74 Gpc. Cosmologists call this the **outward limit of reachability**. Beyond this distance, for a bit we can see them (distant objects in space) but we'll never be able to communicate with them. Funny place, the Universe.

The other fantastic bit of the story is that the Universe hasn't always been so big. That's what Hubble's constant is telling us. Stephen Hawking's sequel to a 'A Brief History of Time' was called 'The Universe in a Nutshell'. At one time the entire Universe was as big as a nutshell now is. Distinguishing science fact from science fiction certainly isn't a matter of just selecting what 'common sense' would have us believe. Science fact can give science fiction a good run for its money in the extra-ordinary stakes.

Meaning of the expanding Universe

There's a very obvious temptation to think that if everything is expanding away from us in all directions then surely that's good evidence that we are at the centre of the Universe. That conclusion isn't true. An analogy often used is to imagine a large currant loaf that has just been made with yeast. As the loaf warms and expands in all directions, every current moves away from every other current. If you imagined yourself sitting on a current and tracking the movement of other currents then, ignoring the ones near the edge of the loaf, it wouldn't matter which one you chose as a reference, your view of the other currents would be of them all expanding away from you. None would be coming towards you.

Perhaps an easier analogy to draw is to imagine that you live in a two-dimensional world on the surface of a large sphere. You measure all distances in your world along the surface of the sphere. Unknown to you at first, your sphere is being inflated. As you watch other spots, you'll see each spot moves away from you. From the outside it is easy to see that every spot moves away from every other spot. No spot is central. Indeed the viewpoint of your universe from any spot is the same and the cosmological principle will apply in the 2D world of your sphere.

The bottom line is that in an expanding Universe obeying the cosmological principle you would expect every point to be moving away from every other point, no matter where you observe from. Experimentally, the Hubble constant is found to be the same in all directions, as near as can be told. This is quite strong supporting evidence for the cosmological principle.

Local matters

Hold on a minute – surely that means that I'm expanding as I sit here, albeit by a very small amount, and the Earth's orbit is expanding, and so on? That's **not** what the expansion of the Universe means. Expansion is a property of the Universe at large, the result of the average density of the Universe. The expansion of the Universe does not apply to locally gravitationally bound objects, or to molecularly bound objects such as the material around us. It doesn't apply to us, to the solar system, to our galaxy or even to our local system. These are but specks of dust in the Universe, specks of dust that are held together by their own internal forces without participating in the expansion. To make another analogy, when a balloon of gas expands as you hold it in front of a fire, it is only the space between the molecules that expands significantly, not the molecules themselves.

Another way of putting the argument is to say that expansion of the Universe is noticeable over large volumes where the average density of matter is comparable with the average density of matter in the Universe. What is that? Well, to get a rough idea suppose that our galaxy and its surroundings contains 10^{11} solar masses (often written $M_\odot$) within a cube of side 1 Mpc. Putting in the figures, $M_\odot \approx 10^{30}$ kg and 1 Mpc $\approx 3 \times 10^{22}$ m gives an estimate for the density of matter in the Universe as ~ 10^{-26} kg m^{-3}. This is the same as a few protons per m^{-3} or about 10^{-29} times smaller than the density of you and I. The cosmologists argue that the mutual attraction of the matter we are made of is so big because our density is so much bigger than that of the Universe at large that we experience no effect from the expansion of the Universe. This result can be made more rigorous by General Relativity.

The cosmic scale factor

Suppose we set up a grid to measure where galaxies are in the Universe. The vector **r** will measure in 3 dimensions the position of a galaxy. After some (long) time *t* the distances on the grid will all be bigger by a factor representing the expansion of the Universe. It makes sense to describe **r** by two quantities, one that measures its position on a grid (this will be called **x)** and a second that takes into account the expansion of the grid (this will be written a(t)). Hence:

$$\mathbf{r} = a(t)\,\mathbf{x} \qquad \mathbf{(2.7)}$$

a(t) is called the **cosmic scale factor** or a name like 'the scale factor of the Universe'. It measures the expansion rate of the Universe with time. The accompanying slide shows what a(t) represents in pictorial form. The coordinate **x** is often called the **co-moving coordinate** because the grid expands with the expansion of the Universe. The figure that I quoted earlier for the size of the visible Universe of 14.3 Gpc is its size in co-moving coordinates.

