Five lectures & five sets of lecture notes

- Kinetic theory
  - Distribution functions*
  - Boltzmann equation*
  - Transport equations
- Lattice-Boltzmann (LB) method
  - Discrete space, time & velocity
  - An LB algorithm
  - Chapman-Enskog analysis*
- Practical aspects of the LB method
  - Dimensional analysis
  - Boundary conditions
  - Coding
- Forces, collision operators
- Multiphase flow
  - Free energy LBM & interfaces*
  - Volume-averaged Navier-Stokes equation

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* mathematically demanding

Distribution function

mass of molecules at location **x** at moment *t* traveling with velocity ξ

$$f(\mathbf{x}, \boldsymbol{\xi}, t)$$

& its discrete counterpart

$f_i(\mathbf{x}, t)$ with a velocity set $\mathbf{c_i} = (c_{ix}, c_{iy}, c_{iz})$

integrations become summations: $\rho = \sum_i f_i \quad \rho\mathbf{u} = \sum_i \mathbf{c_i} f_i$

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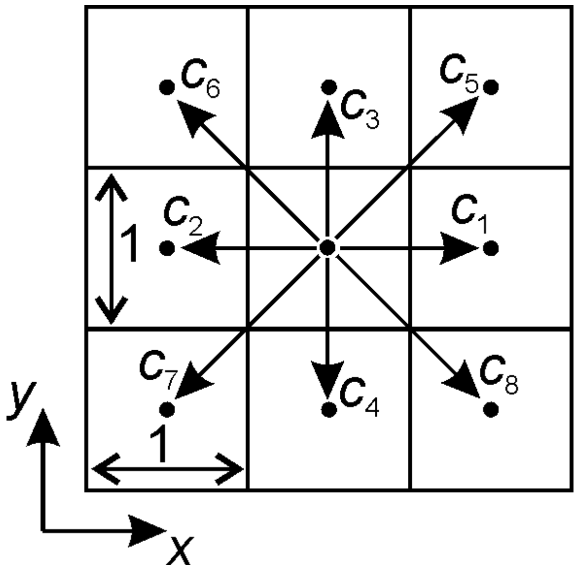

D2Q9

$$\Delta t = 1$$

$$\Delta x = 1$$

streaming: form lattice site to lattice site

collisions $$f_i^*(\mathbf{x},t) = f_i(\mathbf{x},t) + \Omega(\mathbf{x},t)$$

post-collision pre-collision collision operator

streaming & collisions $$f_i(\mathbf{x}+\mathbf{c_i},t+1) = f_i(\mathbf{x},t) + \Omega(\mathbf{x},t)$$

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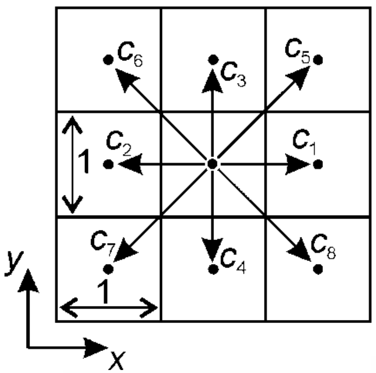
BGK

$$\Omega_i(\mathbf{x},t) = \Omega_i(f) = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right)$$

need a discrete version of the equilibrium distribution function

$$f_i^{eq} = w_i \rho \left[1 + \frac{u_\alpha c_{i\alpha}}{c_s^2} + \frac{(u_\alpha c_{i\alpha})^2}{2c_s^4} - \frac{u_\alpha u_\alpha}{2c_s^2}\right]$$



D2Q9

$$w_0 = 4/9 \quad w_{1-4} = 1/9 \quad w_{5-8} = 1/36 \quad c_s^2 = 1/3$$

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LBE to “Navier-Stokes”

$$f_i(\mathbf{x}+\mathbf{c_i},t+1) = f_i(\mathbf{x},t) - \frac{1}{\tau}\left(f_i(\mathbf{x},t) - f_i^{eq}(\mathbf{x},t)\right)$$

Chapman-Enskog analysis

$$\frac{\partial}{\partial t}(\rho u_\beta) + \frac{\partial}{\partial x_\alpha}(\rho u_\alpha u_\beta) = -\frac{\partial p}{\partial x_\beta} + \nu \frac{\partial}{\partial x_\alpha}\left(\rho\left[\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right]\right)$$

with $p = c_s^2 \rho \qquad \nu = c_s^2\left(\tau - \tfrac{1}{2}\right)$

if ρ were constant, this would be incompressible Navier-Stokes

….but ρ is not constant

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(in)compressibility

$$\rho \approx \text{constant if Ma} = |\mathbf{u}|/c_s \ll 1$$

keep flow velocities *in lattice units* well below speed of sound in *lattice units*