A deep-field image from the Hubble Space Telescope showing a vast field of galaxies. The galaxies are of various colors, including yellow, orange, red, and blue, and are scattered across a dark background. The image is a composite of many individual observations, showing a rich diversity of galaxy shapes and sizes.

3

# General Relativity, geometry and the density of the Universe

**Hubble Deep Field**

**HST · WFPC2**

PRC96-01a · ST ScI OPO · January 15, 1996 · R. Williams (ST ScI), NASA

# What is General Relativity all about?



- Matter affects space and time
- A new way of thinking
- General Relativity evolved from Special Relativity
- GR focuses on accelerated frames of reference

# Einstein's definitive 1916 paper

- The foundation of General Relativity Theory

1916.

№ 7.

## ANNALEN DER PHYSIK. VIERTE FOLGE. BAND 49.

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### 1. *Die Grundlage der allgemeinen Relativitätstheorie;* *von A. Einstein.*

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Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als „Relativitätstheorie“ bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren „spezielle Relativitätstheorie“ und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathe-

# General Relativity in a nutshell

- Einstein's 'field equations' contain the mathematical description of GR
  - they take a lot of explaining!

$$R_{\mu\nu} - (1/2)g_{\mu\nu}R - \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$$

- the left-hand-side describes the curvatures of space-time
  - the  $\Lambda$  term is the 'cosmological constant' term
- the right-hand-side describes the distribution of mass and energy

# Accelerations

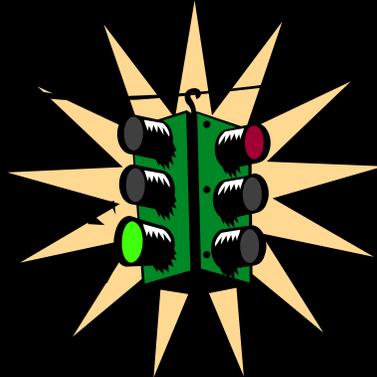
- Acceleration ( $a_c$ ) describes the rate of change of velocity  $v$

