Quasidiagonality of nuclear C^*-algebras

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Joint work with Stuart White and Wilhelm Winter

Quasidiagonality

Definition

A (separable) C^*-algebra A is *quasidiagonal* if there exists a sequence of c.p.c. maps $\phi_n : A \to M_{k_n}$ that are:
(i) approximately multiplicative ($\|\phi_n(a)\phi_n(b) - \phi_n(ab)\| \to 0$, for $a, b \in A$); and
(ii) approximately isometric ($\|\phi_n(a)\| \to \|a\|$, for $a \in A$).

Let $\mathcal{Q}$ be the universal UHF algebra, $\mathcal{Q} := \bigotimes_{n\in\mathbb{N}} M_n$.

Let $\mathcal{Q}_\omega$ be the ultrapower of $\mathcal{Q}$ with respect to a free ultrafilter ω.

Fact: A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

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Quasidiagonality: obstructions

A is quasidiagonal if there exists an injective *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $A \to \prod \mathcal{Q}$.

Quasidiagonal C^*-algebras are stably finite.

Proposition (Rosenberg)

Let G be a discrete group. If $C_r^*(G)$ is quasidiagonal, then G must be amenable (equivalently, $C_r^*(G)$ must be nuclear).

Question (Blackadar–Kirchberg)

Is every nuclear, stably finite C^*-algebra quasidiagonal?

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A trace τ on a C^*-algebra A is *quasidiagonal* if there exists a *-homomorphism $\phi : A \to \mathcal{Q}_\omega$ with a c.p.c. lift $(\phi_n)_n : A \to \prod \mathcal{Q}$ such that

$$\tau(a) = \tau_{\mathcal{Q}_\omega} \circ \phi(a) \ (= \lim_\omega \tau_{\mathcal{Q}} \circ \phi_n(a)), \quad a \in A.$$

Proposition

If A is unital and quasidiagonal then A has a quasidiagonal trace.

If A has a faithful quasidiagonal trace then A is quasidiagonal.

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Quasidiagonal traces and covering dimension

Decomposition rank (Kirchberg–Winter) is a marriage of Lebesgue's covering dimension, Lance's completely positive approximation property, and quasidiagonality.

Definition

A has decomposition rank at most d if:

$$\begin{tikzcd} A \arrow[rr, "\mathrm{id}"] \arrow[dr, "\psi"'] & & A \\ & F^{(0)} \oplus \cdots \oplus F^{(d)} \arrow[ur, "\phi"'] & \end{tikzcd}$$

point-$\|\cdot\|$ approximately commuting, ϕ and ψ are both c.p.c., and $\phi|_{F^{(i)}}$ is orthogonality-preserving (order zero).

E.g., $\mathrm{dr}(C(X)) = \dim(X)$; $\mathrm{dr}(A) = 0$ iff A is AF.

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Can arrange that ψ witnesses quasidiagonality of A (i.e., ψ is approximately multiplicative and approximately isometric); even that ψ witnesses quasidiagonality of traces of A. That is, $\mathrm{dr}(A) < \infty$ implies that A **is quasidiagonal and all traces on A are quasidiagonal.**

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Finite decomposition rank is a powerful property. Characterizing this property is desirable, particularly for Elliott algebras:

Definition

An *Elliott algebra* is a C^*-algebra that is simple, separable, nuclear, unital, and infinite dimensional.

Conjecture (Toms–Winter)

Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
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Let A be an Elliott algebra. Then:
(i) A has finite decomposition rank if and only if A is finite and $\mathcal{Z}$-stable;
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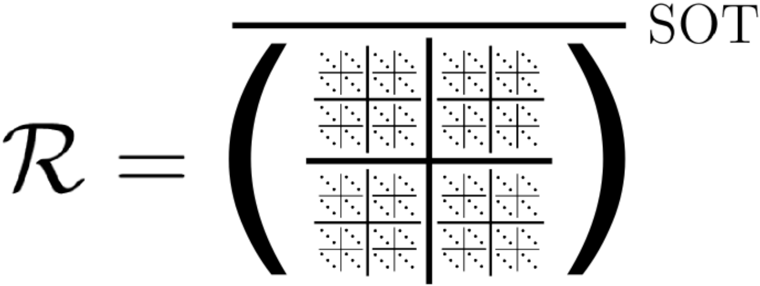
$\mathcal{Z}$ is a C^*-algebraic analogue of $\mathcal{R}$.



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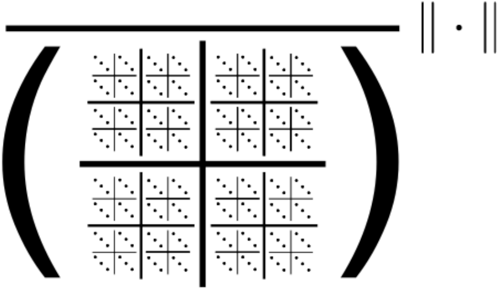

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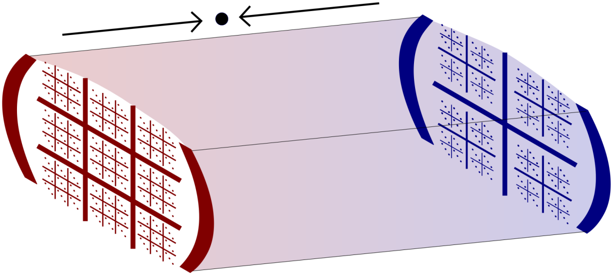
$\mathcal{Z} =$ 

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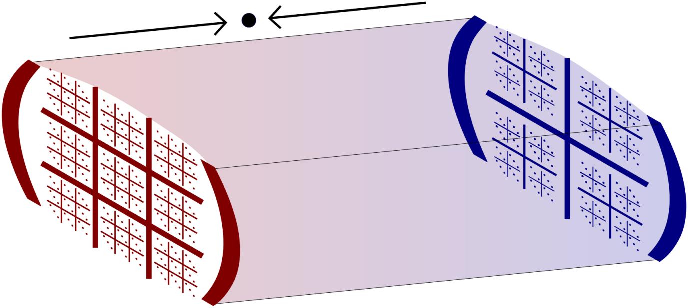
$\mathcal{Z} =$ 

$\mathcal{Z}$ is an Elliott algebra with unique trace and no nontrivial projections, which satisfies $K_*(\mathcal{Z}) \cong K_*(\mathbb{C})$.

Quasidiagonal traces and covering dimension

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Classification of C^*-algebras

The goal of C^*-algebra classification is to show that nuclear C^*-algebras that agree on K-theoretic invariants are isomorphic.

The K-theoretic invariants are ordered K-theory, traces, and the pairing between these:

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Main result

Theorem (T–White–Winter)

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Hence A is quasidiagonal.

Corollary (with Rosenberg, Tu)

A group G is amenable if and only if $C^*_r(G)$ is quasidiagonal.

Proof. The canonical trace given by $\tau(\lambda_g) = \delta_{g,e}$ is faithful.

(Indeed, the left regular representation is the GNS representation with respect to this trace.)

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