On Gravity

John S. Reid

Gravity isn't as simple as you thought it was

Undergraduate courses on electromagnetism are widely recognised as among the hardest that students of Physics and Engineering will encounter. Yet electromagnetism is recognizably simple in comparison with gravity. Four equations suffice to describe the electromagnetic fields of **E** and **B**. These fields are generated by four sources, namely the distribution of charge and the three spatial components of the distribution of current (which describe the time variation of charge). Electromagnetic fields obey a superposition principle in that if two separate sources each generate fields, then the total field is the sum of the two fields. In this sense the theory is linear and the fields are non-interacting. Roger Penrose, a mathematical physicist well known for his work with gravity is want to describe Maxwell's electromagnetic equations as *a mathematical scheme of remarkable elegance and essential simplicity*. Most people would say that electromagnetism is not simple but it is certainly a theory that is widely used and applied in a huge range of practical applications.

Students of Physics are told about Newtonian gravity well before the details of electricity and magnetism are revealed. Gravity, they are told, is caused by mass; when you raise a body it gains gravitational potential energy that can be recovered when you let it fall; all objects fall on Earth with the same acceleration in the absence of air resistance; objects projected on Earth have parabolic orbits; planets and satellites have circular or elliptical orbits. A few other facts and deductions involving gravity are in standard introductory texts. Most people would say that gravity is simple and that Newtonian gravity is a very useful concept for a huge range of practical applications.

The dark truth uncovered by Einstein is that Newtonian gravity is but the welcoming face of a much more complex reality. 10 equations, not 1 or 4, are necessary to describe the gravitational field, which has 10 component sources. Mass is not the only source of gravity. All energy is the source of gravity, including potential energy. Something like this step you might have expected, given that Einstein pointed out in the theory of Special Relativity that all energy has a mass equivalent (remember $E = mc^2$). The gravitational field is not linear, for a field itself is energy and generates a further field contribution, which in turn generates further ones in a conceptual series that goes on to infinity in principle but does converge. Gravitational fields interact with each other, unlike electromagnetic fields. In short, General Relativity, Einstein's theory of gravity, introduces new mathematics, new physics and new philosophy. The pay-off for time spent on General Relativity is an understanding of the nature of the Universe that can't be gained without it.

The situation of having Newtonian gravity working well for everyday life is comparable to that of having Newtonian dynamics working well in everyday life. Newtonian dynamics works well because daily speeds are slow when compared with the speed of light. As a description of how nature works, Newtonian dynamics is hugely flawed. Newtonian concepts of space, time, matter and motion cannot be used successfully to understand nature as a whole. Einstein's Special Relativity theory shows both why Newtonian concepts fail when extended beyond everyday activities and how to put in place very different ideas that do work on a truly universal scale.

The idea that mass alone is the source of gravity is repeated so often in textbooks that deal only with Newtonian gravity that many people who want to know more about gravity find it difficult to believe, as Einstein said, that gravity is produced by other forms of energy. Looking back in history some half century before Einstein's development of General Relativity, a comparable situation arose in the theory of electricity and magnetism. Everyone thought they understood electric current. It is caused by the motion of charge. However, James Clerk Maxwell when he was formulating the mathematics of electricity and magnetism realised that current could also be caused by the change of electric field with time. He called this 'displacement current', extending the concept of the sources of current beyond the 'conduction current' that was familiar to everyone. It was the realisation of how displacement current fitted in to the phenomena of electricity and magnetism that allowed Maxwell to predict the existence of electromagnetic waves that travelled with the known speed of light, thereby identifying optics as an electromagnetic phenomenon. In that sense, the appreciation of a 'new' current source brought with it the prediction of new phenomena. So it was with Einstein's General Relativity.

Weak and strong gravity

To return to gravity itself, if Newtonian theory misses out some truths of the full story, how come it is so useful? One part of the answer is that we live in a place where gravity is weak. It might not seem so when a simple fall to earth from a perch 3 m high is enough to break a bone or two. Even more spectacularly, the continued application of the Earth's gravity on an object a long way away can give it a speed of 11 km s^{-1} by the time it hits the Earth. To get an idea of what constitutes strong gravity and what is weak gravity, think of the simplest case imaginable, namely the gravity experienced by a small mass acted on by a single large mass *M*. Compare the General Relativistic answer with the Newtonian answer.

The details for this simplest of cases implicit in Einstein's equations were first worked out by Karl Schwarzschild not long after Einstein published his equations, and have been elaborated on since. Newtonian gravity is the 'zero-order' field, the first approximation with no additional gravity produced by the field itself. Additional effects depend on GM/c^2r, where r is the distance from the centre of the mass M and G is the gravitational constant that occurs in both Newton's and Einstein's accounts; c is the usual universal constant for the speed of light in vacuum. [GM/c^2r is just the ratio of the gravitational potential of a small mass μ kg to its rest mass energy, i.e. the ratio of $GM\mu/r$ to μc^2]. If M is the mass of the Sun, then at the distance of the innermost planets, say 10^8 km, a back-of-the-envelope estimate for the size of GM/c^2r is $(10^{-10}\times10^{30}/10^{17}\times10^{11}) = 10^{-8}$. [If you want to be more accurate, the figure comes to 1.5×10^{-8}]. This means that the effect of Einstein's complications on the Newtonian inverse-square law of gravity amount to only 1 part in 10^8 at the distance of the innermost planets. This is a measurable amount but it's very small. What about effects near the surface of the Earth due to the Earth's own gravity? Taking M as the mass of the Earth, now, and r as the radius of the Earth, GM/c^2r is 7×10^{-10}, less than 1 part in 10^9. It is in this sense that gravity on Earth is weak. The novel details implied by General Relativity hardly influence the daily application of Newtonian gravity. Thanks to this weakness, many aspects of gravity can be promoted from the most complex subject in the physicists' book of the universe to the introductory chapters of an undergraduate text. If you still find it hard to imagine that gravity on Earth is a weak force in physics, remember that it takes the entire matter in the whole Earth, some 6×10^{24} kg or 3.6×10^{51} nucleons, all acting in unison to produce the modest gravity that keeps us standing on the ground.