$$a_c = \dot{v} = \ddot{r}$$

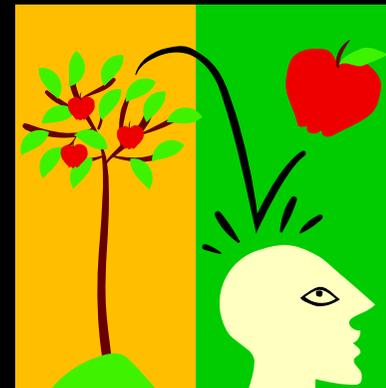
starting



stopping



falling

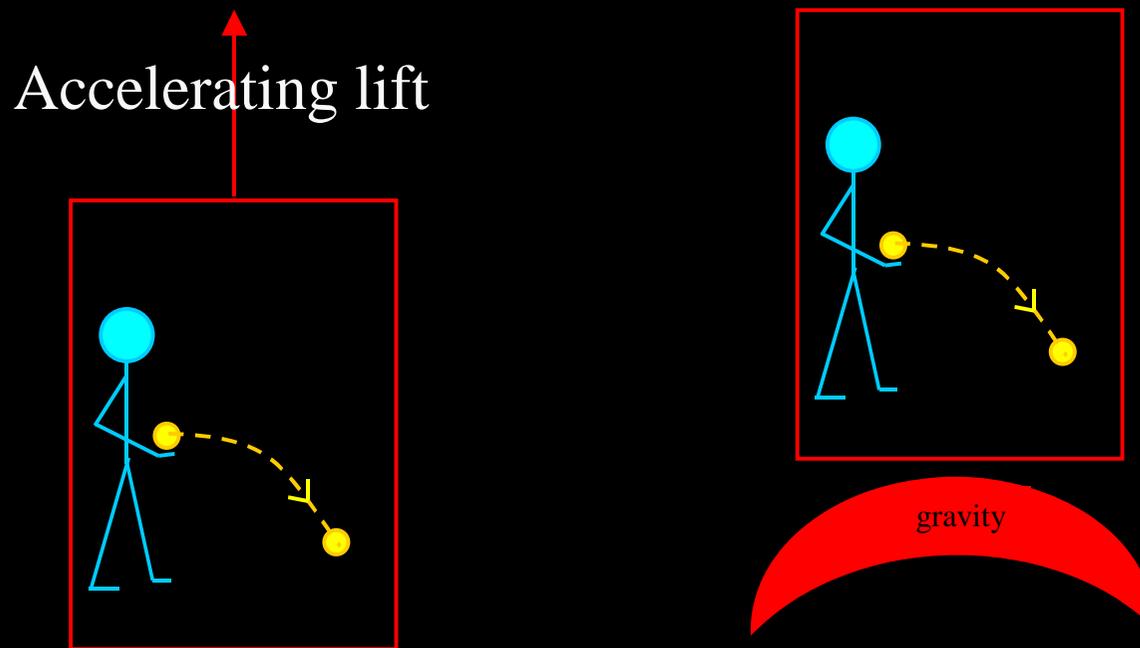


# Principle of equivalence

- A physical principle
- *To an observer in free-fall in a gravitational field the results of all local experiments are completely independent of the magnitude of the field*
  - free-fall is equivalent to an inertial frame
- General Relativity links space, time, gravity and light
  - examples of how on the next slides

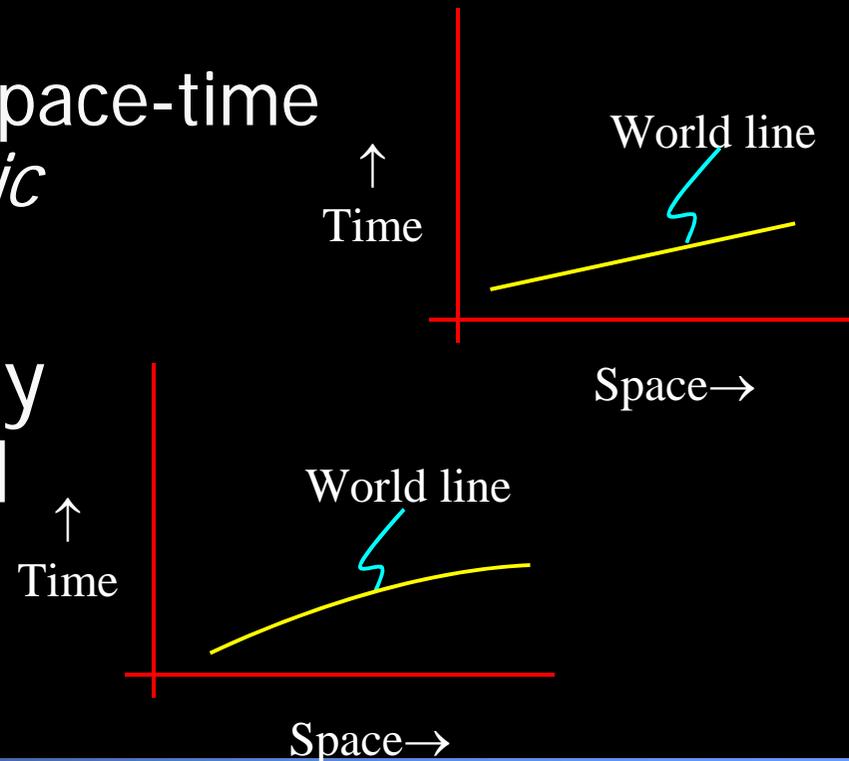
# Einstein's 'lift' thought experiments

- Example 1: an accelerating frame of reference and a fixed frame in a gravitational field show the same effects



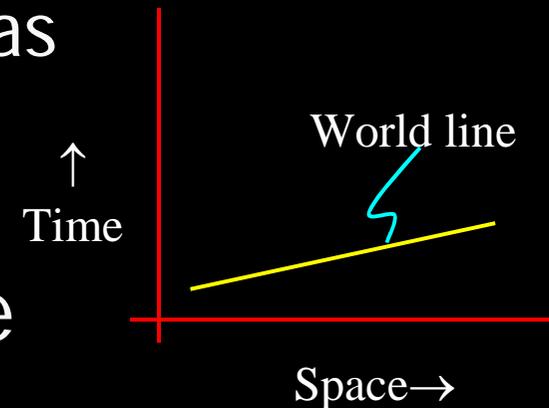
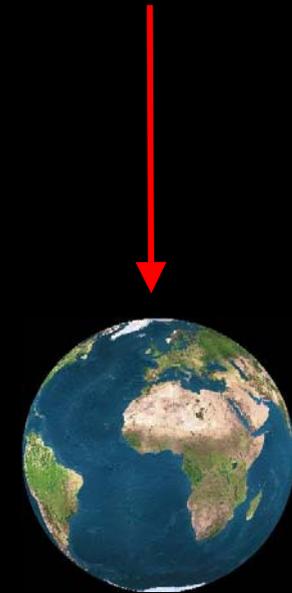
# Space-time diagrams

