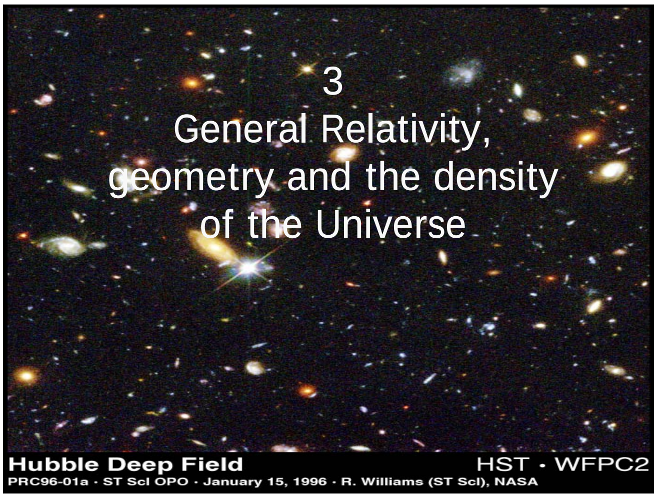


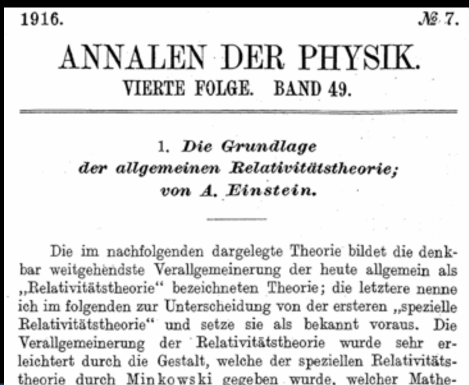
3 General Relativity, geometry and the density of the Universe

What is General Relativity all about?

- Matter affects space and time
- A new way of thinking
- General Relativity evolved from Special Relativity
- GR focuses on accelerated frames of reference

Einstein's definitive 1916 paper

- The foundation of General Relativity Theory



1916. № 7.

ANNALEN DER PHYSIK.

VIERTE FOLGE. BAND 49.

1. *Die Grundlage der allgemeinen Relativitätstheorie; von A. Einstein.*

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathe-

General Relativity in a nutshell

- Einstein's 'field equations' contain the mathematical description of GR
  - they take a lot of explaining!

$$R_{\mu\nu} - (1/2)g_{\mu\nu}R - \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

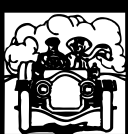
  - the left-hand-side describes the curvatures of space-time
    - the Λ term is the 'cosmological constant' term
  - the right-hand-side describes the distribution of mass and energy

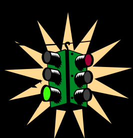
Accelerations

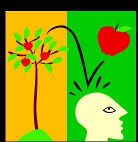
- Acceleration (a_c) describes the rate of change of velocity v

$$a_c = \dot{v} = \ddot{r}$$

starting | stopping | falling





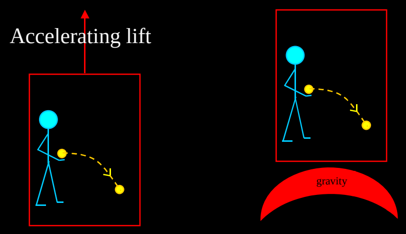


Principle of equivalence

- A physical principle
- *To an observer in free-fall in a gravitational field the results of all local experiments are completely independent of the magnitude of the field*
  - free-fall is equivalent to an inertial frame
- General Relativity links space, time, gravity and light
  - examples of how on the next slides

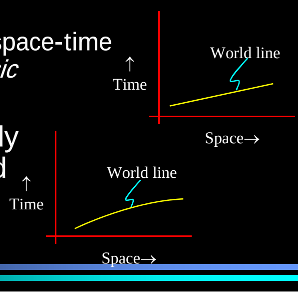
Einstein's 'lift' thought experiments

- Example 1: an accelerating frame of reference and a fixed frame in a gravitational field show the same effects



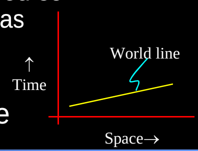
Space-time diagrams

- Subject to no forces, a body travels at uniform speed (Newton's 1st law)
  - its world line on a space-time diagram is straight
  - a straight-line in space-time is called a *geodesic*
- A freely falling (accelerating) body produces a curved world-line

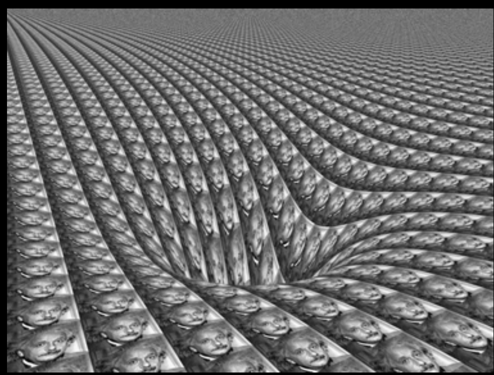


Gravity curves space-time

- In General Relativity we want the same law in the presence of matter (gravity) or when it is absent
  - a body in free fall follows a geodesic
  - this is achieved by making the underlying space-time curved so that the world-line is seen as straight
- Described as gravity curving space-time

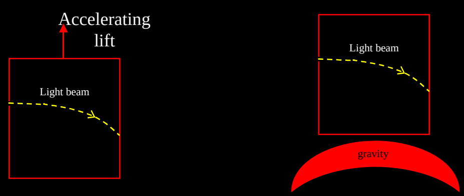


The rubber sheet model



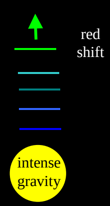
Gravity affects light

- Example 3: Shining a light across an accelerating lift, and in gravity
  - the light beam appears bent, in both circumstances
    - gravity bends the light beam



