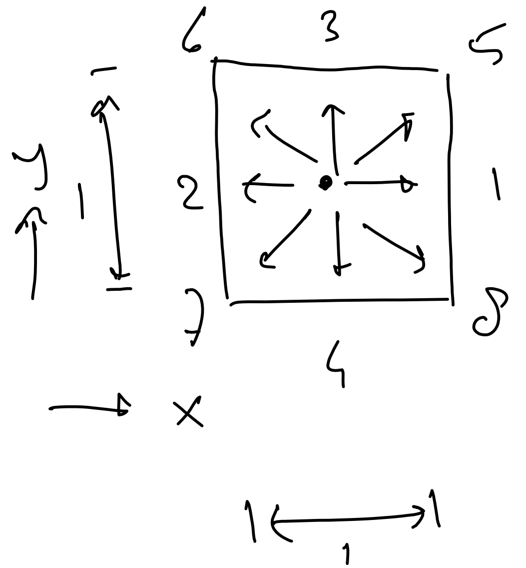
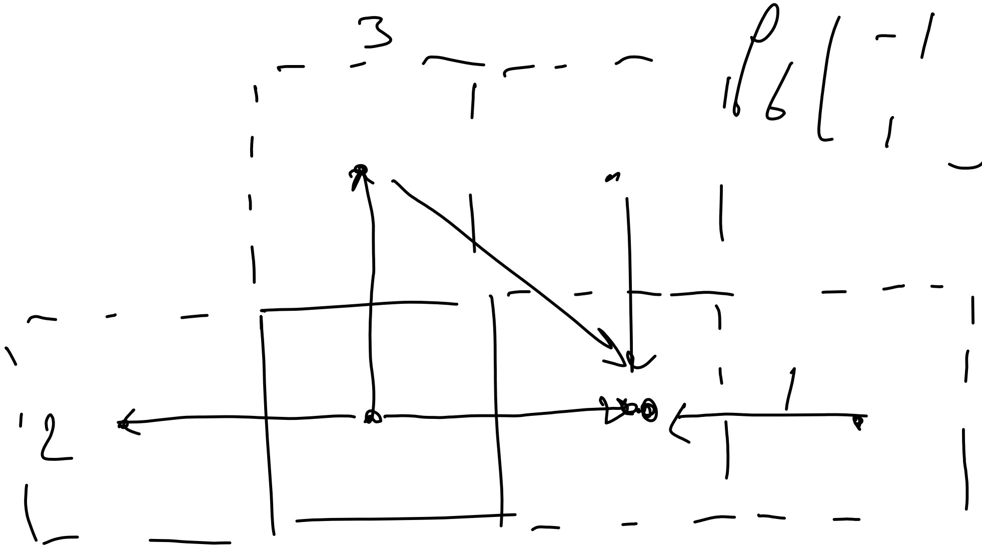


$f_i \qquad i = 0 \ldots 8$

$f_0 \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad f_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad f_2 \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$f_3 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad f_4 \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad f_5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \vec{c}_i$

$f_6 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad f_7 \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad f_8 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$



$f_i(\vec{x}, t)$

$\to f_i^*(\vec{x}, t) = f_i(\vec{x}, t) + \Omega_i(\vec{f})$ collision

$\Omega_i(\vec{x}, t)$

$$f_i(\vec{x} + \vec{c}_i \cdot \Delta t,\ t + \Delta t) = f_i^*(\vec{x}, t) \quad \text{streaming}$$

$$f_i(\vec{x} + \vec{c}_i,\ t+1) = f_i(\vec{x}, t) + \Omega_i(\vec{x}, t)$$

$$f(\vec{x}, \vec{\xi}, t) \longrightarrow f_i(\vec{x}, t)$$

$$\rho = \iiint f(\vec{x}, \vec{\xi}, t)\, d^3\xi \longrightarrow \rho = \sum_{i=0}^{8} f_i(\vec{x}, t)$$

$$\rho\vec{u} = \iiint \vec{\xi} f(\vec{x}, \vec{\xi}, t)\, d^3\xi \longrightarrow \rho\vec{u} = \sum_{i=0}^{8} \vec{c}_i f_i(\vec{x}, t)$$

collision operator

BGK $\Omega(f) = -\frac{1}{\tau}(f - f^{eq})$

$$\Omega_i(\vec{f}) = -\frac{1}{\tau}(f_i - f_i^{eq}) \leftarrow$$

$$- f_i^{eq} = w_i \rho \left[1 + \frac{u_\alpha c_{i\alpha}}{c_s^2} + \frac{(u_\alpha c_{i\alpha})^2}{2 c_s^4} - \frac{u_\alpha u_\alpha}{2 c_s^2} \right] -$$