- Subject to no forces, a body travels at uniform speed (Newton's 1<sup>st</sup> law)
  - its world line on a space-time diagram is straight
  - a straight-line in space-time is called a *geodesic*
- A freely falling (accelerating) body produces a curved world-line

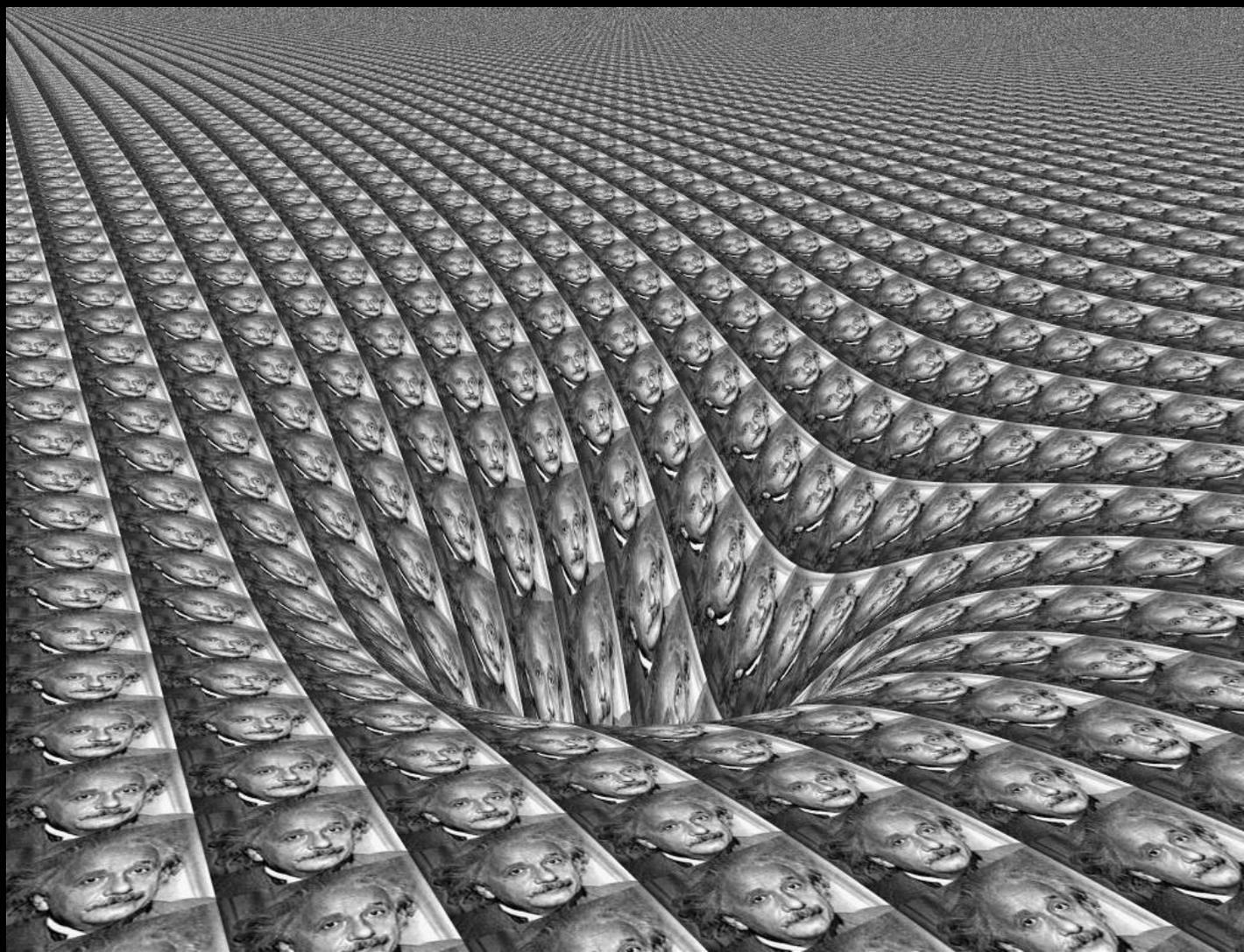


# Gravity curves space-time

- In General Relativity we want the same law in the presence of matter (gravity) or when it is absent
  - a body in free fall follows a geodesic
  - this is achieved by making the underlying space-time curved so that the world-line is seen as straight
- Described as gravity curving space-time