Expansion velocity

Remember that Hubble's constant represents the velocity of galaxies **ignoring their particular velocities** that describe local motion. Ignoring local motion means that on our giant grid, the x values, the co-ordinates of the galaxies, are constant. Hence any observed velocity that represents a change in **r** is due to a change in a(t). There is a well-known notation introduced into physics by Isaac Newton of representing the rate of change of a quantity by putting a dot over it. Thus $\dot{\mathbf{r}}$ represents just the rate of change of **r**, which is nothing other than the velocity of a galaxy. Using this notation, which you'll see used in cosmological textbooks,

$$v = \dot{r} = \dot{a}(t)\,x = \dot{a}(t)\,r / a(t)\,. \qquad (2.8)$$

Now if you compare this with the definition of Hubble's constant you'll see that, since v = H r, then:

$$H = \frac{\dot{a}(t)}{a(t)}\,. \qquad (2.9)$$

Hubble's constant is given by the fractional rate of expansion of the Universe. You'll see that Hubble's constant in general depends on time and so would be better written H(t). Sometimes H(t) is called the Hubble parameter. It is a constant in space rather than time. You might expect that as gravity slowed the expansion of the Universe by the mutual attraction of all the mass in the Universe then H(t) would get smaller with time. We'll talk about this later. All we can do experimentally is to measure its value now. Using '0' to denote time 'now', everyone writes today's Hubble constant as:

$$H_0 = \frac{\dot{a}_0}{a_0} \ . \tag{2.10}$$

This relates the Hubble constant (now) to the rate of change of the cosmic scale factor (now). You can see why the earlier calculation of the Hubble time wouldn't produce quite the right figure for the age of the Universe, if H(t) depends on time. That calculation assumed that the recession velocity remained constant during the expansion of the Universe.

The cosmological red-shift

Although I introduced the red-shift of distant galaxies as a Doppler effect due to the velocity of distant galaxies away from us, and so did Edwin Hubble, now we have a picture of the expanding Universe, the interpretation of the red-shift is different from the normal Doppler effect. The cosmological red-shift is actually due to the stretching of the space between us and the source during the journey time of the light, rather than due to the effect of the speed of galaxies through a fixed space. As you can see above, the effect comes from the change in the cosmic scale factor and not from motion of a galaxy from one place in the Universe to another.

For example, suppose light was emitted from a galaxy when the Universe was half the size it now is. If that light has only just reached us then its wavelength has doubled along with the doubling in size of the Universe. We see a red-shift factor of z = 1 [remember that $z = (\lambda_{obs} - \lambda_{em})/\lambda_{em} = (2 - 1)/1 = 1$].

In general, the wavelength changes in proportion to the scale factor of the Universe and hence you can see that:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} \ . \tag{2.11a}$$

Applied to light received 'now', then this becomes:

$$1 + z = a_0/a(t_{em}). \tag{2.11b}$$

Cosmologists often turn this relationship around and use z as a measure of the size of the Universe when the light was emitted by a distant object. Remember that a(t) is the cosmic scale factor. What they are doing is using the relationship in the form of:

$$\frac{a(t_{em})}{a_0} = \frac{1}{1+z} \ . \tag{2.12}$$

For example, some pretty distant objects have been detected with z = 6, as shown on an earlier slide. We are therefore looking at these objects in the Universe when the Universe was $1/7^{th}$ of the size it is now and close to $1/7^{th}$ of the age it is now. In short, distant galaxies show us the Universe as it was at a distant epoch in time.