* D2Q9 $w_0 = \frac{4}{9}$ $w_{1-4} = \frac{1}{9}$ $w_{5-8} = \frac{1}{36}$ $\leftarrow$

* Summation convention $u_\alpha c_{i\alpha} = u_x c_{ix} + u_y c_{iy}$

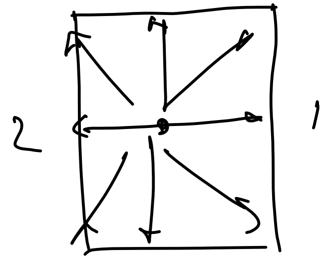
* $c_s^2 = \frac{1}{3}$ for D_2Q9 lattice speed of sound $c_s = \sqrt{1/3}$ $p = \rho/3$

$$\sum_{i=0}^{8} \Omega_i = 0 \qquad \sum_{i=0}^{8} \frac{1}{\tau}\left(f_i - f_i^{eq}\right) = 0 \qquad \sum_i f_i = \sum_i f_i^{eq} = \rho$$

$$\sum_i f_i^{eq} = \rho$$

Need symmetry properties of the lattice

$$\sum_i w_i = 1 \qquad \sum_i w_i c_{i\alpha} = 0$$



$$\sum_i w_i c_{i\alpha} c_{i\beta} = \frac{1}{3}\delta_{\alpha\beta}$$

$$\nu = c_s^2\left(\tau - \frac{\Delta t}{2}\right)$$

$\Delta t = 1$ D2Q9 $c_s^2 = \frac{1}{3}$

$$\nu = \frac{2\tau - 1}{6}$$

$p = c_s^2 \rho$ $\quad p = \rho/3 \quad \nu = \alpha g$

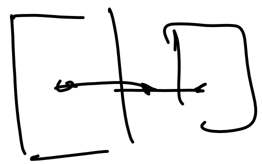
$\vec{x} \quad f_i \quad i = 0 \dots 8$

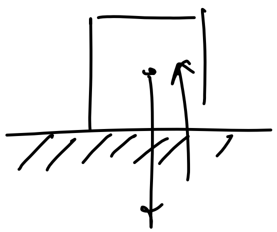
collide $\quad f_i^*(\vec{x}, t) = f_i(\vec{x}, t) + \Omega_i(\vec{x}, t)$

$$\Omega_i = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right)$$

$w_i, \; c_{i\alpha}, \; \rho = \sum f_i, \; u_\alpha = \frac{1}{\rho}\sum c_{i\alpha} f_i$

stream $\quad f_i(\vec{x} + \vec{c}_i, t+1) = f^*(\vec{x}, t)$





LBE
BGK
lu

$$f_i(\vec{x}+\vec{c}_i, t+1) - f_i(\vec{x}, t) = -\frac{1}{\tau}\left[f_i(\vec{x},t) - f_i^{eq}(\vec{x},t)\right]$$

① expand f_i around f_i^{eq} ε

$$f_i = f_i^{eq} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} + \cdots$$

$$\sum_i (f_i - f_i^{eq}) = 0 \rightarrow \sum_i f_i^{(n)} = 0 \quad n \geq 1$$

② derivatives are small

$$c_{i\alpha}\frac{\partial f_i}{\partial x_\alpha} = \varepsilon\, c_{i\alpha}\frac{\partial^{(1)}}{\partial x_\alpha} f_i$$

$$\frac{\partial f_i}{\partial t} = \varepsilon \frac{\partial^{(1)}}{\partial t} f_i + \varepsilon^2 \frac{\partial^{(2)}}{\partial t} f_i + \cdots$$

③ Taylor expansion of the LBE

$$\frac{\partial f_i}{\partial t} + c_{i\alpha}\frac{\partial f_i}{\partial x_\alpha} = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right) + \frac{1}{2\tau}\left\{\frac{\partial}{\partial t}\left(f_i - f_i^{eq}\right) + c_{i\alpha}\frac{\partial}{\partial x_\alpha}\left(f_i - f_i^{eq}\right)\right\}$$

ε^1 $\quad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_\alpha}\rho u_\alpha = 0$ continuity equation

ε^2

$$\frac{\partial}{\partial t}(\rho u_\beta) + \frac{\partial}{\partial x_\alpha}(\rho u_\alpha u_\beta) = -c_s^2\frac{\partial \rho}{\partial x_\beta} + c_s^2\left(\tau - \frac{1}{2}\right)\frac{\partial}{\partial x_\alpha}\left\{\rho\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right)\right\}$$

if $p = c_s^2 \rho$ $\quad -\frac{dp}{dx_\beta}$ $\quad c_s^2(\tau - \frac{1}{2}) = \nu$

only if $\rho \approx$ constant $\quad \sim$ NS eq.

$$Ma = \frac{|\bar{u}|}{c_s} << 1$$