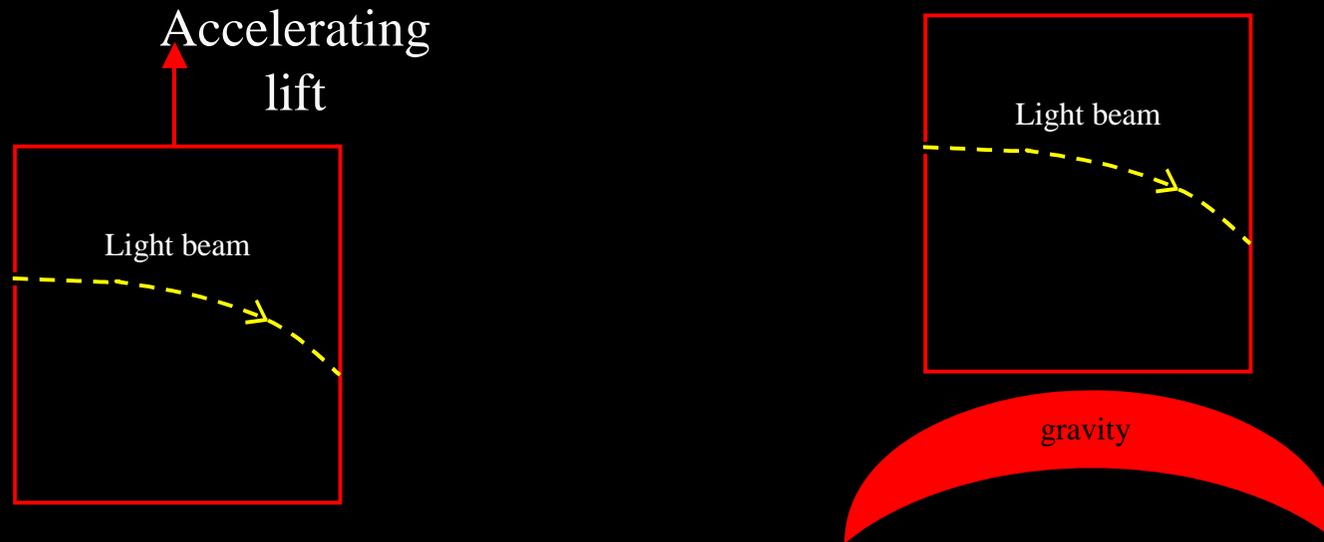


# The rubber sheet model



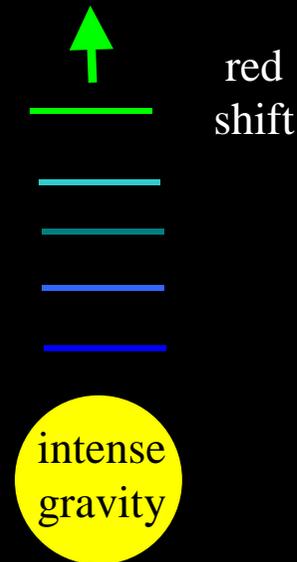
# Gravity affects light

- Example 3: Shining a light across an accelerating lift, and in gravity
  - the light beam appears bent, in both circumstances
    - gravity bends the light beam



# Gravity red-shifts light

- Photons lose energy as they escape from a gravitational field of strength  $g$ 
  - result: a gravitational red-shift
  - blue shift as they descend into a gravitational field
  - the effect depends on  $M/r$ 
    - ratio of mass/radius
  - the effect is noticeable from white dwarves and neutron stars



# Pound & Rebka experiment (1960)

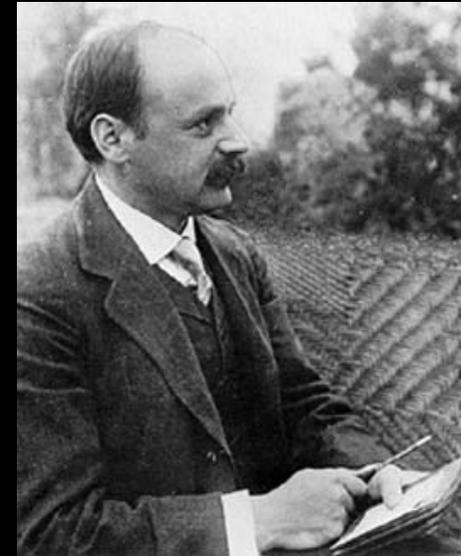


Photos: W. Toley, Harvard

# Black holes

- The Schwarzschild radius,  $r$ , is the distance from a point mass  $M$  that light cannot escape

