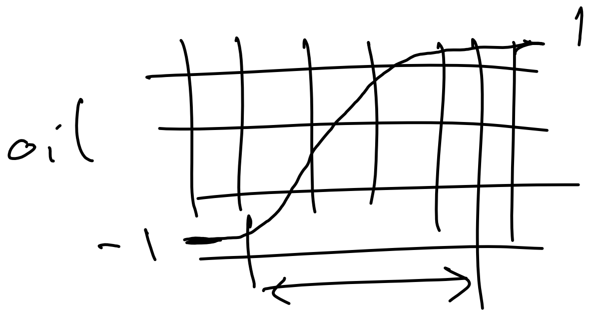
ϕ order parameter

$\phi = 1$ water ; $\phi = -1$ oil



water   diffuse interface method

transport equation

$$\rightarrow \frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x_\beta}(\phi u_\beta) = M \frac{\partial^2 \mu}{\partial x_\beta^2} \qquad \mu = A\phi - A\phi^3 - k\frac{\partial^2 \phi}{\partial x_\beta^2} -$$

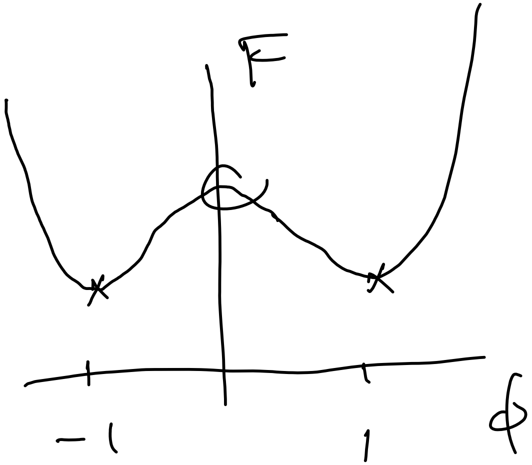
chemical potential

$$\cancel{= \Gamma \frac{\partial^2 \phi}{\partial x_\beta^2}}$$

$A < 0$

$$\mu = \frac{DF}{D\phi} \quad \text{if} \quad k = 0 \qquad \frac{DF}{D\phi} = 0 \quad \text{for} \quad \phi = -1$$

$$\phi = 1$$



mass balance

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_\alpha}{\partial x_\alpha} = 0$$

$$\rho = \rho_c \frac{1}{2}(1-\phi) + \rho_d \frac{1}{2}(1+\phi)$$

$$\nu = \nu_c \frac{1}{2}(1-\phi) + \nu_d \frac{1}{2}(1+\phi)$$

momentum

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(\rho u_\alpha u_\beta) = -\frac{\partial P^{th}_{\alpha\beta}}{\partial x_\beta} + \frac{\partial}{\partial x_\beta}\left[\nu\left(\frac{\partial \rho u_\alpha}{\partial x_\beta} + \frac{\partial \rho u_\beta}{\partial x_\alpha}\right)\right]$$

$$P^{th}_{\alpha\beta} = \left[\underbrace{\frac{1}{3}c_s^2\rho}_{ig} + \underbrace{A\left(\frac{1}{2}\phi^2 - \frac{3}{4}\phi^4\right)} - k\phi\frac{\partial^2\phi}{\partial x_\gamma^2} - \frac{1}{2}\left(\frac{\partial\phi}{\partial x_\gamma}\right)^2\right]\delta_{\alpha\beta} + k\frac{\partial\phi}{\partial x_\alpha}\frac{\partial\phi}{\partial x_\beta}$$

k, A $\quad \gamma = \frac{2}{3}\sqrt{2k|A|}$; $\xi = \sqrt{2k/|A|}$

LBM BGK (MRT)

$$\rightarrow \; f_i(\vec{x} + \vec{c}_i, t+1) = f_i(\vec{x}, t) - \frac{1}{\tau_f}\left[f_i(\vec{x}, t) - \underbrace{f_i^{eq}(\vec{x}, t)}_{} \right]$$

$$\rho = \sum_i f_i \; ; \quad \rho \vec{u} = \sum_i \vec{c}_i f_i \; ; \quad \nu = \frac{2\tau_f - 1}{6}$$

$$\underline{\nu(\phi)}$$

$$\rightarrow \; g_i(\vec{x} + \vec{c}_i, t+1) = g_i(\vec{x}, t) - \frac{1}{\tau_g}\left[g_i(\vec{x}, t) - g_i^{eq}(\vec{x}, t) \right]$$

$$\phi = \sum_i g_i$$

$$f_i^{eq} = w_i \rho \left[1 + \frac{c_{i\alpha} u_\alpha}{c_s^2} + \frac{u_\alpha u_\beta (c_{i\alpha} c_{i\beta} - c_s^2 \delta_{\alpha\beta})}{2 c_s^4} \right] +$$

$$+ \frac{w_i}{c_s^2} \left[\left(p_b - c_s^2 \rho - k \phi \frac{\partial^2 \phi}{\partial x_\alpha^2} \right) + k \left(w_i^{\alpha\beta} \right) \frac{\partial \phi}{\partial x_\alpha} \frac{\partial \phi}{\partial x_\beta} \right]$$

$i \neq 0$ $\qquad f_0^{eq} = \rho - \sum_{i \neq 0} f_i^{eq}$

lattice coefficient

$$p_b = c_s^2 \rho + A \left(\frac{1}{2} \phi^2 - \frac{3}{4} \phi^4 \right)$$

$$g_i^{eq} = w_i \left[\frac{p\mu}{c_s^2} + \frac{\phi u_\alpha c_{i\alpha}}{c_s^2} + \frac{\phi u_\alpha u_\beta (c_{i\alpha} c_{i\beta} - c_s^2 \delta_{\alpha\beta})}{2 c_s^4} \right]$$

$i \neq 0$ $\quad i = 0 \quad g_0^{eq} = \phi - \sum_{i \neq 0} g_i^{eq}$

Γ numerical parameter

1. known f_i g_i
2. determine ρ, $\rho\vec{u}$, ϕ
3. determine the spatial derivatives of ϕ, μ
4. determine f_i^{eq}, g_i^{eq}
5. collide
6. stream

τ_f ; τ_g ; $\underset{3}{A}$; $\underset{3}{k}$; M ; $\underline{\Gamma}$ $\qquad M = \Gamma\left(\tau_g - \frac{1}{2}\right)$

$\tau_\gamma \to \nu$ $\quad \underbrace{\xi = \sqrt{2k/|A|}}_{2-3 \ \mathrm{lu}}$ $\quad \gamma = \frac{2}{3}\sqrt{2k|A|}$ $\quad$ fixes k and A

$\tau_g = 1 \qquad \Gamma = 0.1 - 10$

3 phase contact line