Gravity red-shifts light

- Photons lose energy as they escape from a gravitational field of strength *g*
  - result: a gravitational red-shift
  - blue shift as they descend into a gravitational field
  - the effect depends on M/r
    - ratio of mass/radius
  - the effect is noticeable from white dwarves and neutron stars



Pound & Rebka experiment (1960)

Photos: W. Toley, Harvard

Black holes

Karl Schwarzschild (1873 – 1916)

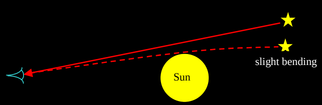
- The Schwarzschild radius, r, is the distance from a point mass M that light cannot escape

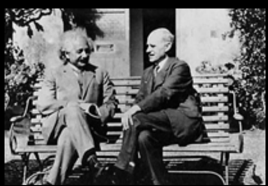
$$r = 2GM/c^2$$

- The density of mass at the Schwarzschild radius $\propto 1/M^2$
  - for $M = 10^6\ M_\odot$, density is ~ neutron star
- Black holes don't suck (unless you're very close)

Starlight is bent by the Sun

- Arthur Eddington tested this in 1919





Eddington & Einstein

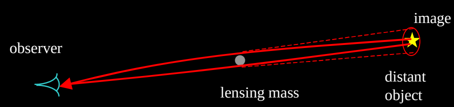
Sir Arthur Eddington (1882 – 1944)

courtesy:http://turnbull.mcs.st-and.ac.uk/~history/PictDisplay/Eddington.html

Gravitational lensing

HST image courtesy: NASA

- Ring imaging or just brightening of distant objects by intervening mass
  - lensing of distant galaxies
  - lensing by MACHOS (dark matter)
  - lensing by main-sequence stars with planets



Precession of Mercury's orbit

- Mercury's orbit swivels in space by ~5600" arc per century
  - cause is interaction with other planets
- 43" arc were not accounted for by Newtonian gravitation
- General Relativity predicted this difference

Gravity probe B

- Experiment to measure two GR predictions

The stronger the gravity, the slower the clock

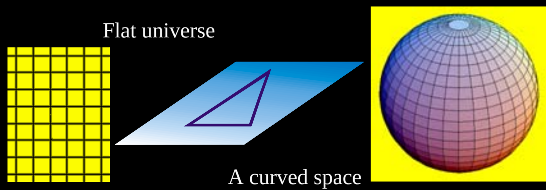
- This effect must be taken into account in the global position satellite (GPS) system
  - satellites ~20,000 km
- Binary pulsar evidence
  - Hulse & Taylor's 1993 Nobel Prize

GPS satellite network

Courtesy: http://geomag.usgs.gov/images/gps_001.jpg

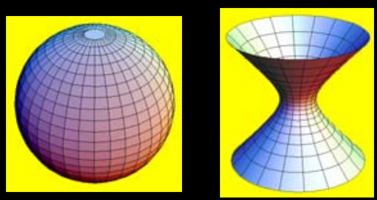
Curved space-time?

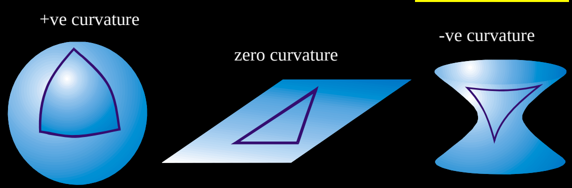
- Curvature is related to the calculation of the interval between neighbouring points
  - for a flat universe $dS^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$



The geometry of the Universe

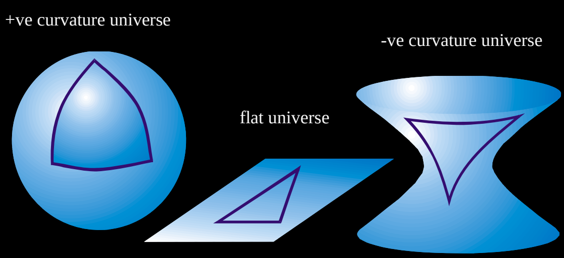
- Curvature affects the geometry of figures



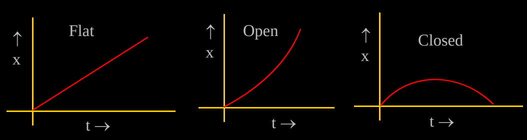


Our Universe

- What is its geometry?
- Controlled by the density of matter



Evolution of the cosmic scale factor



- Future scenarios depend on the geometry of the universe

Critical density ρ_c

- Critical density is that which makes the Universe flat

$$\rho_c = \frac{3H^2}{8\pi G}$$

- This density is ~ 10^{-26} kg m^{-3}, a few hydrogen atoms m^{-3}
- Density parameter $\Omega = \rho(t)/\rho_c$

The “cosmological constant” Λ

- Recent evidence is that the expansion of the universe is accelerating
- Einstein’s *cosmological constant* Λ was introduced to combat expansion
- Today’s option: *dark energy*

Parameters of the Universe

- H_0 Hubble’s constant
- Ω_0 density parameter
- q_0 acceleration parameter

$$q_0 = -\frac{a(t_0)\times\ddot{a}(t_0)}{\dot{a}^2(t_0)}$$

- q_0 is related to the cosmic scale factor and its derivatives
- In a flat universe without Λ, $q_0 = \Omega_0/2$