$$r = 2GM/c^2$$

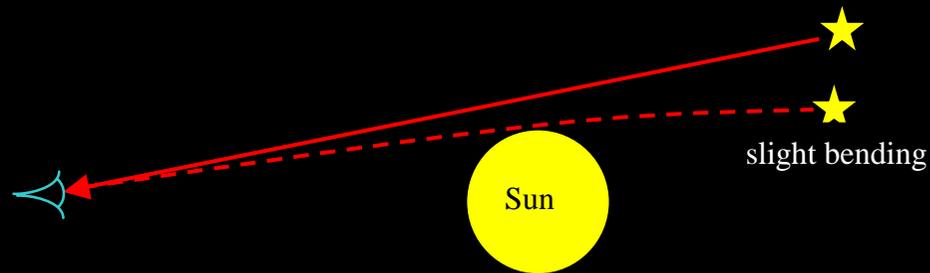


Karl Schwarzschild  
(1873 – 1916)

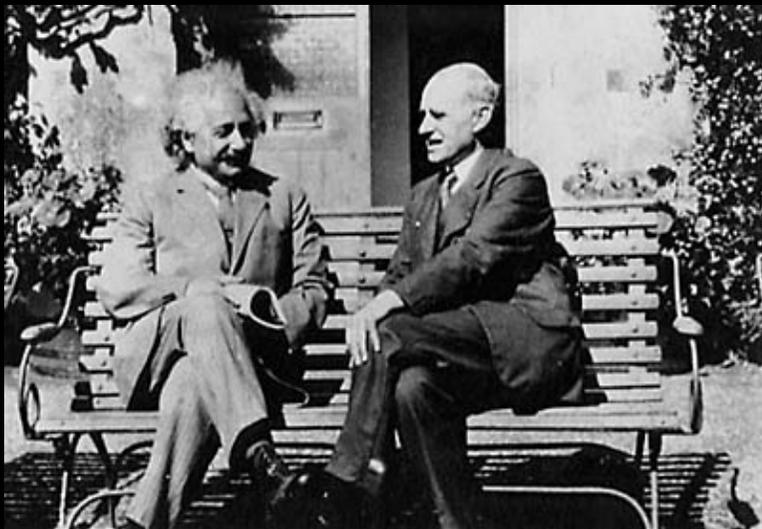
- The density of mass at the Schwarzschild radius  $\propto 1/M^2$ 
  - for  $M = 10^6 M_{\odot}$ , density is  $\sim$  neutron star
- Black holes don't suck (unless you're very close)

# Starlight is bent by the Sun

- Arthur Eddington tested this in 1919



Sir Arthur Eddington  
(1882 – 1944)



Eddington  
& Einstein

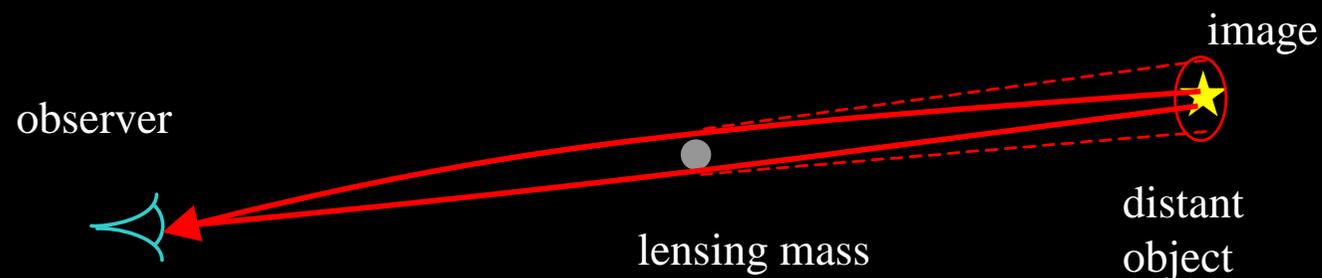
courtesy:<http://turnbull.mcs.st-and.ac.uk/~history/PictDisplay/Eddington.html>

# Gravitational lensing

- Ring imaging or just brightening of distant objects by intervening mass
  - lensing of distant galaxies
  - lensing by MACHOS (dark matter)
  - lensing by main-sequence stars with planets