Gravity, though, is not weak everywhere in the Universe. Also, General Relativity is relevant to the Universe as a whole and hence by implication to all of us in the Universe. Einstein arrived at the principles of General Relativity not by recognizing that tiny unaccounted for effects needed to be explained but because he realised that big issues interpreting nature needed to be explained more satisfactorily than they had been. The conceptual and explanatory nature of General Relativity is therefore in many ways more significant than the new physical phenomena that it introduces, relevant though they are in some cases.

Where is gravity strong? The prime candidates are close to neutron stars and black holes. At the surface of a neutron star of radius say 10 km and 1 solar mass, GM/c^2r is about 0.1. Put another way, r is about $10GM/c^2$ or 5 times the Schwarzschild radius for that mass. At this distance, orbits are still elliptical but precess strongly and gravitational radiation is a significant issue, both effects that Einstein's equations predict but Newtonian gravity doesn't.

A black hole exits if the mass M within does not extend beyond the Schwarzschild radius of $r_s = 2GM/c^2$. r_s is often called the *event horizon*. Light generated within the event horizon cannot escape. Other distances are relevant too. Within a distance of $2r_s$, elliptical orbits no longer exist but orbits become spiral, any matter ending up adding to the mass within the event horizon. An object a long way away but heading within $1.5r_s$ will be captured and hence the black hole has an effective radius of $3M/c^2r$ as far as capture is concerned. For example, there is a black hole at the centre of our galaxy and many other galaxies of at least 10^6 solar masses. For such a black hole, $3M/c^2r$ is about 4.5×10^6 km, giving a diameter of 6 solar diameters for the accretion target. For a common stellar black hole of a few solar masses, the target is nearer 10 km, very small by astronomical distances.

On curvature

Einstein called his theory 'General Relativity' (Allgemeinen Relativitätstheorie) but at first sight, or even second sight, it doesn't look like a theory about relative motion and position or even a theory of gravity. Certainly the gravitational constant G and the speed of light c are the two constants within the equations but the key equations seems to be concerned with space-time curvature. What is going on?

Einstein spent years trying to frame his ideas in the most elegant way using the most appropriate concepts. He extended his Special Relativity in what seemed to him the most natural and unforced way. He recognised of course that gravity affects the motion of bodies but he realised that motion, being a purely dynamical concept was implicitly bound up with the underlying geometry of space-time. Motion is measured by how much you move from one place to another in a set time interval. Displacement from one place to another is just what geometry describes. Mathematicians not long before Einstein had recognised that there was a range of possible geometries associated with a mathematical space. If a 3-dimensional space is 'Euclidian', then the displacement dS that results from small motions dx, dy and dz along perpendicular coordinate axes is determined by $dS^2 = dx^2 + dy^2 + dz^2$. However, non-Euclidean spaces are possible where the displacement is given by a more complex expression. Such spaces possess mathematical 'curvature'. Euclidean space doesn't and is said to be 'flat'.

Einstein recognised that the effect of gravity was to alter motion compared with what it would be in the absence of gravity but that the altered motion could be described by a non-Euclidean geometry. The effect of gravity can be taken account of by calculating the equivalent non-

Euclidean geometry and from then on making no explicit mention of gravity. It is in this sense that Einstein describes gravity as curving space-time. He found when he did so that he obtained a very elegant set of equations describing the curvature of space-time in terms of what he identified as the cause of the curvature (and hence the cause of gravity) namely the stress-energy tensor.

The curvature of a line on a plane is described by only one quantity. That is simple. In 3-dimensions, however, 6 quantities are needed to describe all the degrees of freedom of curvature at one point and in 4-dimensional space-time, the space-time we live in, 20 quantities are needed to describe the most general kind of curvature. Fortunately in the most general case of interest there are some equalities among these curvature components so only 10 general curvature quantities are needed in Einstein's field equations. Hence Einstein's field equations have 10 components, as mentioned earlier.

In short, Einstein pursued the logical consequences of his idea that gravity produces effects identical to accelerations (the 'equivalence principle'). Accelerations are a dynamical phenomenon and the dynamics of an accelerating frame of reference, he realised, could be described by non-Euclidean geometry. Hence the effects of gravity can be described as creating curvature in space-time. The resulting equations that naturally describe the effect turned out to imply phenomena that are essentially new, such as the existence of gravitational waves, the propagation of gravitational effects at the speed of light, the precession of elliptical orbits, the bending of light by stars, the gravitational red-shift, black holes and other effects. The existence of all these new phenomena have been investigated by experiment and the size of the effects found to be in agreement with those predicted, to the highest accuracy attainable by experiment. This gives very strong support to the underlying description that Einstein introduced. Such experimental support is certainly needed because Einstein arrived at his theory more by intuition than by fitting theory to known results. Just as Einstein's description had to include the Newtonian predictions of the effects of gravity in everyday circumstances (because they work), so any theory that may eventually supersede Einstein's will have to include all his accurately predicted effects in circumstances where gravity is strong as well as weak. There is clearly a risk that a theory that sets out to compete with General Relativity will simply end up by being a re-statement of it in different language.

JSR