Five lectures & five sets of lecture notes

- Kinetic theory
  - Distribution functions*
  - Boltzmann equation*
  - Transport equations
- Lattice-Boltzmann (LB) method
  - Discrete space, time & velocity
  - An LB algorithm
  - Chapman-Enskog analysis*
- Practical aspects of the LB method
  - Dimensional analysis
  - Boundary conditions
  - Coding
- Forces, collision operators
- Multiphase flow
  - Free energy LBM & interfaces*
  - Volume-averaged Navier-Stokes equation

* mathematically demanding

Distribution function

mass of molecules at location **x** at moment *t* traveling with velocity ξ

$$f\left(\mathbf{x},\boldsymbol{\xi},t\right)$$

& its discrete counterpart

$f_i\left(\mathbf{x},t\right)$ with a velocity set $\mathbf{c_i}=\left(c_{ix},c_{iy},c_{iz}\right)$

integrations become summations: $\rho=\sum_i f_i \quad \rho\mathbf{u}=\sum_i \mathbf{c_i} f_i$

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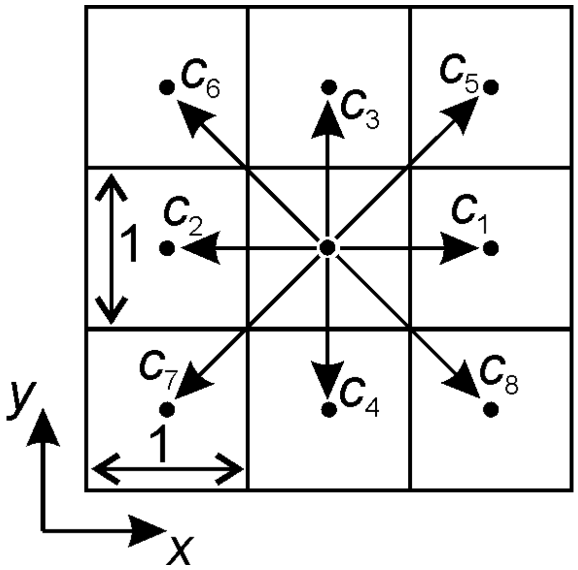


D2Q9

$$\Delta t = 1$$

$$\Delta x = 1$$

streaming: form lattice site to lattice site

collisions $$f_i^*(\mathbf{x}, t) = f_i(\mathbf{x}, t) + \Omega(\mathbf{x}, t)$$

post-collision pre-collision collision operator

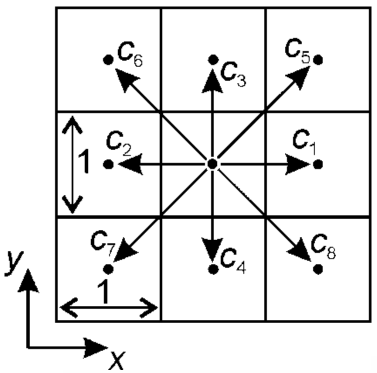
streaming $$f_i(\mathbf{x} + \mathbf{c_i}, t + 1) = f_i^*(\mathbf{x}, t)$$

Collisions: BGK

$$\Omega_i(\mathbf{x},t)=\Omega_i(f)=-\frac{1}{\tau}\left(f_i-f_i^{eq}\right)$$

need a discrete version of the equilibrium distribution function

$$f_i^{eq}=w_i\rho\left[1+\frac{u_\alpha c_{i\alpha}}{c_s^2}+\frac{\left(u_\alpha c_{i\alpha}\right)^2}{2c_s^4}-\frac{u_\alpha u_\alpha}{2c_s^2}\right]$$



D2Q9

$$w_0=4/9\quad w_{1-4}=1/9\quad w_{5-8}=1/36\quad c_s^2=1/3$$

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Collisions: MRT

$$\Omega(\boldsymbol{f}) = -\mathbf{M}^{-1}\mathbf{S}\left[\mathbf{m}(\mathbf{x},t) - \mathbf{m}^{eq}(\mathbf{x},t)\right]$$

$$\mathbf{m} = \mathbf{M}\cdot\boldsymbol{f}$$

$$\mathbf{M} = \begin{bmatrix} m_{00} & m_{01} & \cdots & m_{08} \\ m_{10} & m_{11} & \cdots & m_{18} \\ \vdots & & \ddots & \vdots \\ m_{80} & m_{81} & \cdots & m_{88} \end{bmatrix}$$

m: velocity moments of the distribution function

M: a constant coefficient matrix

$$\mathbf{S} = \begin{bmatrix} \omega_0 & 0 & \cdots & 0 \\ 0 & \omega_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_8 \end{bmatrix}$$