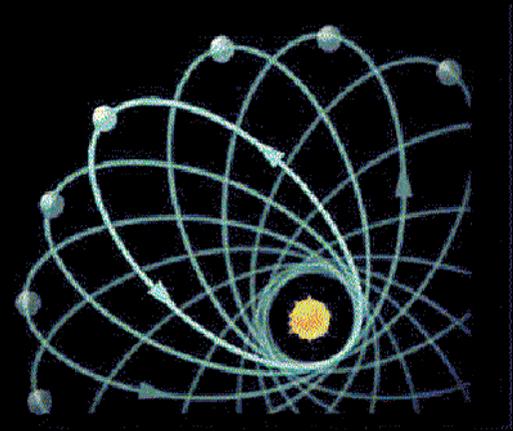


HST image  
courtesy: NASA



# Precession of Mercury's orbit

- Mercury's orbit swivels in space by  $\sim 5600''$  arc per century
  - cause is interaction with other planets
- $43''$  arc were not accounted for by Newtonian gravitation
- General Relativity predicted this difference



# Gravity probe B

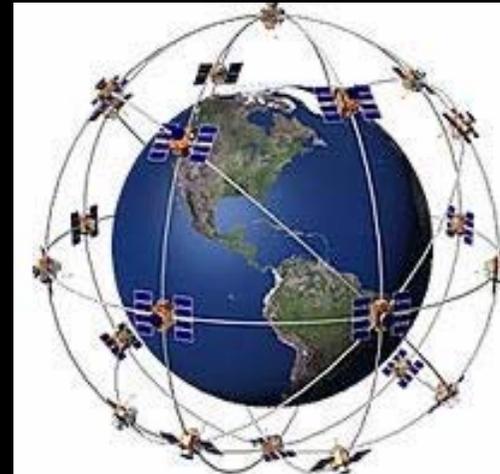
- Experiment to measure two GR predictions



# The stronger the gravity, the slower the clock

- This effect must be taken into account in the global position satellite (GPS) system
  - satellites ~20,000 km
- Binary pulsar evidence
  - Hulse & Taylor's 1993 Nobel Prize

GPS satellite network



Courtesy:

[http://geomag.usgs.gov/images/gps\\_001.jpg](http://geomag.usgs.gov/images/gps_001.jpg)

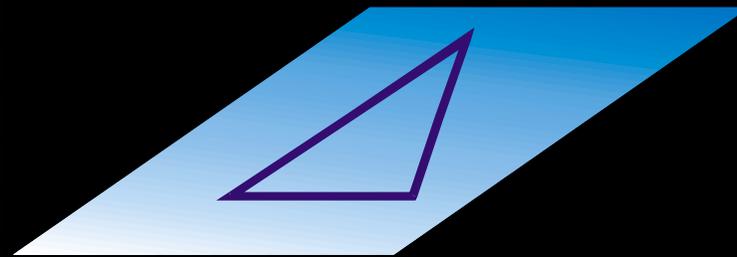
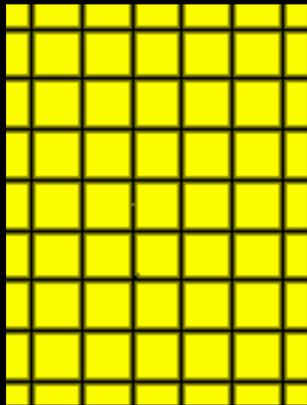
# Curved space-time?

- Curvature is related to the calculation of the interval between neighbouring points