All this just says that there is a very close connection between red-shift, which astronomers can measure relatively easily, and the cosmic scale factor. Measured from the time light was emitted, the red-shift parameter starts at 0 and increases whereas the cosmic scale factor starts at 1 and increases; hence the relationship between a and $z + 1$.

One consequence of this is that we do indeed have a hope of being able to have a cosmology that correctly accounts for the evolution of the Universe, because we can observe some of the Universe at much earlier times. The cosmic microwave background that I'm going to talk about later is red-shifted radiation with $z \approx 1000$ and is a view of the Universe even before any galaxies were formed.

Olbers' paradox

I'm going to finish this section with a topic that you might think has nothing to do with cosmology but in fact it does. Thanks to a publication in 1823, Wilhelm Olbers (1758 – 1840) has his name associated with a question that had occurred to others before him: *why is the sky dark at night?*

"*Because the Sun has set*", you may well reply, but that answer alone is not good enough. If the Universe is big enough, in whatever direction you look your line of sight should come to a star. It's true that a single star is seen as dimmer the further way it is but there is more space further away and hence more stars at a given distance. The two effects just balance out. If the Sun represents a typical star, then the entire celestial sphere should be covered with light of comparable brightness. That argument is correct if the assumption of an infinite, static, universe filled with stars is correct. Since the sky at night isn't as bright as the Sun, then we know that something is wrong with the assumption. Indeed, since we all know the sky is a lot darker at night than the Sun, then there is a lot wrong with the argument. What is it?

Olbers' explanation was that interstellar dust absorbed light from distant stars so that distant stars gradually greyed out. If the dust is uniformly distributed throughout the universe then you can show that the light from a distant star should decrease exponentially with the distance away of the star. This might seem a good solution to the paradox since we can see in astrophotographs that weren't available to Olbers that there are conspicuously dark areas of the sky, made dark by interstellar dust. However, Olbers was wrong in citing this as the chief cause of the dark night sky. One important reason is that if the dust he needed has always been present in the Universe then that dust itself will have heated up with all the radiation it is absorbing and will be glowing as brightly as the sources of light that are illuminating it. Another reason Olbers explanation isn't right is that the amount of dust needed even if there hasn't been time for it to heat up exceeds the amount we can see in the Universe.

The correct explanation is more subtle and hinges on the finite life of the Universe and the finite life of stars. If you look along any line of sight then, as we have seen, you are looking back in time. Now, a star populates a line of sight for only a finite time, namely the lifetime of the star. Because of the finite lifetimes of stars, there aren't enough populated lines of sight to make the night sky any brighter than it is. To put it another way, the density of stars in the universe is sufficiently low that most lines of sight don't intersect a star before the line of sight reaches the boundary of the visible Universe. Stars haven't been around for long enough to populate most lines of sight. There is also the matter that if a line of sight intercepts a star very far away then its light will have been red-shifted out of the visible part of the spectrum, but in fact this is not a major effect. A book covering lots of arguments about Olbers' paradox is E. Harrison *Darkness at Night* [Harvard Univ. Press, 1987].

Olbers' paradox highlights a simple observation that everyone has made unless they've spent their entire life living in a big city. The sky is comparatively dark at night. Even big city dwellers know that you need to put the lights on at night. The reason is that individual stars don't live for ever and there haven't been stars around for ever. The concept of stars having a finite life and evolving in predictable ways according to the known laws of physics is a product of 20th century astrophysics. We'll discuss star evolution later in the course.

In summary, this chapter has introduced a range of useful concepts about the expanding Universe, including the observed red-shifts z, Hubble's constant H_0, the cosmic scale factor a(t), and concepts like the size and age of the Universe. The next chapter covers Einstein's General Relativity and related topics.

First astronomer: "... anyway, the police pulled me over and asked if I realised that I had just run through a red light. I said that I didn't see the light as red because its colour must have been Doppler shifted to the green as I approached it."

Second astronomer: "And he let you go?"

First Astronomer: "No, he booked me for speeding."

JSR