S: relaxation rates (different moment have different rates)

$$\mathbf{S} = \mathrm{diag}\left(0, \omega_e, \omega_\varepsilon, 0, \omega_q, 0, \omega_q, \omega_\nu, \omega_\nu\right)$$

e.g. $\mu = \rho c_s^2 \left(\frac{1}{\omega_\nu} - \frac{1}{2}\right)$

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Forces in LBGK

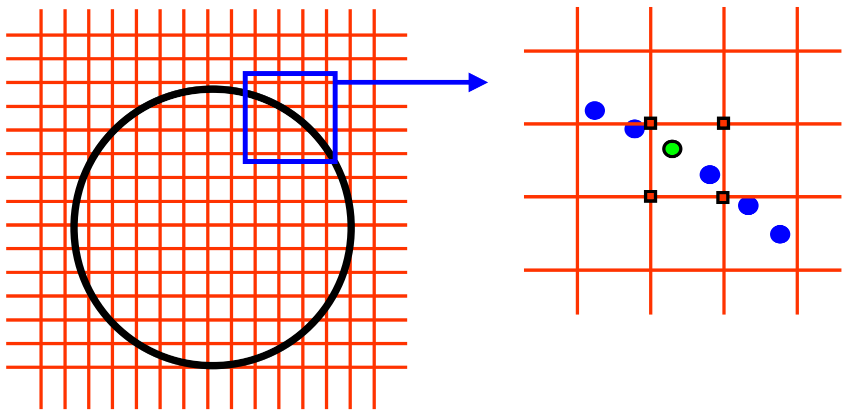
$$\rho \frac{\partial u_\alpha}{\partial t} + \rho u_\beta \frac{\partial u_\alpha}{\partial x_\beta} = -\frac{\partial p}{\partial x_\alpha} + \frac{\partial}{\partial x_\beta}\left[\mu\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right)\right] + F_\alpha$$

include a new term in the LBE equation $f_i^* = f_i + \Omega_i + S_i$

$$S_i = \left(1 - \frac{1}{2\tau}\right) w_i \left(\frac{c_{i\alpha}}{c_s^2} + \frac{\left(c_{i\alpha}c_{i\beta} - c_s^2\delta_{\alpha\beta}\right)u_\beta}{c_s^4}\right) F_\alpha$$

requires a “force correction” for momentum

$$\rho u_\alpha = \sum_i f_i c_\alpha + \tfrac{1}{2} F_\alpha$$



application of forces: immersed boundary method to force fluid to a desired velocity at off-grid locations

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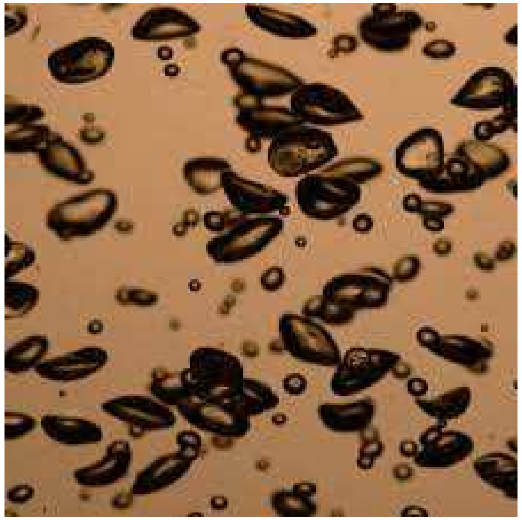
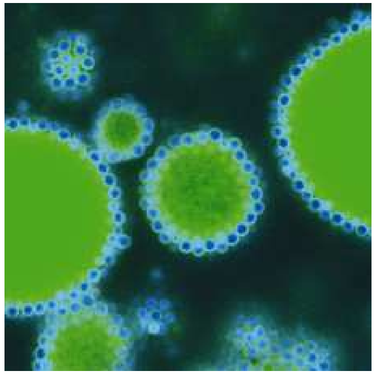
Now something new: multiphase flow