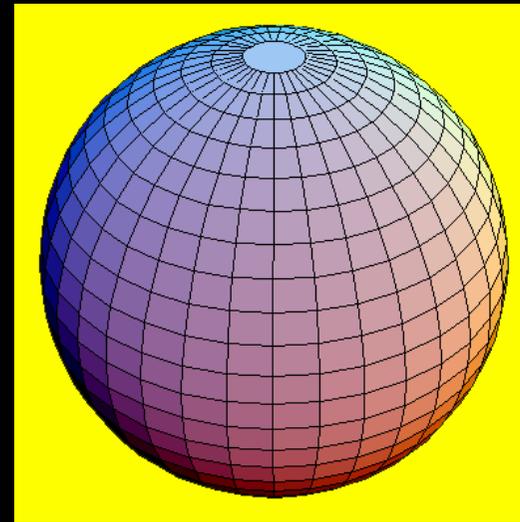
- for a flat universe

$$dS^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

Flat universe

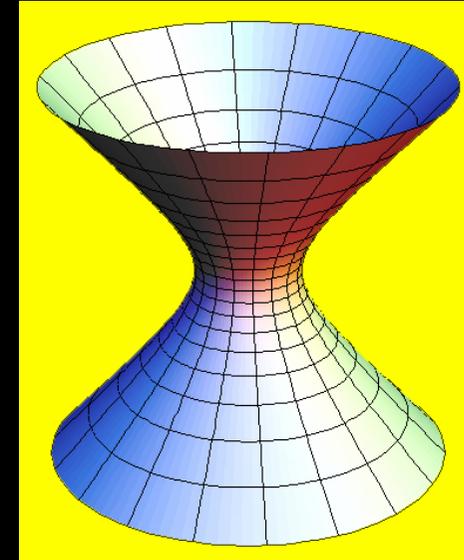
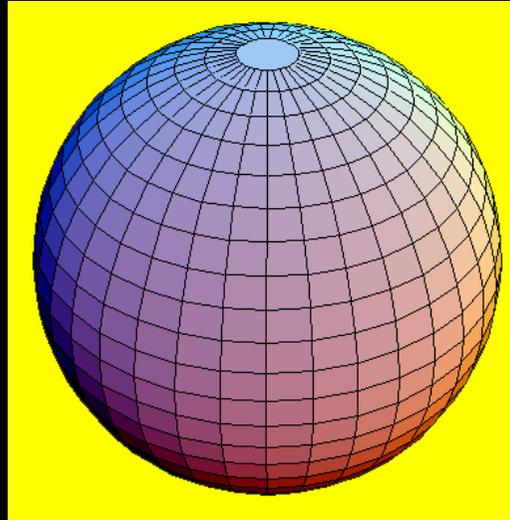


A curved space

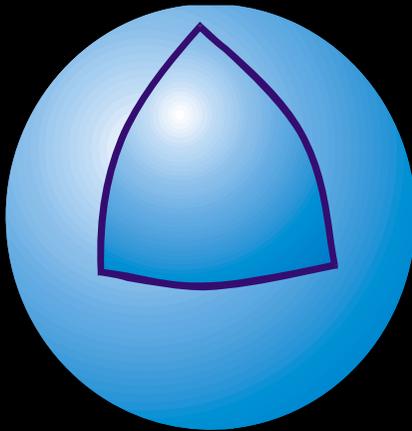


# The geometry of the Universe

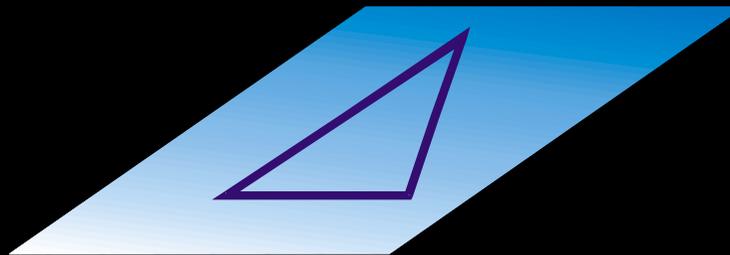
- Curvature affects the geometry of figures



