distribution function

$\rho(\vec{x}, t) \qquad f(\vec{x}, \vec{\xi}, t)$

$$\underbrace{\rho(\vec{x}, t)}_{kg/m^3} = \iiint \underbrace{f(\vec{x}, \vec{\xi}, t)}_{\frac{kg}{m^3 \cdot m^3 s^{-3}}} \underbrace{d^3\xi}_{m^3/s^3}$$

$d^3\xi \rightarrow d\xi_x \, d\xi_y \, d\xi_z$

momentum

$$\rho(\vec{x}, t)\, \vec{u}(\vec{x}, t) = \rho\vec{u} = \iiint \vec{\xi} f(\vec{x}, \vec{\xi}, t)\, d^3\xi$$

$\vec{u}$ bulk velocity

kinetic energy

$$\rho E = \frac{1}{2} \iiint |\vec{\xi}|^2 f(\vec{x}, \vec{\xi}, t)\, d^3\xi = \ldots$$

$$\vec{\xi} = \vec{u} + \vec{v} \rightarrow \vec{v} = \vec{\xi} - \vec{u}$$

$$\rho e = \frac{1}{2}\iiint |\bar{v}|^2 f(\vec{x},\vec{\xi},t)\, d^3\xi$$

$$e = \frac{3}{2} k T \longrightarrow \text{per molecule}$$

per mole $\quad e = \frac{3}{2} k M_{avo} T = \frac{3}{2} R_{ig} T \qquad R = \frac{R_{ig}}{m}$

ig. law per mole

$$pV = R_{ig} T \longrightarrow p = \frac{1}{V} R_{ig} T = \frac{m}{V} R T = \rho R T$$

$$p = \rho R T = \frac{2}{3}\rho e = \frac{1}{3}\iiint |\bar{v}|^2 f(\vec{x},\vec{\xi},t)\, d^3\xi$$

Equilibrium distribution function $\quad \vec{\xi} = \vec{v}$

$$f^{eq}(\vec{x},\vec{v},t) = \rho\left(\frac{1}{2\pi R T}\right)^{3/2} \exp\left\{-\frac{|\bar{v}|^2}{2RT}\right\}$$

Evolution of a distribution function

$f(\vec{x}, \vec{\xi}, t)$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \underbrace{\frac{\partial f}{\partial x_\beta}\frac{dx_\beta}{dt}}_{\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}} + \frac{\partial f}{\partial \xi_\beta}\frac{d\xi_\beta}{dt}$$

summation convention

$$\frac{\partial f}{\partial u}\frac{du}{dt} + \frac{\partial f}{\partial v}\frac{dv}{dt} + \frac{\partial f}{\partial w}\frac{dw}{dt}$$

$$\boxed{\frac{\partial f}{\partial t} + \xi_\beta \frac{\partial f}{\partial x_\beta} + \frac{F_\beta}{\rho}\frac{\partial f}{\partial \xi_\beta} = \Omega(f)} \qquad \rho \frac{\partial \xi_\beta}{\partial t} = F_\beta$$

Boltzmann equation

$$\Omega(f) = -\frac{1}{\tau}\left(f - f^{eq}\right) \qquad \text{BGK operator}$$

$$\frac{\partial f}{\partial t} = -\frac{1}{\tau}\left(f - f^{eq}\right) \qquad f = f^{eq} + \left[f(t=0) - f^{eq}\right] e^{-t/\tau}$$

$$\frac{\partial \rho}{\partial t} ; \iiint \xi_\beta \frac{\partial f}{\partial x_\beta} d^3\xi = \frac{\partial}{\partial x_\beta} \iiint \xi_\beta f \, d^3\xi = \underbrace{\frac{\partial}{\partial x_\beta} (\rho u_\beta)}$$

$$\iiint \frac{F_\beta}{\rho} \frac{\partial f}{\partial \xi_\beta} d^3\xi = 0 \qquad \frac{\partial \rho}{\partial t} + \underbrace{\frac{\partial}{\partial x_\beta} (\rho u_\beta)} = 0$$

$$\iiint \Omega(f) \, d^3\xi = 0 \qquad \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

continuity equation

$$\iiint \underbrace{\xi_\alpha \frac{\partial f}{\partial t}} + \xi_\alpha \xi_\beta \frac{\partial f}{\partial x_\beta} + \xi_\alpha \frac{F_\beta}{\rho} \frac{\partial f}{\partial \xi_\beta} = \underbrace{\xi_\alpha \Omega(f)}$$

$$\to \underbrace{\frac{\partial}{\partial t} (\rho u_\alpha)} \qquad \underbrace{\frac{\partial}{\partial x_\beta} \iiint \xi_\alpha \xi_\beta f \, d^3\xi} = \Pi_{\alpha\beta} \leftarrow \text{momentum flux tensor}$$

$$\frac{F_\beta}{\rho} \iiint \xi_\alpha \frac{\partial f}{\partial \xi_\beta} d^3\xi = \underbrace{-F_\beta \delta_{\alpha\beta}}_{F_\alpha}$$

kronecker delta

$$\delta_{\alpha\beta} = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \alpha \neq \beta \end{cases}$$

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta} \Pi_{\alpha\beta} = F_\alpha$$

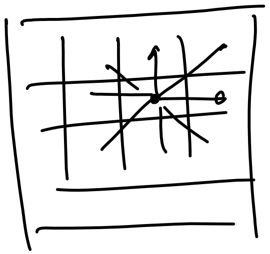
$$\xi_\alpha \xi_\beta = (u_\alpha + v_\alpha)(u_\beta + v_\beta) = u_\alpha u_\beta + v_\alpha v_\beta + \underline{u_\alpha v_\beta + u_\beta v_\alpha}$$

$$\Pi_{\alpha\beta} = \rho u_\alpha u_\beta + \underbrace{\iiint v_\alpha v_\beta f \, d^3\xi}_{-\sigma_{\alpha\beta} \text{ stress tensor}}$$

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(\rho u_\alpha u_\beta) = \frac{\partial}{\partial x_\beta}(\sigma_{\alpha\beta}) + F_\alpha$$

$$\sigma_{\alpha\beta} = \sigma'_{\alpha\beta} - p\,\delta_{\alpha\beta}$$

$$\frac{\partial \rho u_\alpha}{\partial t} + \frac{\partial}{\partial x_\beta}(\rho u_\alpha u_\beta) = -\frac{\partial p}{\partial \alpha} + \frac{\partial}{\partial x_\beta}(\sigma'_{\alpha\beta}) + F_\alpha$$



$f_i(\vec{x}, \vec{c}_i, t)$

$\rho = \sum_i f_i$

$\rho u = \sum_i \vec{c}_i f_i$