blood

sediment transport



bubbly flow



(Pickering) emulsion

freeboard of a fluidized bed

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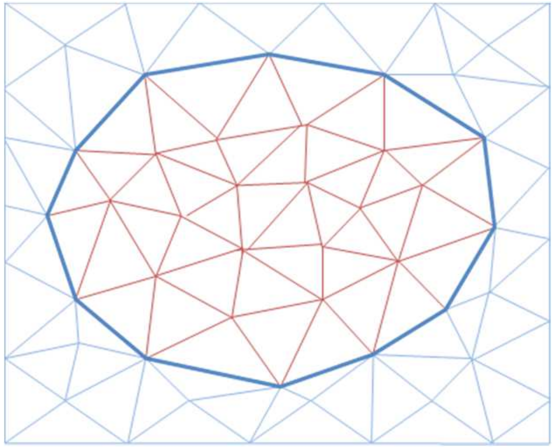
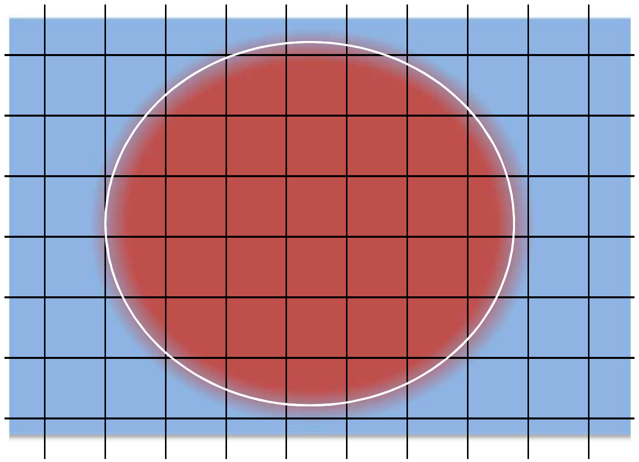
Dealing with interfaces (in general)



explicit interface tracking

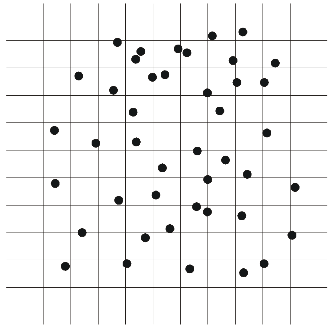
Explicit and implicit interface tracking methods

- topological changes are difficult with explicit methods
- fine grid required for implicit methods



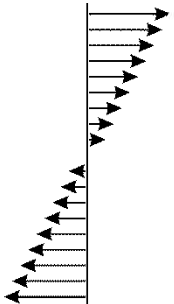
implicit interface tracking

This lecture

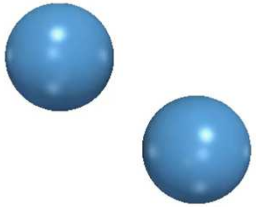


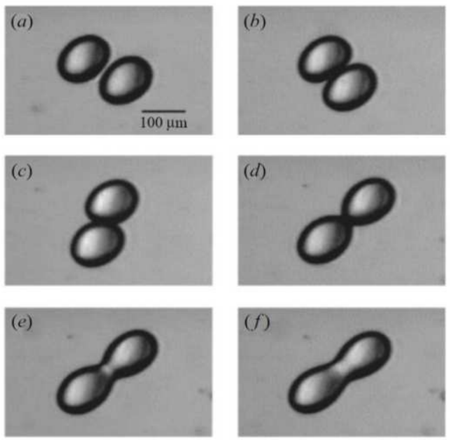
Two applications involving LBM

- liquid-liquid systems
- solid-liquid systems with ***unresolved*** particles



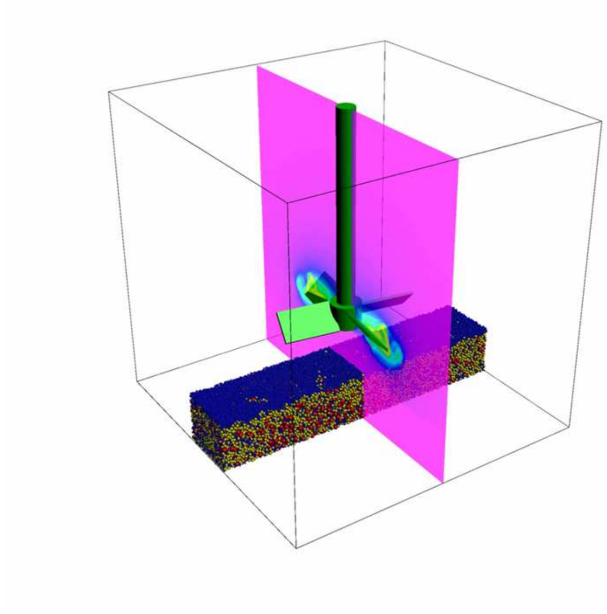
$G = \partial u_x / \partial y$





coalescence in shear flow

solid particles (of various sizes) getting suspended by stirring



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