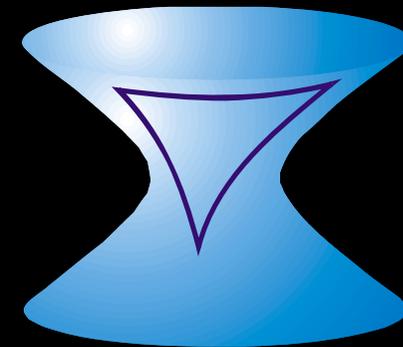
+ve curvature



zero curvature



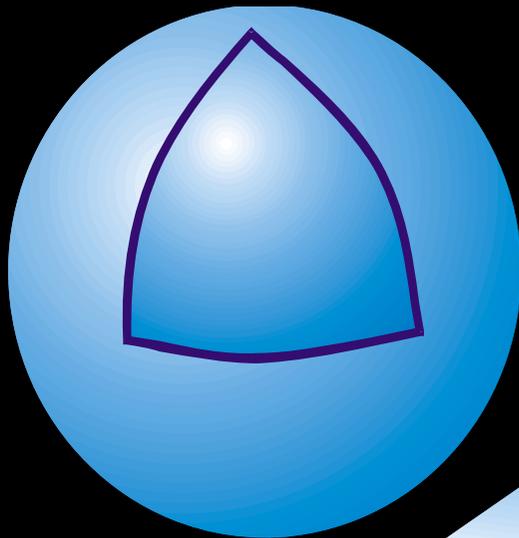
-ve curvature



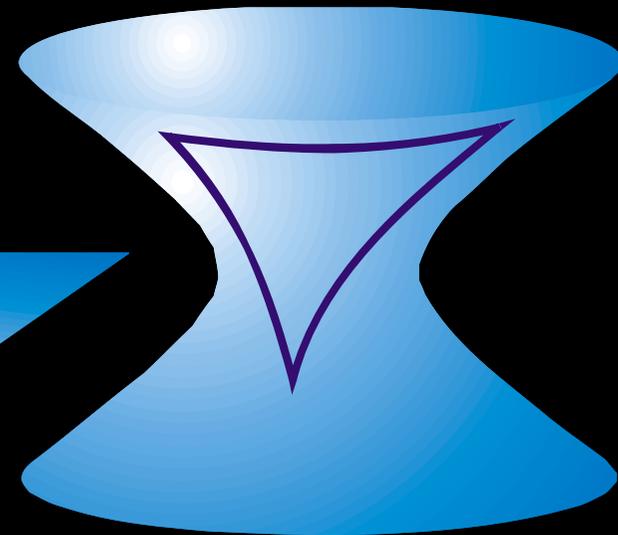
# Our Universe

- What is its geometry?
- Controlled by the density of matter

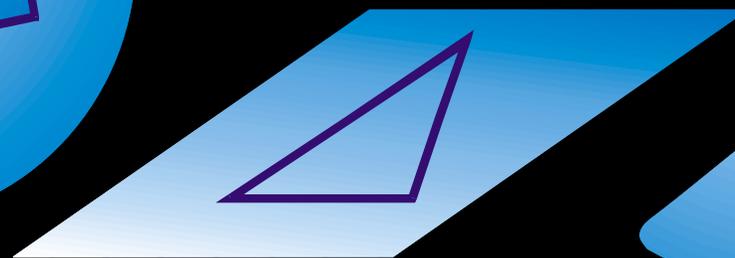
+ve curvature universe



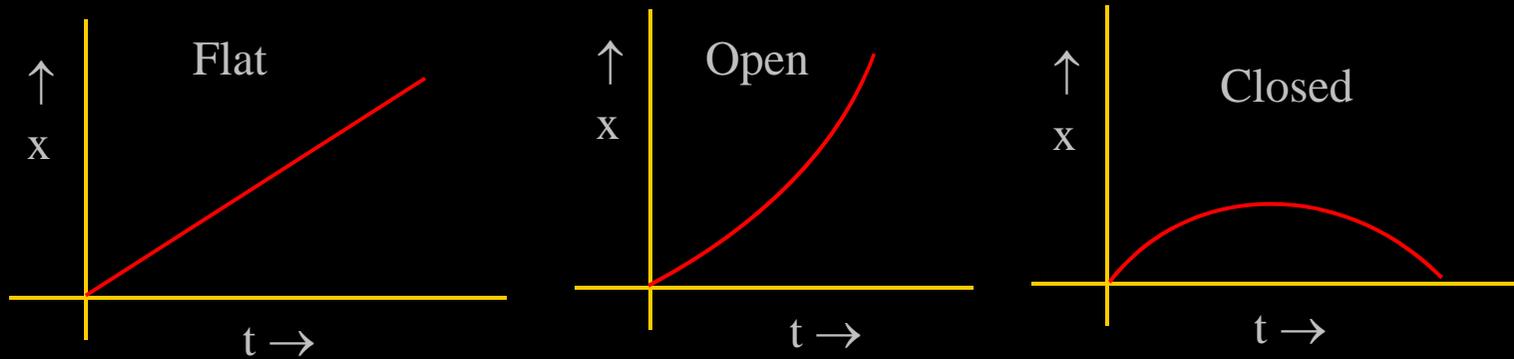
-ve curvature universe



flat universe



# Evolution of the cosmic scale factor



- Future scenarios depend on the geometry of the universe

# Critical density $\rho_c$

- Critical density is that which makes the Universe flat

$$\rho_c = \frac{3H^2}{8\pi G}$$

- This density is  $\sim 10^{-26} \text{ kg m}^{-3}$ , a few hydrogen atoms  $\text{m}^{-3}$
- Density parameter  $\Omega = \rho(t)/\rho_c$

# The “cosmological constant” $\Lambda$

- Recent evidence is that the expansion of the universe is accelerating
- Einstein's *cosmological constant*  $\Lambda$  was introduced to combat expansion
- Today's option: *dark energy*



# Parameters of the Universe

- $H_0$  Hubble's constant
- $\Omega_0$  density parameter
- $q_0$  acceleration parameter

$$q_0 = \frac{a(t_0) \times \ddot{a}(t_0)}{\dot{a}^2(t_0)}$$

- $q_0$  is related to the cosmic scale factor and its derivatives
- In a flat universe without  $\Lambda$ ,  $q_0 = \Omega_0/2$